Measuring Distance with Sound & Light

(28)MDSL43a.tex

March 15, 2006

1 Principles

- The strain gauged cantilever and torsion bar are used to measure small distances and angles.
- Large distance is often measured using the "time of flight" (ToF) of an acoustic (sound) or electromagnetic (*e.g.*, light) wave from the measuring station by waiting for the bounced "echo" to return.
- This time interval, from wave emission to return, is recorded as proportional to twice the distance from the station to the object from which the wave bounced.
- Acoustic ranging is moderately accurate for distances from 1m to 20m. In rooms, multiple reflections cause problems. You can buy an inexpensive unit from, say, Can. Tire to sketch-map rooms in a house, *e.g.*
- Speed of sound is about 343.7_5m/s at about 21C while light travels in vacuum at about 3×10^8m/s,
- Because of the wide range of available frequency in the electromagnetic spectrum, all the way from relatively long wavelength λ, low frequency f provided by VHF radio waves of $\lambda = 1\text{m} \equiv f = 300\text{MHz}$ to visible light $\lambda = 5500\text{nm} \equiv f = 5.45 \times 10^{13}\text{Hz}$, range-finding with EMR is used to find distances from a few metres all the way up to bouncing a radar signal off the moon.
- If one desires a resolution of 1mm with this sort of optical rangefinder our "clock" would have to resolve $\frac{1}{3} \times 10^{-11}$s or run at 300GHz; about 100 times faster than today's state-of-the-art computer oscillator.

The speed of sound in a gas (air) is given by

$$v = \sqrt{\gamma R T}\text{m/s}$$

and depends on

- γ, the ratio of specific heat at constant pressure to that at constant volume, *i.e.*, for air
$$\gamma = \frac{c_p}{c_v} = 1.4$$
- R, the perfect gas constant for air, about $287 \frac{\text{m}^2}{\text{Ks}^2}$
- T, the absolute temperature in Kelvin. At room temperature of 21C this is about 273+21=294K. In this case $v = 344$m/s.

2 A Little Problem

A 500Hz acoustic transducer measures distance to a wall 10m away. How many pulses would be counted by the time the echo returns and stops this 500Hz "clock" that started when the sound pulse was emitted? If, instead, a 100kHz crystal oscillator were used as a time base, how many "ticks" would be recorded during this 20m sound pulse transit? Take a look at the enclosed illustration to see how to build such an oscillator.

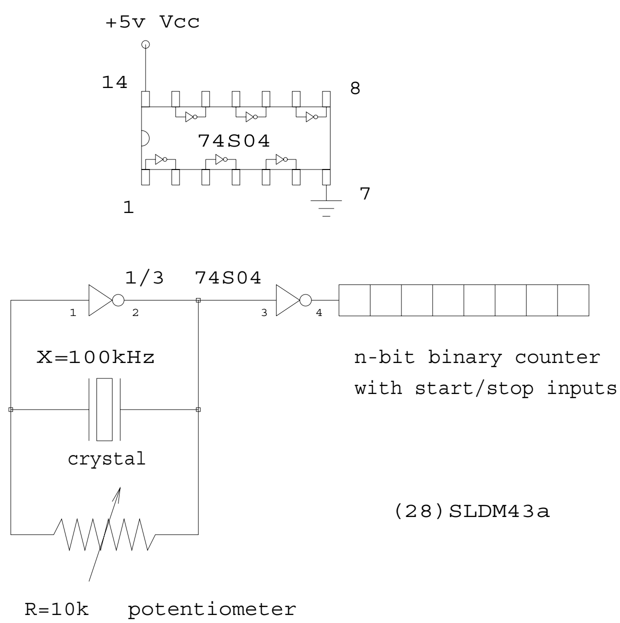

Figure 1: A Simple Crystal Oscillator

3 GPS

Global **P**ositioning **S**ystem measurements may be modelled as the intersection of three spheres -four, to unequivocally resolve up/down ambiguity- that represent ToF distances transmitted from three -or four- satellites to the receiving station that sorts out the geometric problem and reports the result as either a latitude/longitude coordinate set or maybe a spot on a map display stored in the unit. The steps to solve the geometric problem are summarized belw

- Write three sphere equations.

$$e_i: \ (x-x_{si})^2+(y-y_{si})^2+(z-z_{si})^2-r_i^2=0, \ \ i=1,2,3$$

- Create two plane equations with, say,

$$e_4: \ e_1-e_2=0, \ \ e_5: \ e_2-e_3=0$$

- Create two points on a line with, say,

$$A: \ e_4 \cap e_5 \cap x=0, \ \ B: \ e_4 \cap e_5 \cap y=0$$

- With, say, sphere $i=1$, e_1, find P, the position of the receiving unit.
- Resolve ambiguity by intersecting with a fourth sphere

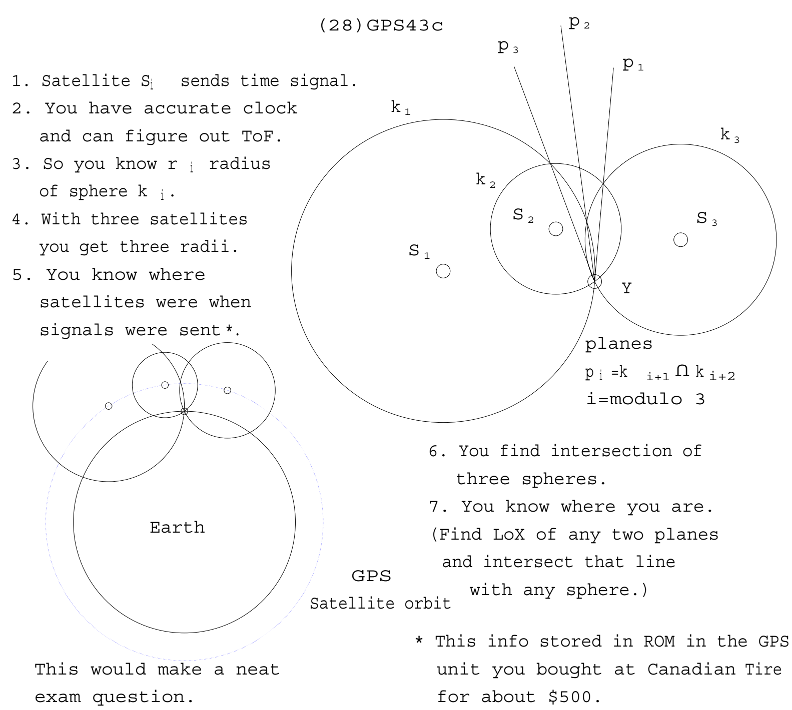

Figure 2: Three **GPS** Spheres