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On some applications of primitive Schönflies-motion generators

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ABSTRACT

In this paper, we focus on some examples of practical applications of the primitive Schönflies-motion generators that are named also the primitive *X*-motion generators for conciseness. We address especially the serial SCARA-type robots and the 3-dof translational parallel manipulators. Based on the limb architectures of primitive *X*-motion generators, we systematically introduce general architectures of serial or parallel manipulators for the purpose of practical applications. The brief account of various achievements of some of the *X*-motion generators that are already described in the recent literature is also proposed and their obviously singular postures are preliminarily obtained.

1. Introduction

The Schönflies-motion generators [2,11,12] have many applications [1–10]. For instance, all the kinematic pairs or parallelogram couplings can be actuated thus making up a serial 4-dof manipulator often called a SCARA robot. The motion set $\{\boldsymbol{X}(\mathbf{u})\}$ of Schönflies motions includes 3-dof translation and 1-dof rotation around any axis that is parallel to $\mathbf{u}$. In most of practical applications, the orienting vector $\mathbf{u}$ is vertical and the tasks are pick-and-place operations of small objects like chocolates, parts of a device, etc. In this tendency, a SCARA manipulator was studied at the McGill University [9,10].

In addition to the serial SCARA robots, in order to account for the increasing interest on the parallel manipulators, the sequel will focus on translational parallel manipulators (TPMs), which potentially can be mechanisms of machine-tools like milling machines. The manipulators introduced hereinafter are the serial SCARA-type robots with general architectures and the overconstrained symmetrical translational parallel manipulators with three 4-dof limbs. Several manipulators are synthesized systematically by using the limb architectures of primitive *X*-motion generators that are proposed in [2] and further addressed in [11,12]. Additionally, some already-known manipulators, such as Delta robot [1], Star robot [3,4], Orthoglide [5], MEL micro-finger [6], Tripteron TPM [8], SCARA-type McGill manipulator [9,10] etc., are verified through the parallel arrangement of primitive *X*-motion generators.

2. Serial SCARA-type Robots

The SCARA acronym stands for Selective Compliance Assembly Robot Arm. This robot was first developed at Yamanashi University in Japan for assembly tasks. It is constructed with four joints with parallel axes. It includes two vertical revolute pairs

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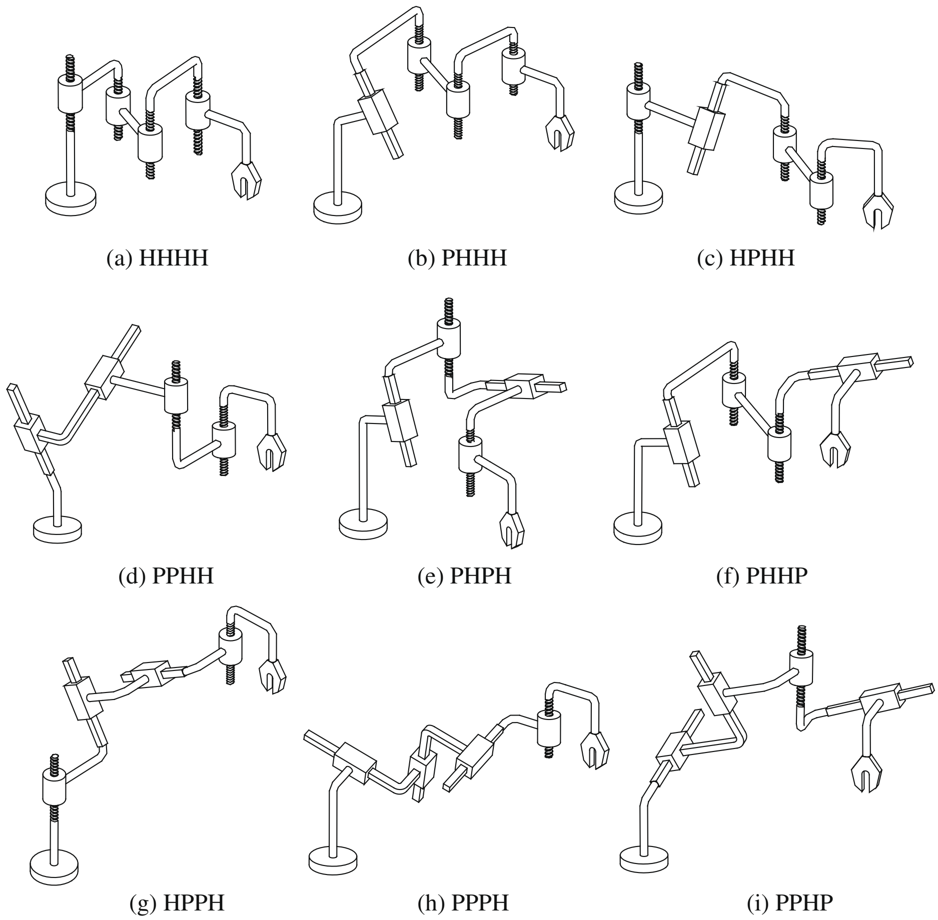


Fig. 1. General architectures of serial SCARA robots.

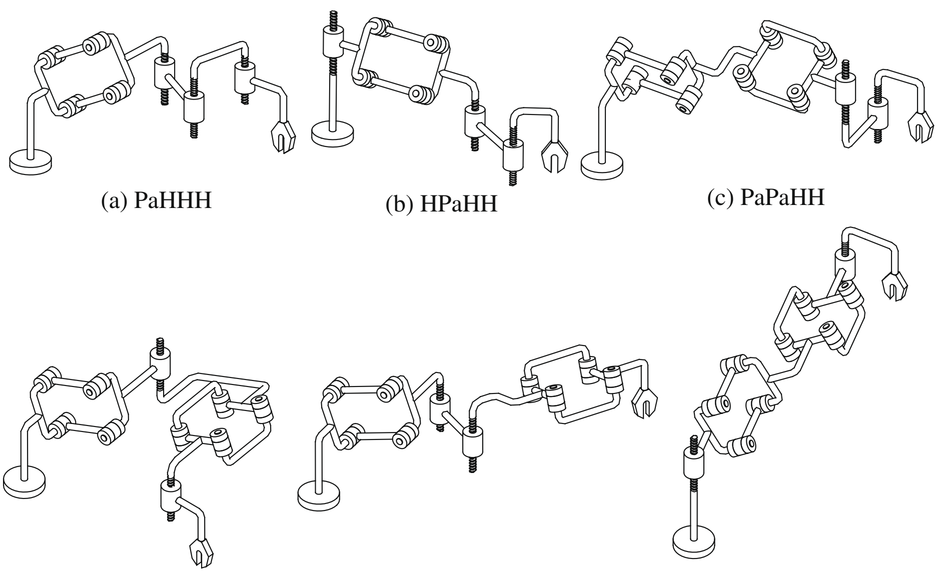


Fig. 2. Six general architectures of serial SCARA robots with Pa.

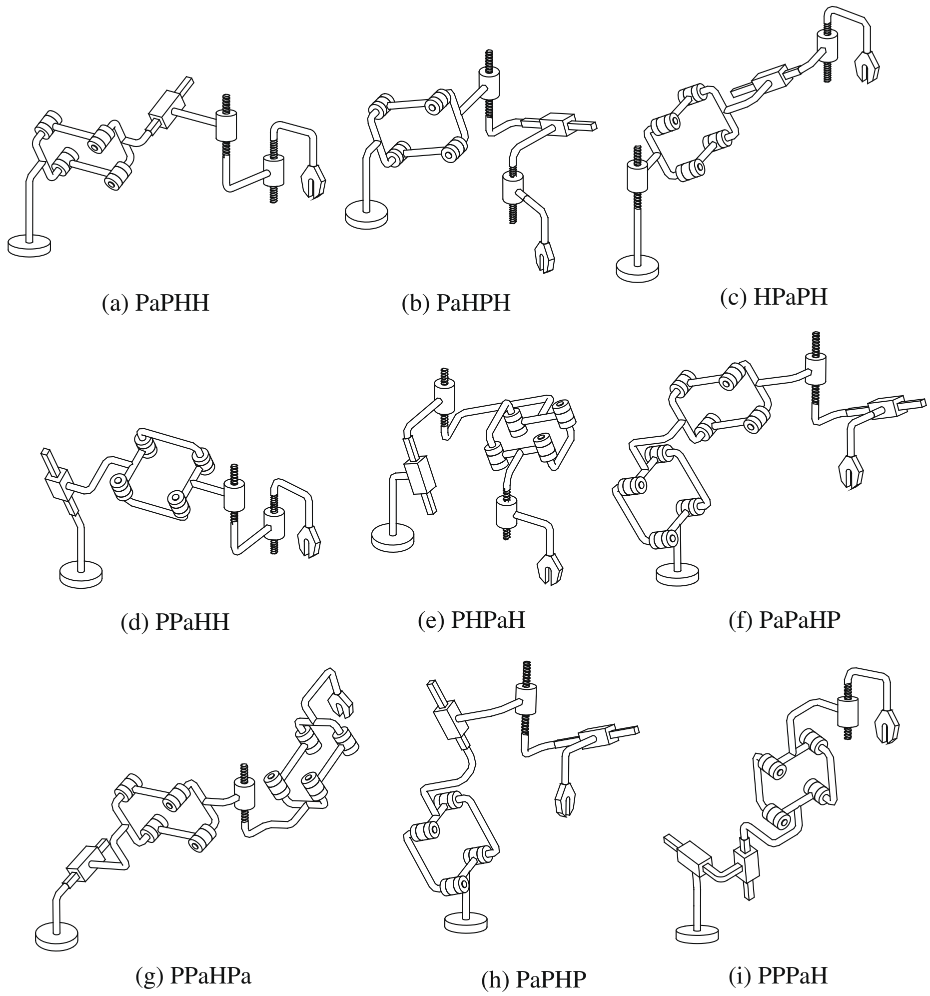


Fig. 3. Nine general architectures of serial SCARA robots with at least one Pa.

and a vertical prismatic pair. The fourth joint is also a vertical revolute pair that controls the rotation in the wrist. Now, several SCARA robots are commercially available, such as the Adept One robot, the IBM 7545 robot, the Intelledex 440 robot, and the Rhino SCARA robot. The SCARA-type serial robot designed at McGill University includes two hinged parallelograms. These special versions of 4-dof serial robots implement one among the primitive Schönflies-motion generators. However, more architectures of serial SCARA robot can be synthesized by using all the primitive *X*-motion generators [12]. These robots are obtained by fixing the first link and attaching a gripper (or an end-effector) to the last link. We show the general forms of the novel SCARA-type robots in Fig. 1. When R pairs replace one, two or three of H pairs, there are forty-three general types of serial SCARA-type robot. Reversing the order of joints in any of these architectures produces also an SCARA-type robot.

Furthermore, implementing the *X* motion generators with hinged parallelograms, we can design more SCARA-type robots with at least one parallelogram. Fig. 2 shows six robots derived from them by replacing all prismatic pairs P by hinged parallelograms Pa. Nine typical robots with hinged parallelograms in Fig. 3 have at least one prismatic pair. Once more, the kinematic inversions of these robots are also SCARA-type serial manipulators. The locution of ''kinematic inversion'' designates a mechanical operation on a kinematic chain, which means the change of fixed body into a moving end-body and vice versa. Based on the above findings, the original SCARA robot is a special geometric choice of the inversion of Fig. 1b with three Rs (R-joints) instead of three Hs (H-joints); the P is chosen parallel to the R axes for sake of simple motion control. Furthermore, McGill SCARA-type serial robot is a special configuration of the inversion of Fig. 2c with two Rs in place of Hs.

3. Translational parallel manipulators

The Schönflies-motion subgroup plays a key role in the comprehensive enumeration of overconstrained parallel manipulators producing 3-dof translation of the moving platform through the parallel setting of two or three 4-dof limbs. As a matter of fact, the limbs generate Schönflies-motions [2,11,12]. Based on the architectures of all the primitive mechanical generators of *X*-motion, we will systematically synthesize symmetrical TPMs. The parallel layout of three *X*-motion generators with at least two distinct orienting vectors, between a fixed base and a moving platform, leads to a particular kind of TPMs.

The set of feasible displacements between the base and the moving platform is the intersection of the limb bonds. In order to achieve the 3-DOF translation of the moving platform in a fully parallel manner [13], we implement in parallel three limbs that generate three distinct *X* motions. For simplicity, the rotation axis in each *X*-motion is parallel to one of three unit vectors of an orthonormal vector base (**u**, **v**, **w**). Then, the three limbs generate three kinematic bonds $\{\boldsymbol{X}(\mathbf{u})\}, \{\boldsymbol{X}(\mathbf{v})\}$ and $\{\boldsymbol{X}(\mathbf{w})\}$. The platform can undergo motions that are represented by $\{\boldsymbol{X}(\mathbf{u})\} \cap \{\boldsymbol{X}(\mathbf{v})\} \cap \{\boldsymbol{X}(\mathbf{w})\}$. Each of the *X*-bonds can be equated to a product of a set of spatial translations and a set of rotations:

$$\{\boldsymbol{X}(\mathbf{u})\} = \{\boldsymbol{T}\}\{\boldsymbol{R}(A, \mathbf{u})\}, \quad \forall A \tag{1}$$

$$\{\boldsymbol{X}(\mathbf{v})\} = \{\boldsymbol{T}\}\{\boldsymbol{R}(B, \mathbf{v})\}, \quad \forall B \tag{2}$$

$$\{\boldsymbol{X}(\mathbf{w})\} = \{\boldsymbol{T}\}\{\boldsymbol{R}(C, \mathbf{w})\}, \quad \forall C \tag{3}$$

These three equalities are valid for any point A, B, or C. Hence, we can choose $A = B = C = N, N$ being any given point, and then we will have

$$\{\boldsymbol{X}(\mathbf{u})\} \cap \{\boldsymbol{X}(\mathbf{v})\} \cap \{\boldsymbol{X}(\mathbf{w})\} = \{\boldsymbol{T}\}[\{\boldsymbol{R}(N, \mathbf{u})\} \cap \{\boldsymbol{R}(N, \mathbf{v})\} \cap \{\boldsymbol{R}(N, \mathbf{w})\}] \tag{4}$$

$\{\boldsymbol{R}(N, \mathbf{u})\} \cap \{\boldsymbol{R}(N, \mathbf{v})\} \cap \{\boldsymbol{R}(N, \mathbf{w})\}$ represents the motion of a body that is connected to a fixed base by three limbs, each limb having only one R pair. All the R axes intersect at *N* and the whole mechanism is a 3-R parallel spherical chain, which is not movable. As a matter of fact, the parallel system 3-R can be regarded as a parallel 3-dof wrist of structural type 3-RRR with six locked R pairs. Hence we have

$$\{\boldsymbol{R}(N, \mathbf{u})\} \cap \{\boldsymbol{R}(N, \mathbf{v})\} \cap \{\boldsymbol{R}(N, \mathbf{w})\} = \{\boldsymbol{E}\} \tag{5}$$

and, therefore,

$$\{\boldsymbol{X}(\mathbf{u})\} \cap \{\boldsymbol{X}(\mathbf{v})\} \cap \{\boldsymbol{X}(\mathbf{w})\} = \{\boldsymbol{T}\}\{\boldsymbol{E}\} = \{\boldsymbol{T}\} \tag{6}$$

The motion set of the moving platform is the 3D group $\{\boldsymbol{T}\}$ of spatial translations.

All the combinations delineated in the enumeration of primitive *X*-motion generators [12] can be used as limbs in the construction of TPMs. In this way, avoiding the presence of inactive or idle pairs, all possible architectures of symmetrical TPMs can be synthesized. Some representative architectures are graphically displayed in Fig. 4, in which H pairs can be arbitrarily replaced by R pair but avoiding the 3-RRRR architecture. The limbs including three P (or Pa) are of minor interest because in such limbs the H or R pair plays no role in the production of translation and, therefore, remains inactive. The kinematic inversion of these tripods, are also valid TPMs. For brevity, they are omitted in the figures hereinafter.

Fig. 5 graphically displays eight typical TPMs with hinged parallelograms. In these combinations, an R pair can replace any H pair. In addition, the kinematic inversion brings forth more valid TPMs.

4. Already described parallel manipulators

Various achievements of some of the primitive generators of *X*-motion are already published in the recent literature [1–10]. In this section, they are verified via the primitive *X*-motion generators that are enumerated in [11,12].

4.1. Delta robot

Probably, the well-known Delta robot [1] is historically the first TPM. This noteworthy robot was successfully used in the industry. A limb RRPaR of this Delta robot [1] in Fig. 6a is derived by kinematic inversion of the *X*-motion generator HPaHH, which appears in Fig. 2b or is an *II*16 generator of Fig. 6 in [12], and by choosing H pairs with a zero pitch. One can notice that a Delta limb can reach singular poses (or configurations). The singularity of the flattened parallelogram, Fig. 6b should be avoided and the infinitesimal singularity may happen when the three R axes are in the same plane as shown in Fig. 6c.

4.2. Star and H robots

The Star robot is a TPM having three limbs as shown in Fig. 7a [3,4]. A Star limb has a $\underline{\text{RH}}$PaR architecture, which can be derived from the *X*-motion generator HPa$\underline{\text{PH}}$ (*III*12 generator) of Fig. 9 in [12] or Fig. 3c in this paper when the $\underline{\text{P}}$ is parallel to the $\underline{\text{H}}$ axis. The fixed $\underline{\text{R}}$ and the adjacent $\underline{\text{H}}$ are chosen collinear thus being equivalent to a C pair and making up a kind of jack. Ignoring the singularity of the flattened parallelogram, the limb can become singular when, in the $\underline{\text{RH}}$-R sub-chain, the $\underline{\text{R}}$ axis coincides with the R axis. Moreover, the coaxial $\underline{\text{R}}$ and $\underline{\text{H}}$ are also equivalent to an array $\underline{\text{RP}}$ with a $\underline{\text{P}}$ that is parallel to the $\underline{\text{R}}$ axis. Hence, the singularity of two parallel P can occur when the translation provided by the Pa is parallel to the fixed $\underline{\text{R}}$ axis, Fig. 7b.

The H robot implements two geometric arrangements of PRPaR limbs, Fig. 8a and b. Both belong to the general category PHPaH of *X*-motion generators that is depicted as being an *III*8 generator of Fig. 10 in [12] or a serial SCARA robot in Fig. 3e. In addition, Fig. 8c shows a possible singular posture.

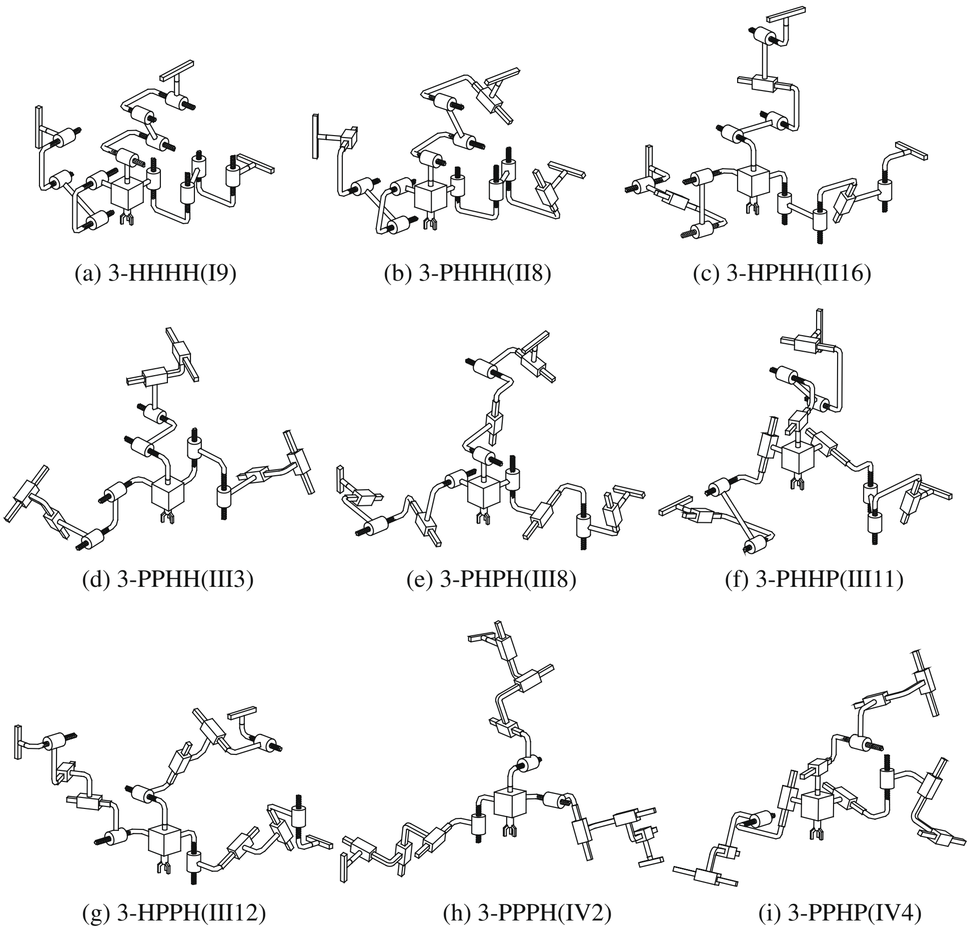


Fig. 4. Representative TPMs with primitive *X*-motion generators.

4.3. Orthoglide

A PRPaR limb of the Orthoglide TPM [5], which is shown in Fig. 9a, is a special geometric choice in the general category PHPaH of primitive *X*-motion generators [12]. Ignoring the singularity of the flattened parallelogram, two limb singularities can happen: two R become coaxial and the translation provided by Pa becomes parallel to the fixed P. Actually, such a situation corresponds also to the parallelogram flattening. Another limb singularity can happen when the sub-chain PR-R is a singular generator of planar motion; then the P is perpendicular to the plane of the R axes, as shown in Fig. 9b.

4.4. MEL micro-finger

A TPM used as a micro-finger with flexure R joints [6] was designed at the Mechanical Engineering Laboratory (MEL) of Tsukuba. The MEL micro-finger has the limb structure RPaPaR belonging to the general category HPaPaH (*III*12 generator) depicted in Fig. 7 of [12] or Fig. 2f. This limb kind is displayed in Fig. 10a. What is more, a patented device for a three-axis machine that prevents rotational movement [7] has the same architecture. The singular poses of the MEL limb may happen if the translations of the two Pa become locally parallel, Fig. 10b and also if the two Rs can become coaxial.

4.5. Tripteron

A 3-$\underline{C}$RR TPM [8] also named "Tripteron" whose limb architecture is shown in Fig. 11a is an improved version, namely a special TPM with Cartesian control of the platform translation, of a more general architecture prior proposed in [2]. The limbs are derived from the general category PHHH (*II*8 generator) of Fig. 3 in [12] or Fig. 1b: the P is parallel to the parallel H axes and the H pitches are zero. The limb singularity happens when the three pair axes lie in the same plane, which is displayed in Fig. 11b.

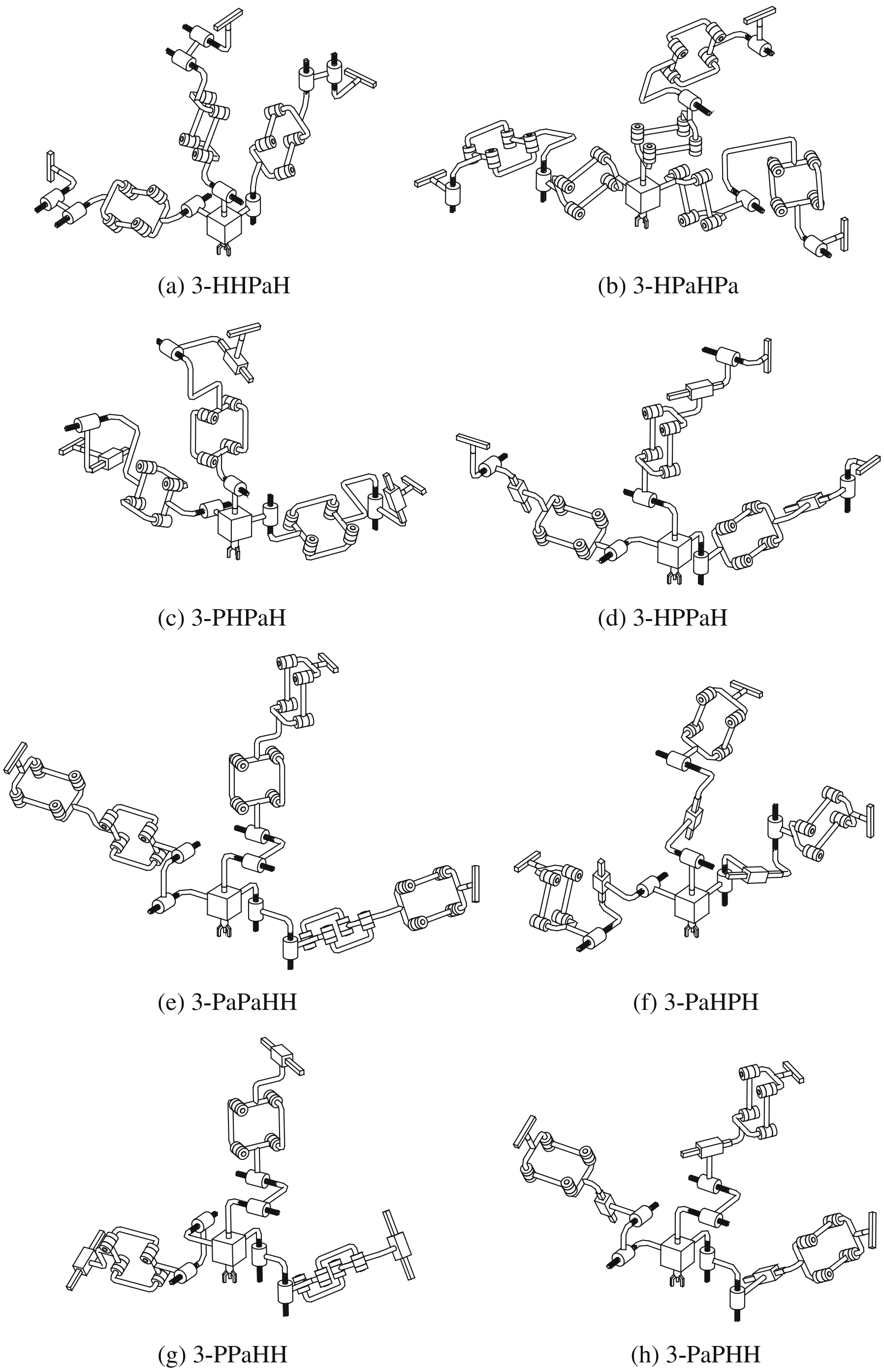


Fig. 5. General TPMs with hinged parallelograms.

4.6. Prism robot

The Prism robot [4] is a TPM implementing CPR limbs belonging to the general category PHPH (*III*8 generator) of Fig. 4 in [12] or Fig. 1e. A simple control of the platform translation is obtained by using two geometric arrangements shown in Fig. 12a and b of the CPR architecture. Limb singularity may happen if the C and the R can become coaxial.

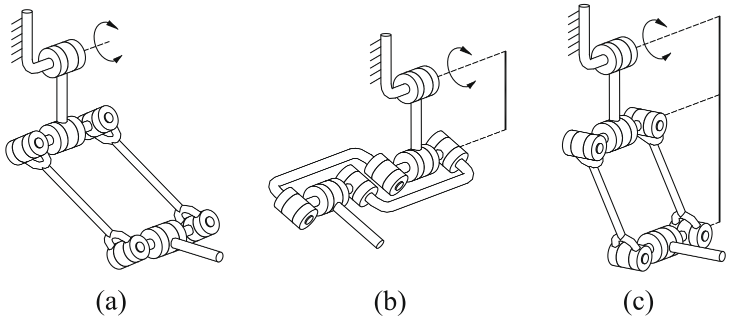


Fig. 6. Limb architecture of Delta robot.

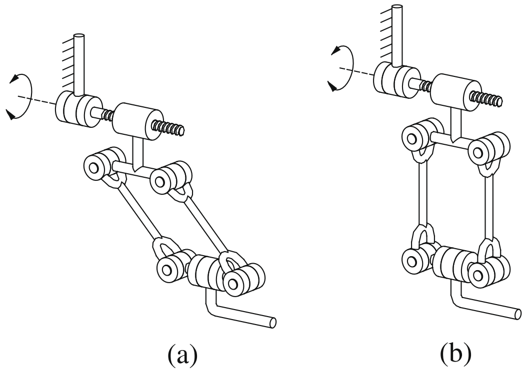


Fig. 7. Limb architecture of Star robot.

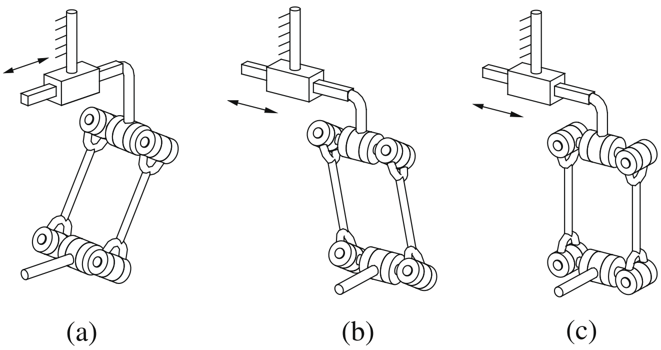


Fig. 8. Limb architectures of H-robot.

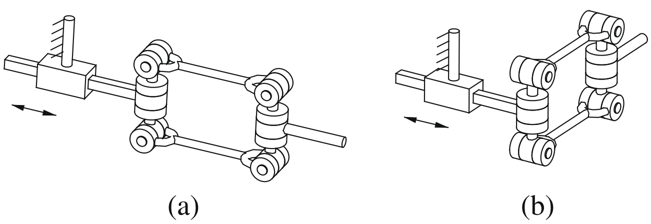
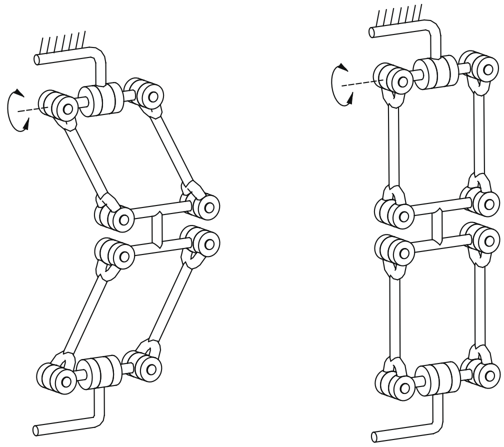


Fig. 9. Limb architecture of Orthoglide manipulator.

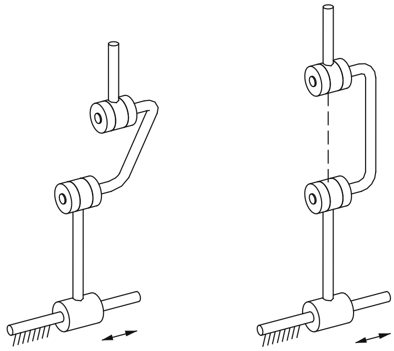
4.7. McGill robot

At McGill University, a serial generator and a parallel generator with two limbs of X motion were designed [9]. The chosen limbs RPaRPa of these "robust" parallel manipulators depicted in Fig. 13a are derived from an inversion of the chain HPaHPa



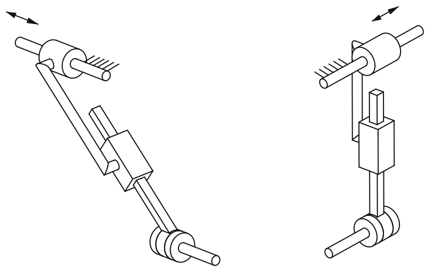
(a) (b)

Fig. 10. Limb architecture of MEL micro-finger.



(a) (b)

Fig. 11. Limb architecture of a 3-C̲RR TPM.



(a) (b)

Fig. 12. Limb architectures of Prism-robot.

with two hinged parallelograms of Fig. 2d or of Fig. 7 (*III*8 generator) in [12]. The singularity of two R pairs transitorily coaxial can happen, such as Fig. 13b. The singularity of two locally parallel translations can also happen, Fig. 13c.

Besides these obvious singular poses, more special singular poses of the generators of $\{\boldsymbol{X}(\mathbf{u})\}$ motion have to be derived through the study of a possible linear dependency of twists of joints. Any singular pose of the chain studied at McGill University can be obtained by kinematic inversion from the singular poses of PaRPaR. Each Pa produces 1-dof translation with equal circular trajectories, which is called a circular translation; when its amplitude is small, such a motion is similar to linear translation parallel to the local tangent to a circular trajectory. Hence, the singular postures of the foregoing chain can be

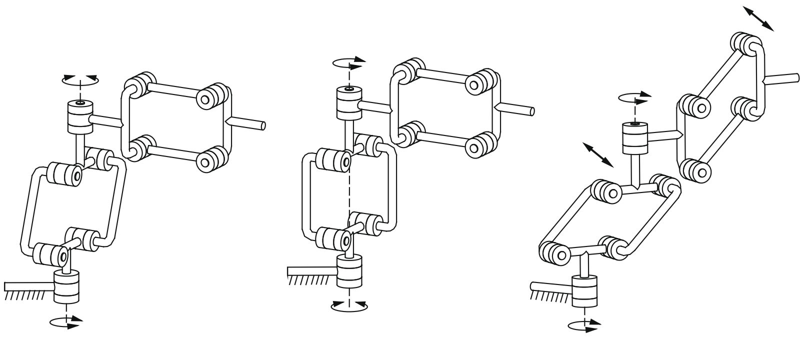


Fig. 13. Generator of X-motion at McGill university (a) obviously singular poses (b) and (c).

derived from those of a PRPR chain. Moreover, the singular PRPR chain is the special case with zero pitches of the singular PHPH chain. As explained in the pseudo-planar motion generators [14], a singularity of X-motion generators is attained if and only if the twist of the moving H is a linear combination of the three twists in the other three pairs that are P, H and P. Hence, in a general approach, the resultant twist of the sub-chain PHP is studied and this resultant twist is the twist of an H pair with an axis designated by $(N_R, \mathbf{u})$ and a pitch depending on N_R. For instance, the twist of a first P parallel to the unit vector $\mathbf{s}_1$ is expressed as $\mathbf{d}_1\mathbf{M} = \beta_1\mathbf{s}_1$. The twist of a second P is $\mathbf{d}_2\mathbf{M} = \beta_2\mathbf{s}_2$. The twist of the H with the axis $(N, \mathbf{u})$ and the pitch $p = 2\pi k$ is $\mathbf{d}_3\mathbf{M} = \alpha[\mathbf{u} \times (\mathbf{OM}) + (\mathbf{ON}) \times \mathbf{u} + k\mathbf{u}]$. For simplicity, the point N is chosen in the plane $Pl(O, \perp \mathbf{u})$ that contains the origin O and is perpendicular to $\mathbf{u}$. Then, the resultant twist of a serial array PHP (PPH or HPP) is

$$\mathbf{d}_R\mathbf{M} = \alpha[\mathbf{u} \times (\mathbf{OM}) + (\mathbf{ON}) \times \mathbf{u} + k\mathbf{u}] + \beta_1\mathbf{s}_1 + \beta_2\mathbf{s}_2 = \alpha_R[\mathbf{u} \times (\mathbf{OM}) + \mathbf{t}_R] \tag{7}$$

where $\alpha_R = \alpha$ and $\mathbf{t}_R = [(\mathbf{ON}) \times \mathbf{u} + k\mathbf{u}] + (\beta_1\mathbf{s}_1 + \beta_2\mathbf{s}_2)/\alpha$. This equation stands for the twist of a H pair with an axis parallel to $\mathbf{u}$. Its pitch $p_R(= 2\pi k_R)$ is provided by $k_R = \mathbf{u} \cdot \mathbf{t}_R = k + [\beta_1(\mathbf{u} \cdot \mathbf{s}_1) + \beta_2(\mathbf{u} \cdot \mathbf{s}_2)]/\alpha$. Its axis $(N_R, \mathbf{u})$ is determined by

$$(\mathbf{ON}_R) = \mathbf{u} \times \mathbf{t}_R = (\mathbf{ON}) + (\beta_1/\alpha)\mathbf{u} \times \mathbf{s}_1 + (\beta_2/\alpha)\mathbf{u} \times \mathbf{s}_2 \tag{8}$$

The vectors $\mathbf{u} \times \mathbf{s}_1$ and $\mathbf{u} \times \mathbf{s}_2$ are perpendicular to $\mathbf{u}$ and, therefore, are parallel to the plane $Pl(O, \perp \mathbf{u})$. Let $\mathbf{d}_1$ and $\mathbf{d}_2$ be unit vectors that are parallel to $\mathbf{u} \times \mathbf{s}_1$ and $\mathbf{u} \times \mathbf{s}_2$ respectively. For instance, $\mathbf{d}_1 = \mathbf{u} \times \mathbf{s}_1/\|\mathbf{u} \times \mathbf{s}_1\| = \mathbf{u} \times \mathbf{s}_1/|\sin(\mathbf{u}, \mathbf{s}_1)|$ and $\mathbf{d}_2 = \mathbf{u} \times \mathbf{s}_2/\|\mathbf{u} \times \mathbf{s}_2\| = \mathbf{u} \times \mathbf{s}_2/|\sin(\mathbf{u}, \mathbf{s}_2)|$. Placing the origin O at N yields

$$(\mathbf{NN}_R) = (\beta_1/\alpha)\mathbf{u} \times \mathbf{s}_1 + (\beta_2/\alpha)\mathbf{u} \times \mathbf{s}_2 \tag{9}$$

$$(\mathbf{NN}_R) = |\sin(\mathbf{u}, \mathbf{s}_1)|(\beta_1/\alpha)\mathbf{d}_1 + |\sin(\mathbf{u}, \mathbf{s}_2)|(\beta_2/\alpha)\mathbf{d}_2 \tag{10}$$

The parameters β_1/α and β_2/α can be derived from the datum of any point N_R lying in the plane $Pl(O, \perp \mathbf{u})$. In a general case, $\mathbf{d}_2 \neq \mathbf{d}_1; (N, \mathbf{d}_1, \mathbf{d}_2)$ is a reference frame for the plane $Pl(O, \perp \mathbf{u}) = Pl(N, \perp \mathbf{u})$. In this frame, the coordinates of N_R are $a_1 = |\sin(\mathbf{u}, \mathbf{s}_1)|\beta_1/\alpha$ and $a_2 = |\sin(\mathbf{u}, \mathbf{s}_2)|\beta_2/\alpha$. By using a_1 and a_2, which can be readily derived from the geometry of the chain posture, the pitch p_R of the resultant twist is expressed as follows

$$p_R = 2\pi k_R = 2\pi\{k + [a_1/|\sin(\mathbf{u}, \mathbf{s}_1)|](\mathbf{u} \cdot \mathbf{s}_1) + [a_2/|\sin(\mathbf{u}, \mathbf{s}_2)|](\mathbf{u} \cdot \mathbf{s}_2)\} \tag{11}$$

Maintaining the notations of the general case, the resultant twist of the special chain PaRPa is considered, Fig. 14. The translations generated by the two Pas replacing the two Ps of the general case are instantaneously parallel to $\mathbf{s}_1$ and $\mathbf{s}_2$, respectively. The unit vectors $\mathbf{d}_1$ and $\mathbf{d}_2$ are defined by $\mathbf{d}_1 = \mathbf{u} \times \mathbf{s}_1/\|\mathbf{u} \times \mathbf{s}_1\| = \mathbf{u} \times \mathbf{s}_1/|\sin(\mathbf{u}, \mathbf{s}_1)|$ and $\mathbf{d}_2 = \mathbf{u} \times \mathbf{s}_2/\|\mathbf{u} \times \mathbf{s}_2\| = \mathbf{u} \times \mathbf{s}_2/|\sin(\mathbf{u}, \mathbf{s}_2)|$. These vectors are perpendicular to the two parallelogram planes and, therefore, are parallel to the two sets of parallelogram hinges. The point N is chosen in the plane $Pl(O, \perp \mathbf{u})$. This point N determines the axis $(N, \mathbf{u})$ of the R in the PaRPa chain. Generally $\mathbf{d}_1 \neq \mathbf{d}_2$, and $(N, \mathbf{d}_1, \mathbf{d}_2)$ is a frame of reference of $Pl(O, \perp \mathbf{u})$. The resultant twist of PaRPa open chain characterizes a H pair whose axis $(N_R, \mathbf{u})$ is determined by $(\mathbf{NN}_R) = a_1\mathbf{d}_1 + a_2\mathbf{d}_2$ and its pitch is derived as

$$p_R = 2\pi k_R = 2\pi\{0 + [a_1/|\sin(\mathbf{u}, \mathbf{s}_1)|](\mathbf{u} \cdot \mathbf{s}_1) + [a_2/|\sin(\mathbf{u}, \mathbf{s}_2)|](\mathbf{u} \cdot \mathbf{s}_2)\} \tag{12}$$

In the current special case, $p_R = 0$ is laid down by the choice of a PaRPaR generator of X motion. The prescribed condition $p_R = 0$ implies $[a_1/|\sin(\mathbf{u}, \mathbf{s}_1)|](\mathbf{u} \cdot \mathbf{s}_1) + [a_2/|\sin(\mathbf{u}, \mathbf{s}_2)|](\mathbf{u} \cdot \mathbf{s}_2) = 0$. Hence, we have

$$a_2 = -a_1|\sin(\mathbf{u}, \mathbf{s}_2)|(\mathbf{u} \cdot \mathbf{s}_1)/[|\sin(\mathbf{u}, \mathbf{s}_1)|(\mathbf{u} \cdot \mathbf{s}_2)] \tag{13}$$

The point N_R cannot be chosen arbitrarily in $Pl(O, \perp \mathbf{u})$ and has to be on the straight line, Fig. 14,

$$N_R = N + (\mathbf{NN}_R) = N + a_1\{\mathbf{d}_1 - \mathbf{d}_2|\sin(\mathbf{u}, \mathbf{s}_2)|(\mathbf{u} \cdot \mathbf{s}_1)/[|\sin(\mathbf{u}, \mathbf{s}_1)|(\mathbf{u} \cdot \mathbf{s}_2)]\} \tag{14}$$

Generally the axis $(N', \mathbf{u})$ of the last R in PaRPaR does not intersect the previous straight line, $N' \neq N_R$, where N' is the position of N after moving and the PaRPaR chain is not singular. Furthermore, let us assume that $(\mathbf{NN}') \perp \mathbf{d}_2$, which means that the

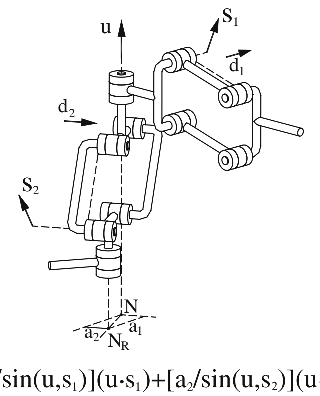


Fig. 14. A more special singular pose of McGill robot.

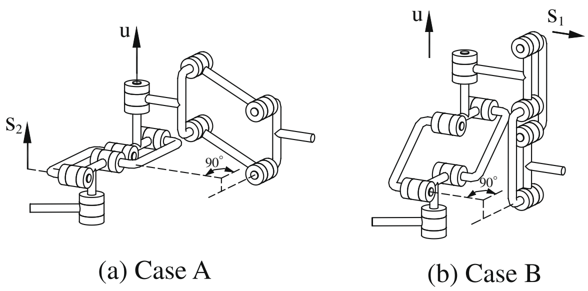


Fig. 15. Singular poses of McGill robot.

parallelogram plane is parallel to the two R axes plane, and that $N' = N_R$, which characterizes the chain singularity. Then $(\mathbf{NN}_R) \cdot \mathbf{d}_2 = 0$, and

$$a_1\{(\mathbf{d}_1 \cdot \mathbf{d}_2) - |\sin(\mathbf{u}, \mathbf{s}_2)|(\mathbf{u} \cdot \mathbf{s}_1)/[|\sin(\mathbf{u}, \mathbf{s}_1)|(\mathbf{u} \cdot \mathbf{s}_2)]\} = 0 \tag{15}$$

Generally $(\mathbf{d}_1 \cdot \mathbf{d}_2) - |\sin(\mathbf{u}, \mathbf{s}_2)|(\mathbf{u} \cdot \mathbf{s}_1)/[|\sin(\mathbf{u}, \mathbf{s}_1)|(\mathbf{u} \cdot \mathbf{s}_2)] \neq 0$ and necessarily $a_1 = 0$. Then $N_R = N$, and the two R axes are collinear as already detected, Fig. 13b. Even though this singular pose is avoided, the equality $(\mathbf{d}_1 \cdot \mathbf{d}_2) - |\sin(\mathbf{u}, \mathbf{s}_2)|(\mathbf{u} \cdot \mathbf{s}_1)/[|\sin(\mathbf{u}, \mathbf{s}_1)|(\mathbf{u} \cdot \mathbf{s}_2)] = 0$ can be locally or instantaneously achieved. For instance, let us assume the particular condition $(\mathbf{d}_1 \cdot \mathbf{d}_2) = 0$, which means that the two parallelogram planes are perpendicular. The resulting special equation is $|\sin(\mathbf{u}, \mathbf{s}_2)|(\mathbf{u} \cdot \mathbf{s}_1)/[|\sin(\mathbf{u}, \mathbf{s}_1)|(\mathbf{u} \cdot \mathbf{s}_2)] = 0$, which is satisfied with either $\sin(\mathbf{u}, \mathbf{s}_2) = 0$ or $(\mathbf{u} \cdot \mathbf{s}_1) = 0$.

Case A $\sin(\mathbf{u}, \mathbf{s}_2) = 0$

In this situation, we have $\mathbf{s}_2 = \mathbf{u}$, the translation parallel to $\mathbf{s}_2$ is parallel to $\mathbf{u}$. Fig. 15a illustrates this kind of local or infinitesimal singularity.

Case B $\mathbf{u} \cdot \mathbf{s}_1 = 0$

This condition means $\mathbf{s}_1 \perp \mathbf{u}$, i.e. the translation parallel to $\mathbf{s}_1$ is perpendicular to $\mathbf{u}$. This singular pose is shown in Fig. 15b. One can notice that, in this configuration, the sub-chain R-RPa of the chain RPaRPa is instantaneously a singular generator R-RP (P $\perp$ plane R axes) of $\{\boldsymbol{G}(\mathbf{u})\}$, what confirms the singularity.

More generally, when $(\mathbf{d}_1 \cdot \mathbf{d}_2) \neq 0$, a singular pose is attained when we have $|\sin(\mathbf{u}, \mathbf{s}_2)|/(\mathbf{u} \cdot \mathbf{s}_2) = (\mathbf{d}_1 \cdot \mathbf{d}_2)|\sin(\mathbf{u}, \mathbf{s}_1)|/(\mathbf{u} \cdot \mathbf{s}_1)$. Then,

$$\tan(\mathbf{u}, \mathbf{s}_2) = \pm(\mathbf{d}_1 \cdot \mathbf{d}_2)\tan(\mathbf{u}, \mathbf{s}_1) \tag{16}$$

The datum through $(\mathbf{d}_1 \cdot \mathbf{d}_2)$ of the relative angular position between the two planes of parallelograms and the datum through $\tan(\mathbf{u}, \mathbf{s}_1)$ of the translation direction in one parallelogram yield the singular poses characterized by the translation direction in the other parallelogram given by the previous formula in Eq. (16). That way, we found out an infinity of singular poses for the McGill robot. A special example is obtained with $\mathbf{d}_1 \cdot \mathbf{d}_2 = 1$. Then, the two parallelogram planes are parallel. Eq. (16) becomes $\tan(\mathbf{u}, \mathbf{s}_2) = \pm \tan(\mathbf{u}, \mathbf{s}_1)$ which has the solutions as follows,

$$angle(\mathbf{u}, \mathbf{s}_2) = \pm\ angle\ (\mathbf{u}, \mathbf{s}_1) + n\pi\ (n\ is\ integer). \tag{17}$$

The vectors $\mathbf{s}_1$ and $\mathbf{s}_2$ are parallel and, therefore, the translations generated by the two Pa couplings are parallel. This obvious type of singularity is illustrated by Fig. 13c.

5. Conclusions

The *X* motion generators are very useful for the synthesis of novel serial or parallel manipulators. In this paper, a brief account of some applications of the *X*-motion generators is addressed. All serial SCARA-type robots and all fully parallel symmetrical TPMs with 4-dof limbs are systematically introduced. Various achievements of some of the generators of *X*-motion are already described in the present-day literature. These mechanisms highlight and confirm the effectiveness of the enumeration of primitive Schönflies-motion generators, which was further studied with full details in [12]. Additionally, the singularity is detected in some of the known limbs recalled in this paper. Beyond the theoretical findings of this work, we expect that specialization in general architectures will be considered for the synthesis of practical parallel mechanisms.

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