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Fully-Isotropic Parallel Manipulators with Schönflies Motions and Complex Legs with Rhombus Loops

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***Abstract* - This paper presents a new family of fully-isotropic parallel manipulators with Schönflies motions and complex legs containing up to *m* rhombus loops. The moving platform of a parallel manipulator with Schönflies motions (PMSM) has four degrees of freedom, which are three independent translations and one rotation about an axis of fixed direction. A one-to-one correspondence exists between the actuated joint velocity space and the external velocity space of the moving platform. The Jacobian matrix mapping the two vector spaces of fully-isotropic PMSMs presented in this paper is the identity 4×4 matrix throughout the entire workspace. The condition number and the determinant of the Jacobian matrix being equal to one, the manipulator performs very well with regard to force and motion transmission capabilities. As far as we are aware the family of *$14m^4$* solutions of fully-isotropic PMSMs introduced in this paper is presented for the first time in the literature. These solutions are derived from a family of *$7m^4$* PMSMs with decoupled motions and complex legs with rhombus loops also presented for the first time.**

Index Terms - fully-isotropic, parallel manipulators, rhombus loops, Schönflies motions.

I. INTRODUCTION

Parallel manipulators (PMs) with Schönflies motions are the parallel counterparts of the well-known SCARA robots. The end-effector of these robots has four degrees of freedom, which are three independent translations (T3) and one rotation (R1) about an axis of fixed direction. This motion T3R1-type was study by the German mathematician Arthur Moritz Schönflies and is usually called Schönflies motion [1]. Several types of parallel robots with four degrees of mobility T3R1-type have been proposed and investigated [2]-[19]. The most known PMs T3R1-type are H4 [6]-[7], [16], Manta and Kanuk [8], 4-*URU* and 4-*UPU* [14]. We note that all these solutions have coupled motions. The following joints are used in these solutions: revolute (*R*), prismatic (*P*), universal joint (*U*), cylindrical (*C*) as well as the parallelogram loop (*Pa*) which can be considered as a complex pair of circular translation [14], [20]-[22].The first solutions of fully-isotropic parallel manipulators with Schönflies motions (PMSMs) have been very recently proposed [23]-[25]. Fully-isotropic PMSMs have a very simple command and achieve important energy-saving due to the fact that for a unidirectional motion only one motor works and the other are locked. For parallel manipulators, the velocities of the moving platform are usually related to the velocities of the actuated joints [$\dot{q}$] by the general equation:

$$ {}^{p}\begin{bmatrix} v \\ \omega \end{bmatrix}_{H} = [J][\dot{q}] \quad (1)$$

where : $[v]=[v_x\ v_y\ v_z]^T$ is the velocity of a point *H* belonging to the moving platform, $[\omega]=[\omega_x\ \omega_y\ \omega_z]^T$ - angular velocity of the moving platform, [*J*] – Jacobian matrix and *p* is the coordinate system in which the velocities of the moving platform with respect to the fixed platform are expressed.

Isotropy of a robotic manipulator is related to condition number of its Jacobian matrix, which can be calculated as the ratio of the largest and the smallest singular values. A robotic manipulator is fully-isotropic if its Jacobian matrix is isotropic throughout the entire workspace, i.e., the condition number of the Jacobian matrix is one. The condition number of the Jacobian matrix is an interesting performance index characterizing the distortion of a unit hypersphere under the linear mapping (1). Condition number of the Jacobian matrix was first used to design mechanical fingers [26] and developed in [27] as a kinetostatic performance index of robotic mechanical systems. The isotropic design aims at ideal kinematic and dynamic performance of the manipulator [28].

Four types of PMs are distinguished in [29] by taking into consideration the form of the Jacobian matrix: (i) fully-isotropic PMs, if the Jacobian *J* is an diagonal matrix with identical diagonal elements throughout the entire workspace, (ii) PMs with uncoupled motions if *J* is a diagonal matrix with different diagonal elements, (iii) PMs with decoupled motions, if *J* is a triangular matrix and (iv) PMs with coupled motions if *J* is neither triangular nor diagonal matrix. Fully-isotropic PMs realize a one-to-one mapping between the actuated joint velocity space and the external velocity space. The condition number and the determinant of the Jacobian matrix being equal to one, the manipulator performs very well with regard to force and motion transmission.

The fully-isotropic PMSMs proposed in [23]-[25] use elementary legs to connect the mobile platform to the actuators situated on the fixed base. The reduced rigidity of the elementary legs represents the main drawback of these solutions. To overcome these disadvantages, PMSMs with complex legs containing up to *m* rhombus loops are proposed in this paper. We recall that an elementary leg consists of a serial kinematic chain and a complex leg integrates at least a closed loop. The closed loops integrated in leg architecture contribute to increasing leg rigidity and implicitly the robot precision. Due to space limitations, we reduced our presentation in this paper to decoupled, uncoupled and fully-isotropic overconstrained PMSMs without idle mobilities and complex legs containing rhombus loops. In the standard terminology the rhombus loop, is a planar closed kinematic chain with four revolute pairs connected by four links of the same length. No fixed link exists in the rhombus loop (*Rb*). One or more rhombus loops can be concatenated in a complex leg. With respect to elementary legs the use of complex legs reduces leg torsion loading.

II. PMs with Decoupled Schönflies Motions and Complex Legs Containing Rhombus Loops

In this paper we consider parallel manipulators with Schönflies motions (PMSMs) T3R1-type enabling three independent translations along x, y and z-axes and one independent rotation about y-axis. Rotation about x or z-axis could also be considered. The basic kinematic structure of a PMSMs is obtained by concatenating four legs $A_i(1\equiv 0\text{-}\ldots\text{-}n_{Ai}\equiv n)$, $i=1,2,3,4$ that can be identical or different. The first link (1) of each leg is the fixed platform (0) and the final link is the moving platform ($n_{Ai}\equiv n$). The first joint of each leg A_i ($i=1,\ldots,4$) is actuated. We denote by q_i and $\dot{q}_i$ ($i=1,2,3,4$) the finite displacements and the velocities in the actuated joints and by v_x, v_y, v_z and ω_y the translational and angular velocities of a point H situated on the mobile platform.

Examples of structural solutions of complex legs with 4 or 5 degrees of freedom (dof) containing one or two rhombus loops used in PMs with decoupled Schönflies motions are presented in Figs. 1-2. Table I presents the joint arrangement in each leg A_i ($i=1,\ldots,4$). The notations $\perp$ and $\|$ in Figs. 1-2 and Table 1 indicate the perpendicular or parallel positions of the joint axes. The indexes x, y, z, c, and d associated with the joint symbol denotes the direction of the joint axis. Two consecutive joints with the same index have parallel axes. Two consecutive joints with different indexes have perpendicular axes. The actuated joint of each leg A_i ($i=1,\ldots,4$) is underlined.

By various associations of the four legs A_i presented in Table I we could obtain m^4 basic structural types of PMs with decoupled Schönflies motions without idle mobilities and complex legs containing rhombus loops. Figures 3 and 4 present structural solutions with one and two rhombus loops in each leg. To simplify the notations of the elements e_{Ai} ($i=1,2,3,4$ and $e=1,2,\ldots,n$) by avoiding the double index in Fig. 3 and the following figures we have denoted by e_A the elements belonging to the leg A_1 ($e_A\equiv e_{A1}$), by e_B the elements of the leg A_2 ($e_B\equiv e_{A2}$), by e_C the elements of A_3 ($e_C\equiv e_{A3}$) and by e_D the elements of A_4 ($e_D\equiv e_{A4}$). The axes of revolute joints connecting the legs A_1 and A_3 to the moving platform must be superposed and the reference point H must be situated on this common axis, as we can see in the examples presented in Figs. 3-4. The axis of revolute joint connecting the leg A_2 to the moving platform could be: (i) superposed with the axis of the last revolute joint of the leg A_4, as in Fig. 3 and Fig. 5-a, (ii) superposed with the axis of the last revolute joints of the legs A_1 and A_3, as in Fig. 5-b or (iii) not superposed with the axis of another joint, as in Fig. 5-c. These different positions do not involve structural modifications of the basic solution of PMs with decoupled Schönflies motions. In all cases four revolute joints are adjacent to the mobile platform.

Derived structural solutions of PMs with decoupled Schönflies motions can be obtained from the basic solutions by: integrating in a common element the last elements of the legs (i) A_1 and A_2 (Fig. 6-a), (ii) A_2 and A_3 (Fig. 6-b), (iii) A_2 and A_4 (Fig. 6-c), (iv) eliminating the last revolute joint of the

TABLE I
JOINT ARRANGEMENT IN COMPLEX LEGS WITH RHOMBUS LOOPS

No	Type	$M_A = S^A_{nA/1}$	Joint arrangement	Base ($R^{\beta}_{n/1}$)	No. sol./ leg
1	$\underline{P}RRb\ldots RbR$	4	$\underline{P}\|R\|Rb\|\ldots\|Rb\|R$ Leg A_2: $\underline{P}_y\text{-}R_y\text{-}Rb_y\text{-}\ldots\text{-}Rb_y\text{-}R_y$	(v_x,v_y,v_z,ω_x)	m
2	$\underline{P}RRb\ldots RbRR$	5	$\underline{P}\|R\|Rb\|\ldots\|Rb\|R\perp R$ Leg A_1: $\underline{P}_x\text{-}R_x\text{-}Rb_x\text{-}\ldots\text{-}Rb_x\text{-}R_x\text{-}R_y$ Legs A_3 and A_4 $\underline{P}_z\text{-}R_z\text{-}Rb_z\text{-}\ldots\text{-}Rb_z\text{-}R_z\text{-}R_y$	$(v_x,v_y,v_z,\omega_x,\omega_y)$ $(v_x,v_y,v_z,\omega_y,\omega_z)$	m m

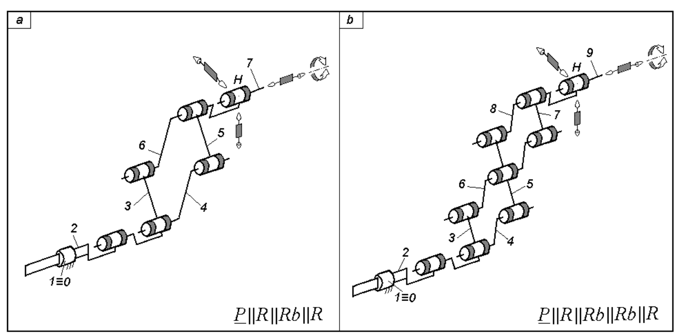

Fig. 1. Complex legs with 4 dof and one rhombus loop $\underline{P}RRbR$-type (a) and two concatenated rhombus loops $\underline{P}RRbRbR$-type (b).

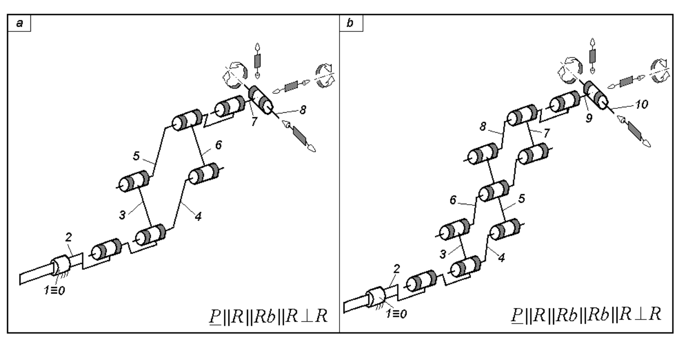

Fig. 2. Complex legs with 5 dof and one rhombus loop $\underline{P}RRbR$-type (a) and two concatenated rhombus loops $\underline{P}RRbRbR$-type (b).

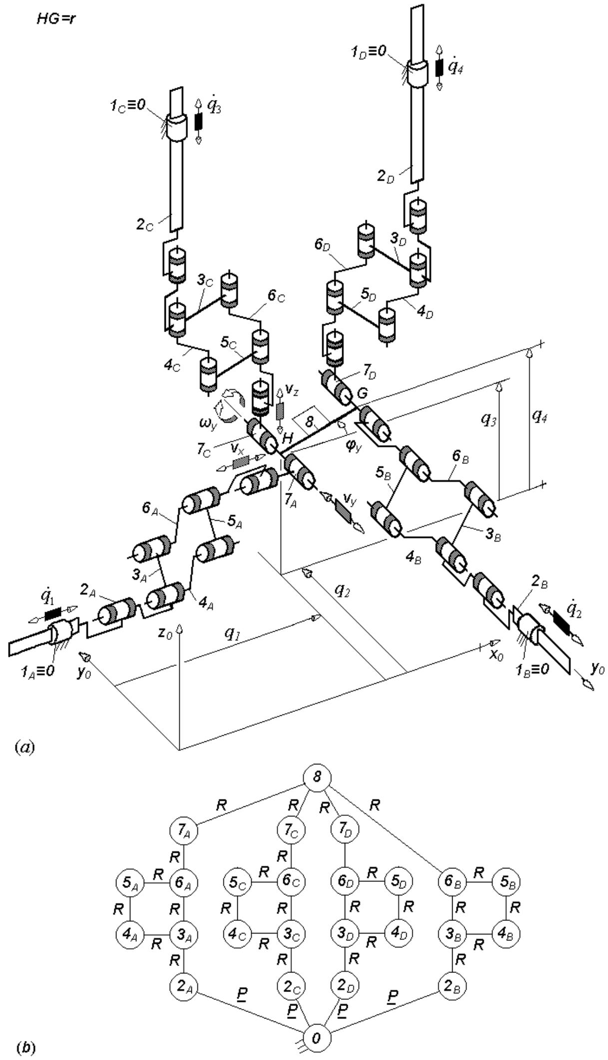

Fig. 3. Basic kinematic structure of PM with decoupled Schönflies motions containing one rhombus loop in each leg (a) and its associated graph (b).

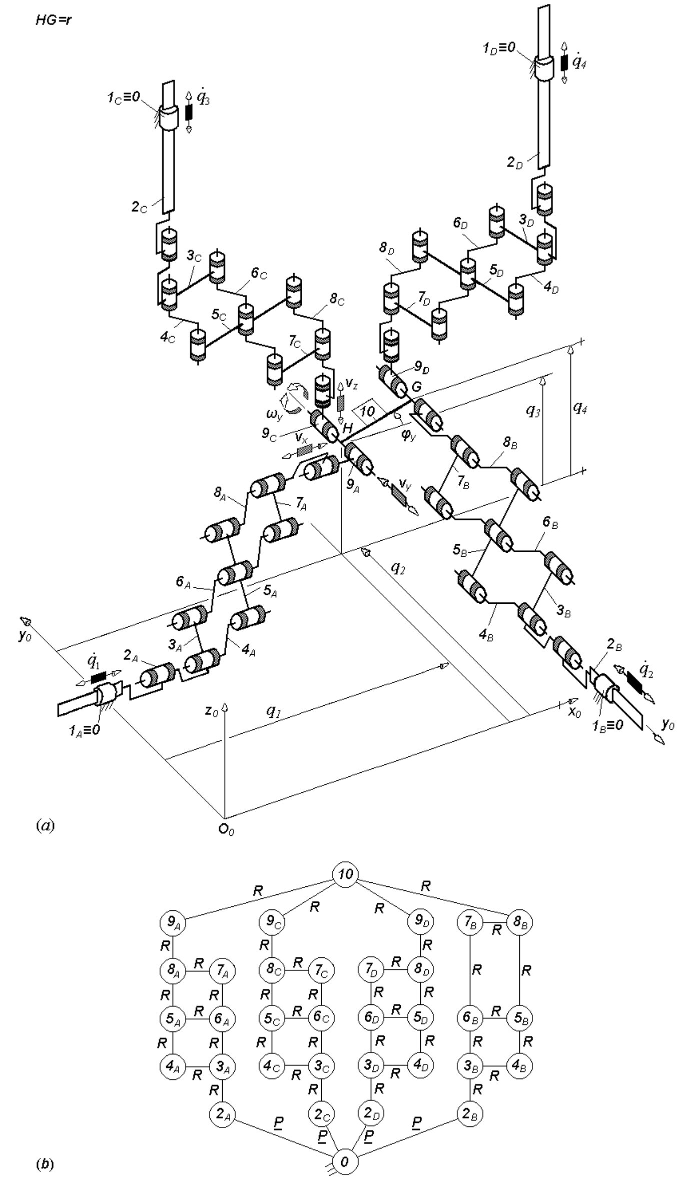

Fig. 4. Basic kinematic structure of PM with decoupled Schönflies motions containing one rhombus loop in each leg (a) and its associated graph (b).

leg A_1 and integrating in a common element the last elements of the legs A_1 and A_3 (Fig. 6-d), (v) eliminating the last revolute joint of the leg A_1 and integrating in a common element the last elements of the legs A_1 and A_3 and in another common element the last elements of the legs A_2 and A_4 (Fig. 6-e), (vi) eliminating the last revolute joint of the leg A_1 and integrating in a common element the last elements of the legs A_1, A_2 and A_3 (Fig. 6,f). Only two or three revolute joints are adjacent to the mobile platform in the derived solutions. In this way we can set up $6m^4$ structural solutions of PMs with decoupled Schönflies motions without idle mobilities and complex legs containing rhombus loops. These PMSMs are obtained by coupling various legs with the mobile platform according to the six connecting solutions presented in Fig. 6.

The $7m^4$ basic and derived solutions obtained in this way have decoupled motions and complex legs actuated by linear motors situated in the fixed base. For these solutions of PMSMs, (1) becomes

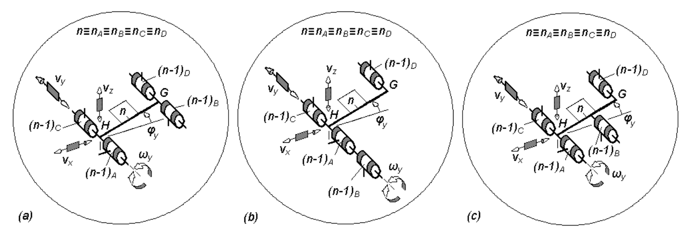

Fig. 5. Different positions of the axis of revolute joint connecting the leg A_2 to the moving platform.

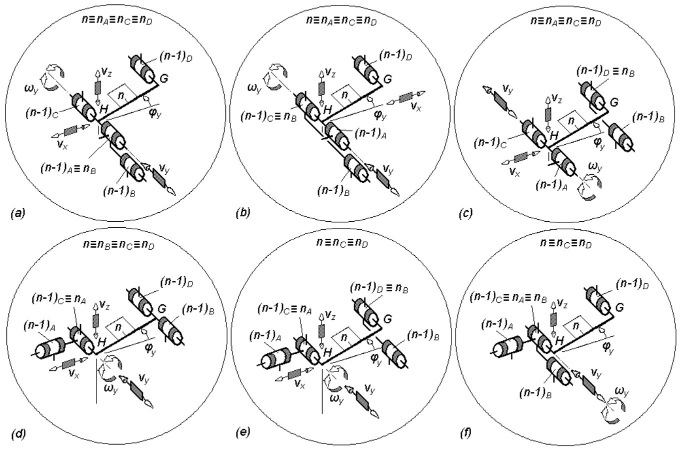

Fig. 6. Six distinct structural solution for coupling the four legs to the moving platform.

$$ {}^{p}\begin{bmatrix} v_x \\ v_y \\ v_z \\ \omega_y \end{bmatrix}_H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -a & a \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \end{bmatrix}, \quad a = \frac{1}{r\cos\varphi_y} \tag{2} $$

where $r=HG$ and φ_y are the length and the rotation angle of the moving platform (see Figs. 3 and 4). The length r is defined by the common normal on the revolute axes connecting the moving platform to the legs A_3 and A_4. We can see that v_x, v_y and v_z are uncoupled motions ($v_x = \dot{q}_1$, $v_y = \dot{q}_2$, $v_z = \dot{q}_3$) and ω_y is a coupled motion depending of $\dot{q}_3$ and $\dot{q}_4$ with $\omega_y = (\dot{q}_4 - \dot{q}_3)/(r\cos\varphi_y)$.

III. PMs with Uncoupled Schönflies Motions and Complex Legs Containing Rhombus Loops

The leg A_4 in the PMs with decoupled Schönflies motions presented in the previous section connects the fixed and the mobile platform. If we connect the leg A_4 between the first kinematic element of the leg A_3 ($1_D \equiv 2_C$) and the mobile platform we can obtain PMs with the four uncoupled Schönflies motions of the mobile platform (Fig. 7). In this way, m^4 basic structural solutions and $6m^4$ derived solutions of PMs with uncoupled Schönflies motions can be set up from the solutions presented in the previous section. For these PMSMs the linear mapping (1) becomes:

$$ {}^{p}\begin{bmatrix} v_x \\ v_y \\ v_z \\ \omega_y \end{bmatrix}_H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & a \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \end{bmatrix}, \quad a = \frac{1}{r\cos\varphi_y} \quad . \tag{3} $$

In these solutions the fourth actuator is not situated on the fixed base.

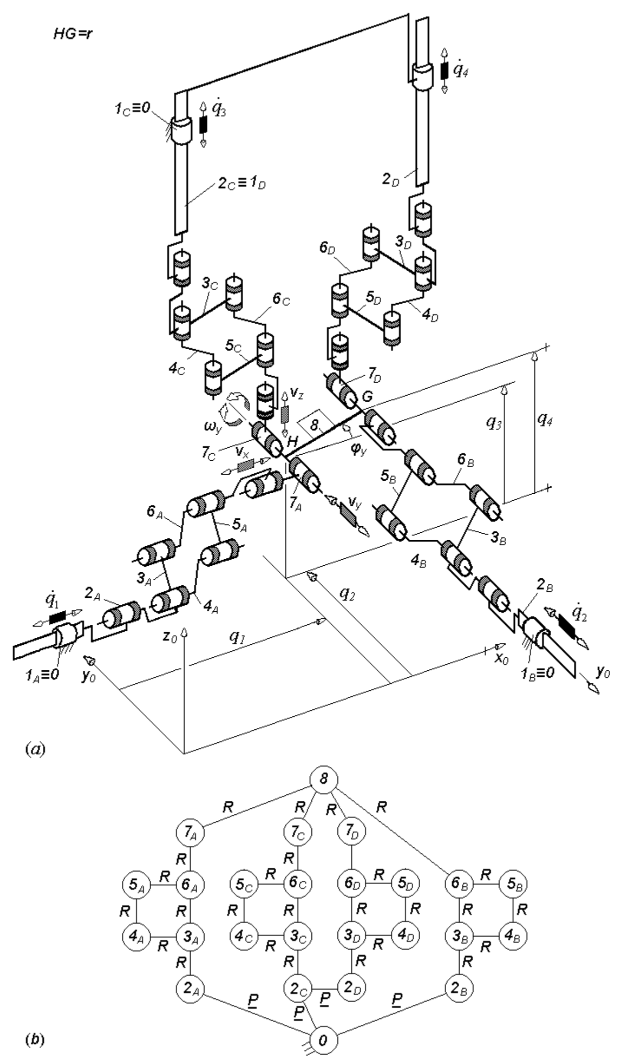

Fig. 7. Example of basic kinematic structure of PM with uncoupled Schönflies motions actuated by four linear motors (a) and its associated graph (b).

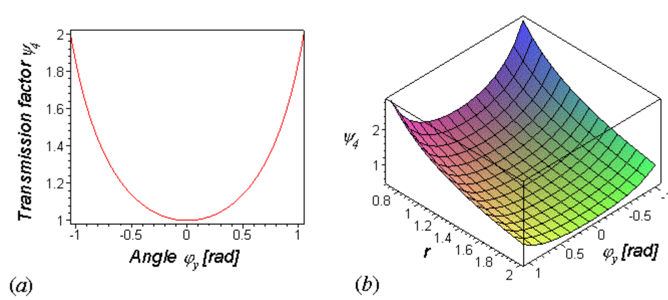

Fig. 8. Variation of the transmission factor ψ_4 with the rotation angle of the platform: for the characteristic length $L_c = r = 1$ (a) and for various values of the platform length (b)

We can see that v_x, v_y, v_z and ω_y are uncoupled motions ($v_x = \dot{q}_1$, $v_y = \dot{q}_2$, $v_z = \dot{q}_3$ and $\omega_y = \dot{q}_3/(r\cos\varphi_y)$.

To compare the singular values of the Jacobian matrix of the linear mapping (1), the elements of this matrix should have the same units. From (3), the elements of the first and the second columns of the Jacobian matrix J are non-dimensional. The third column has the unit of length^{-1}. The characteristic length of the manipulator, i.e., L_c, is used to homogenize the elements of the Jacobian matrix so that the condition number is non-dimensional. The characteristic length renders the Jacobian dimensionally homogeneous and optimally conditioned, i.e., with a minimum condition number [27]. The isotropic configuration of the PMs with uncoupled Schönflies motions given by the isotropy condition of the Jacobian matrix of linear mapping (3) is obtained when $\varphi_y = 0$ and $L_c=r=1$. In this isotropic configuration the Jacobian becomes the identity 4×4 matrix and (3) maps the joint rates belonging to a unit hypersphere into operational velocities belonging to another unit hypersphere. In any other robot configuration for joint rates belonging to a unit hypersphere are mapped into the operational velocities of the moving platform belong to a 4-dimensional ellipsoid. The eigenvectors of the matrix $(JJ^T)^{-1}$ define the direction of the principal axes of this ellipsoid. The square roots ξ_1, ξ_2, ξ_3 and ξ_4 of the eigenvalues of $(JJ^T)^{-1}$ are the lengths of the aforementioned principal axes. The velocity transmission factors in the directions of the principal axes are defined by $\psi_1 = 1/\xi_1$, $\psi_2 = 1/\xi_2$, $\psi_3 = 1/\xi_3$ and $\psi_4 = 1/\xi_4$. These transmission factors can be used to define the joint limits [30]. The PMs with uncoupled Schönflies motions have $\psi_1 = 1$, $\psi_2 = 1$, $\psi_3 = 1$ and $\psi_4 = 1/(r\cos\varphi_y)$. Fig. 8 presents the variation of the transmission factor ψ_4 with the rotation angle of the moving platform. In Fig. 8-a we considered that the platform length is equal to the characteristic length $r=L_c=1$. In Fig. 8-b various values of platform length are considered. For $1 \le r \le 2$ and $\varphi_y \in [-60°, 60°]$ the transmission factor is $0.4 \le \psi_4 \le 2$.

IV. Fully-Isotropic PMSMs

Fully-isotropic PMSMs can be set up from the PMs with uncoupled Schönflies motions, presented in the previous section, by replacing the actuated prismatic joint in leg A_4 by a kinematic chain with two revolute joints (Fig. 9). These two revolute joints have parallel axes situated in a plane perpendicular to the other revolute joints of the leg A_4 i.e., the plane xy. The first revolute joint is actuated and q_4 represents its rotation angle. In these $7m^4$ basic and derived fully-isotropic solutions the fourth actuator is not on the fixed base. Other $7m^4$ solutions of fully-isotropic PMSMs with the four actuators mounted on the fixed base can be set up from the previous fully-isotropic solutions by replacing the kinematic chain of two revolute parallel joints in A_4-leg by an extensible double parallelogram De Roberval scale-type (Fig. 10). We can see that in this case the actuated revolute joint in the fourth leg can also be adjacent to the fixed base. The Jacobian matrix of the linear maping (1) for the $14m^4$ basic and derived fully-isotropic PMSMs is the identity 4×4 matrix throughout the entire workspace. A one-to-one correspondence exists between the actuated joint velocity space and the external

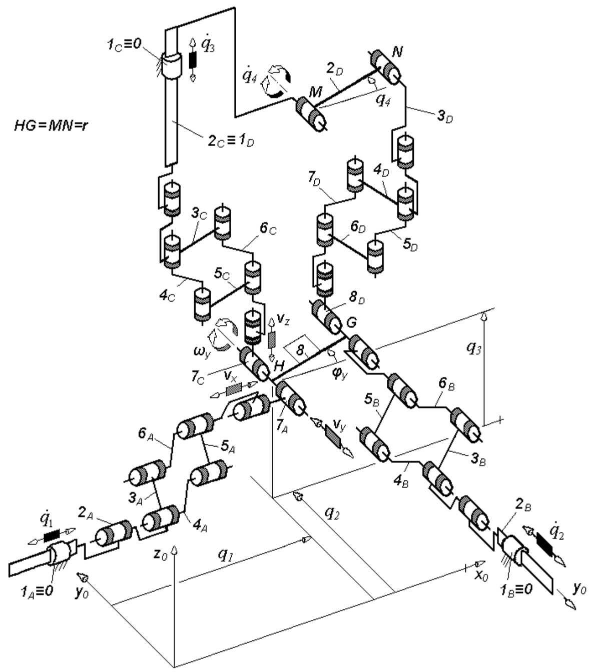

Fig. 9. Example of basic kinematic structure of fully-isotropic PMSM actuated by one rotative and three linear motors.

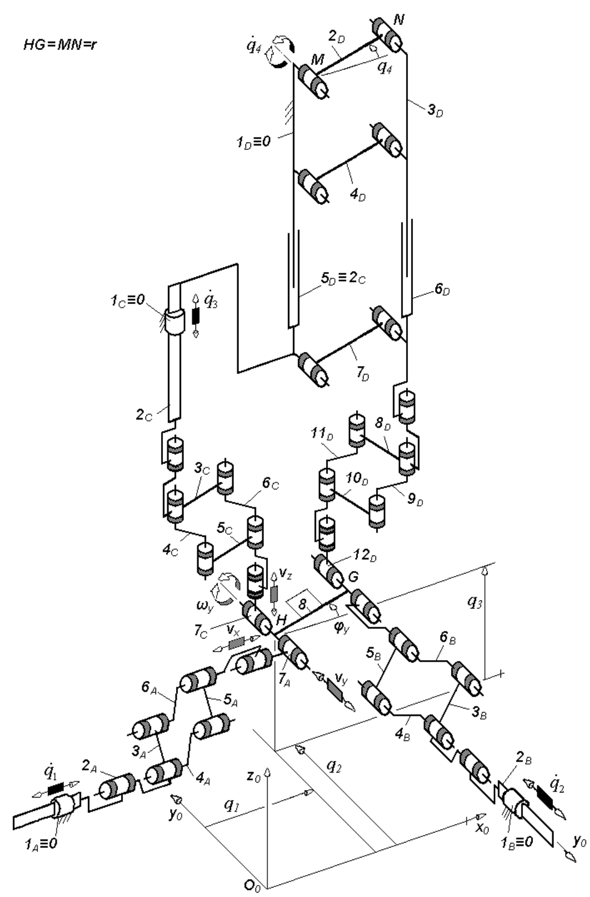

Fig. 10. Example of kinematic structure of fully-isotropic PMSMs with the four actuators mounted in the fixed base.

velocity space of the moving platform ($v_x = \dot{q}_1$, $v_y = \dot{q}_2$, $v_z = \dot{q}_3$ and $\omega_y = \dot{q}_4$).

V. Conclusions

An approach has been proposed for structural synthesis of a new family of over-constrained fully-isotropic parallel manipulators with Schönflies motions and complex legs containing up to m rhombus loops. This family includes $14m^4$ structural solutions that are set up from a family of $7m^4$ parallel manipulators with decoupled Schönflies motions. An intermediary family of $7m^4$ solutions of parallel manipulators with uncoupled Schönflies motions is also set up. The Jacobian matrix mapping the joint and the operational vector spaces of the fully-isotropic parallel manipulators presented in this paper is the identity 4×4 matrix throughout the entire workspace. These solutions realize a one-to-one mapping between the actuated joint velocity space and the operational velocity space. The condition number and the determinant of the Jacobian matrix being equal to one, the manipulator performs very well with regard to force and motion transmission. Moreover, the $7m^4$ solutions of fully-isotropic parallel manipulators with Schönflies motions have the actuators mounted directly on the base. This effectively contributes to the reduction of the weight of the moving parts. The solutions presented in this paper overcome many disadvantages usually affecting parallel manipulators such as complex command and a lower dexterity due to a high motion coupling and multiplicity of singularities inside their workspace. Special legs integrating rhombus loops have been conceived to achieve fully-isotropic conditions. Examples of fully-isotropic solutions and solutions with decoupled and uncoupled motions are presented in this paper to illustrate the proposed approach. A modular prototype is under construction at the French Institute of Advanced Mechanics. As far as we are aware the solutions of parallel manipulators with Schönflies motions and complex legs containing rhombus loops set up in this paper are presented for the first time in the literature.

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