MECH 289
Design Graphics
Geometric Construction Exercise -I-

January 28, 2008

1 Block with Dovetailed Groove and Land

Look at Fig. 1 . . .

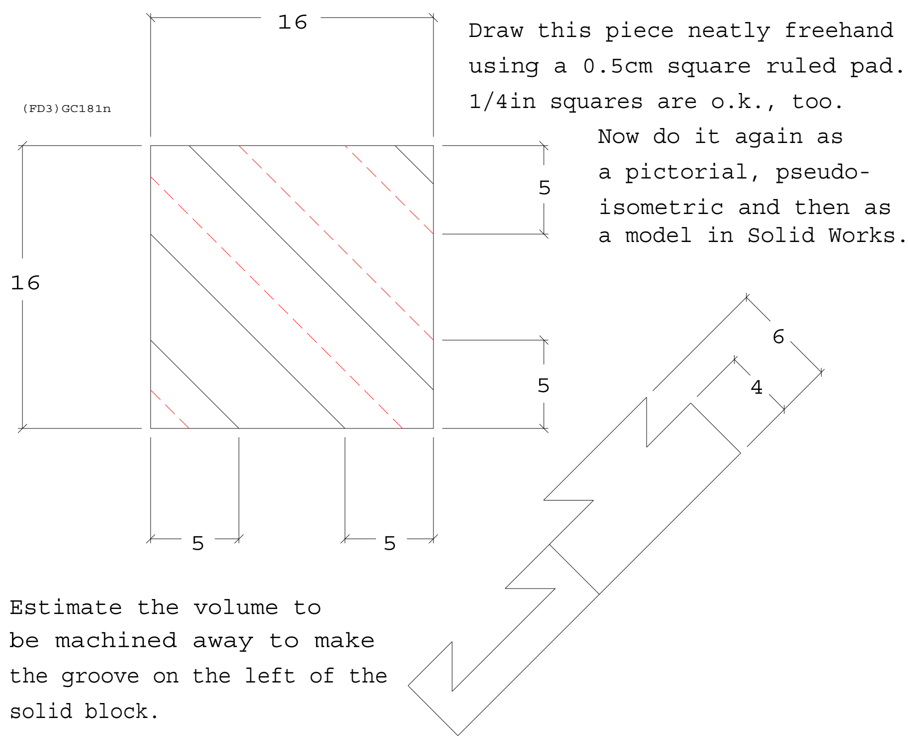

Figure 1: The Block

. . . and follow the instructions thereon.

2 Volume of a Prism of Triangular Cross-Section

To help with your volume calculations, consider Fig. 2 where one sees a prism, in arbitrary pose presented in a Cartesian frame, with three parallel edges AD, BE, CF of arbitrary length, given the position vectors **a**,**b**,**c**,**d**,**e**,**f**, of the six corner points.

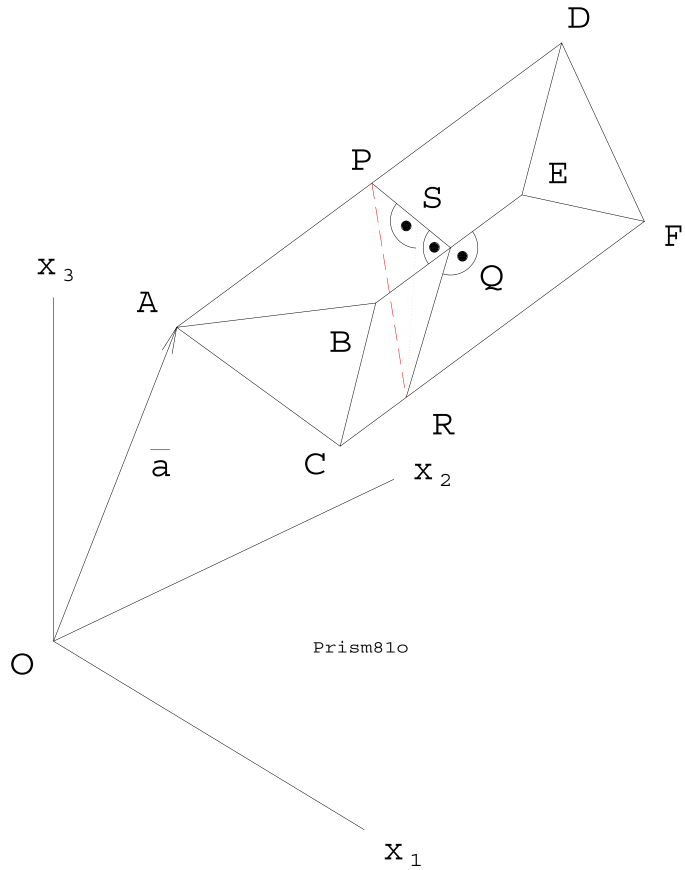

Figure 2: The Prism

A well-known principle of solid mensuration states that such a volume is equal to the product of the average edge length and the area of the normal section.

$$V_{ABCDEF} = \frac{L_{AD} + L_{BE} + L_{CF}}{3} A_{PQR}$$

A typical edge length, say AD, is given by the difference of the end point position vectors.

$$L_{AD} = \left\| \sqrt{(\mathbf{d} - \mathbf{a})^2} \right\|$$

The area of normal section A_{PQR} can be calculated as one half the base L_{PQ} times the height L_{RS}.

$$A_{PQR} = \frac{1}{2} L_{PQ} L_{RS}$$

It does not matter where section PQR is located. Let us cut segments AD, BE, CF with plane n on the frame origin O. Since, *e.g.*, $P \in n$

$$n: \quad (d_1 - a_1)p_1 + (d_2 - a_2)p_2 + (d_3 - a_3)p_3 = 0$$

Since $P \in AD$

$$\mathbf{p} = \mathbf{a} + t(\mathbf{d} - \mathbf{a})$$

or

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} + t\left(\begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix} - \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}\right) - \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

It can be shown that

$$t = -\frac{\sum_{i=1}^{3}(d_i - a_i)a_i}{\sum_{i=1}^{3}(d_i - a_i)^2}$$

After finding position vectors $\mathbf{q}$ and $\mathbf{r}$ in the same way, $\mathbf{s}$ can be found with the following three equations. The first is to find the constant coefficient k in the equation for plane r that must be normal to PQ. The second is that of the plane r so as to contain S while the third constrains $S \in PQ$.

$$r: \quad (q_1 - p_1)r_1 + (q_2 - p_2)r_2 + (q_3 - p_3)r_3 + k = 0$$

$$r: \quad (q_1 - p_1)s_1 + (q_2 - p_2)s_2 + (q_3 - p_3)s_3 + k = 0$$

$$\mathbf{s} = \mathbf{p} + u(\mathbf{q} - \mathbf{p})$$

3 "Grassmannian" Volume of a Tetrahedron

The fundamental element, called a *simplex*, in three-dimensional space is the tetrahedron, just as the planar simplex is the triangle. The following procedure shows how the volume of the dovetail slot, that was divided into two triangular prisms, can be divided into six tetrahedra by trisecting each prism. To aid visualization, the triangular prism, presented in a general configuration in Fig. 2, is regularized, to assume the form of a piece of familiar candy while preserving the same labeling topology, and presented in Fig. 3.

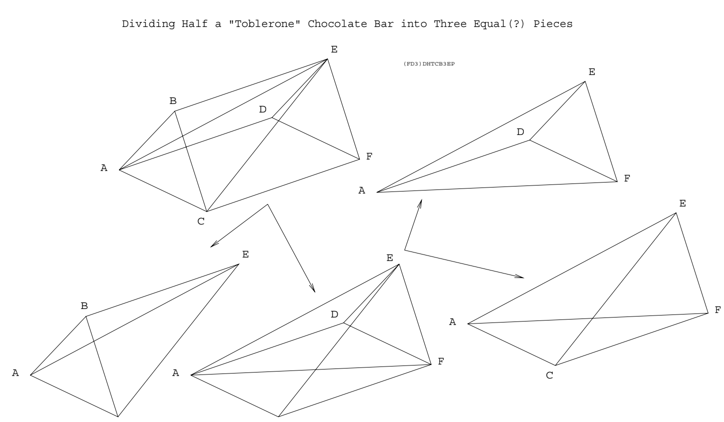

Figure 3: Tetrahedron

To find the volume v_{ABCE} of one of the three pieces, say, the tetrahedron with vertices $ABCE$ and respective point position vectors **abce**, one merely evaluates the determinant expressed below.

$$v_{ABCE} = \frac{1}{6} \begin{vmatrix} 1 & a_x & a_y & a_z \\ 1 & b_x & b_y & b_z \\ 1 & c_x & c_y & c_z \\ 1 & e_x & e_y & e_z \end{vmatrix}$$

If one feels that this procedure is considerably more computationally expensive than the prism volume calculation consider that for finite element mesh modeling any shape may be represented by a composite of simplices. Furthermore the simplicity of the tetrahedron calculation suits it to serve as the finite element kernel running on modern pipelined and parallel architectures used for very big numerical jobs.

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