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Quadratic Polynomials: Completing the Square.

If guessing does not work, "completing the square" will do the job.

A first example.

Let us try to factor $x^2 + 2x - 7$. We will actually consider the equivalent problem of finding the roots, the solutions of the equation

$$x^2 + 2x - 7 = 0.$$

Move the constant term to the other side of the equation:

$$x^2 + 2x = 7.$$

The magic trick of this method is to exploit the binomial formula:

$$x^2 + 2bx + b^2 = (x + b)^2.$$

If we look at the left side of the equation we want to solve, we see that it matches the first two terms of the binomial formula if b=1. Let's write down the binomial formula for b=1:

$$x^2 + 2x + 1 = (x + 1)^2.$$

But the third term of the binomial formula does not show up in our equation; we make it show up by force by adding 1 to both sides of our equation:

$$x^2 + 2x + 1 = 7 + 1.$$

This trick is called "completing the square"! Now we use the binomial formula to simplify the left side of our equation (also adding 7+1=8):

$$(x+1)^2 = 8.$$

Next we take square roots of both sides, but be careful: there are **two** possible cases:

$$x+1 = \sqrt{8}, \text{ or } x+1 = -\sqrt{8}.$$

In both cases $(x+1)^2 = 8$. We are done, once we solve the two equations for *x*.

$$x = -1+\sqrt{8}, \text{ or } x = -1-\sqrt{8}$$

are the two roots of our polynomial. Consequently, our polynomial factors as follows:

$$\begin{aligned} x^2+2x-7 &= \left(x-(-1+\sqrt{8})\right)\cdot\left(x-(-1-\sqrt{8})\right) \\ &= (x+1-2\sqrt{2})\cdot(x+1+2\sqrt{2}) \end{aligned}$$

Another example.

Let us try to factor $2x^2+6x+3$. We will again consider the equivalent problem of finding the roots, the solutions of the equation

$$2x^2+6x+3 = 0.$$

In this example, the leading coefficient (the number in front of the x^2) is 2, and thus not equal to 1; we can fix that by dividing by 2:

$$x^2+3x+\frac{3}{2} = 0.$$

Move the constant term to the other side of the equation:

$$x^2+3x = -\frac{3}{2}.$$

The magic trick of this method is to exploit the binomial formula:

$$x^2+2bx+b^2 = (x+b)^2.$$

If we look at the left side of the equation we want to solve, we see that it matches the first two terms of the binomial formula if *b*=3/2. Let's write down the binomial formula for *b*=3/2:

$$x^2+2\cdot\frac{3}{2}x+\left(\frac{3}{2}\right)^2 = x^2+3x+\frac{9}{4} = \left(x+\frac{3}{2}\right)^2.$$

But the third term of the binomial formula does not show up in our equation; we make it show up by adding $\frac{9}{4}$ to both sides of our equation:

$$x^2+3x+\frac{9}{4} = -\frac{3}{2}+\frac{9}{4}.$$

We have "completed the square"! Now we use the binomial formula to simplify the left side of our equation (also simplifying the right side):

$$\left(x+\frac{3}{2}\right)^2=\frac{3}{4}.$$

The rest is easy: we take square roots of both sides, but be careful: there are **two** possible cases:

$$x+\frac{3}{2}=\sqrt{\frac{3}{4}},\ \text{or}\ x+\frac{3}{2}=-\sqrt{\frac{3}{4}}.$$

In both cases $\left(x+\frac{3}{2}\right)^2=\frac{3}{4}$. We are done, once we solve the two equations for *x*.

$$x=-\frac{3}{2}+\sqrt{\frac{3}{4}},\ \text{or}\ x=-\frac{3}{2}-\sqrt{\frac{3}{4}}$$

are the two roots of our polynomial. Here is the factorization of our polynomial. Be careful: We have to multiply our two "standard" factors by 2 (the leading term we divided by in the beginning!)

$$\begin{aligned}2x^2+6x+3 &= 2\left(x-\left(-\frac{3}{2}+\sqrt{\frac{3}{4}}\right)\right)\cdot\left(x-\left(-\frac{3}{2}-\sqrt{\frac{3}{4}}\right)\right)\\ &= 2\left(x+\frac{3}{2}-\frac{\sqrt{3}}{2}\right)\cdot\left(x+\frac{3}{2}+\frac{\sqrt{3}}{2}\right)\end{aligned}$$

A last example "for the road".

Let us factor x^2-6x+3. We will again consider the equivalent problem of finding the roots, the solutions of the equation

$$x^2-6x+3=0.$$

Move the constant term to the other side of the equation:

$$x^2-6x=-3.$$

The magic trick of this method is to exploit the binomial formula:

$$x^2+2bx+b^2=(x+b)^2.$$

If we look at the left side of the equation we want to solve, we see that it matches the first two terms of the binomial formula if *b*=-3. **You always take half of the term in front of the *x*.** Let's write down the binomial formula for *b*=-3:

$$x^2+2\cdot(-3)x+9=x^2-6x+9=(x-3)^2.$$

But the third term of the binomial formula does not show up in our equation; we make it show up by adding 9 to both sides of our equation:

$$x^2-6x+9=-3+9.$$

We have "completed the square"! Now we use the binomial formula to simplify the left side of our equation (also simplifying the right side):

$$(x-3)^2=6.$$

The rest is easy: we take square roots of both sides, but be careful: there are **two** possible cases:

$$x - 3 = \sqrt{6}, \text{ or } x - 3 = -\sqrt{6}.$$

In both cases $(x - 3)^2 = 6$. We are done, once we solve the two equations for x.

$$x = 3 + \sqrt{6}, \text{ or } x = 3 - \sqrt{6}$$

are the two roots of our polynomial. Here is the factorization of our polynomial.

$$\begin{aligned} x^2 - 6x + 3 &= (x - (3 + \sqrt{6})) \cdot (x - (3 - \sqrt{6})) \\ &= (x - 3 - \sqrt{6}) \cdot (x - 3 + \sqrt{6}) \end{aligned}$$

Exercise 1.

Find the roots of the polynomial $x^2 - 10x + 5$ by completing the square.

Answer.

Exercise 2.

Factor the polynomial $2x^2 - 5x - 8$ by completing the square.

Answer.

There are more exercises on the next page dealing with the quadratic formula.

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