MECH 314
Dynamics of Mechanisms

March 10, 2011

Rolling without Slipping 7-Bar Mechanism Static Analysis

1 Position Analysis

Examine Fig. 1. It shows the mechanism that defied the CGK planar mobility relation. To do a position analysis, given the positions of anchored R-joints A, B, H, the two wheel radii and the link lengths CE, DF, EF, EG, FG and GH is beyond the scope of this course. Therefore the coordinates of all these points are given as shown on the illustration. Furthermore to aid in the static analysis the distances $r_{AC} = 6$ and $r_{BD} = 4$ are given along with $r_{GH} = 4.896$. All dimensions are taken as dm. Specification of all forces will be in N and all moments, torques and couples will be in Nm.

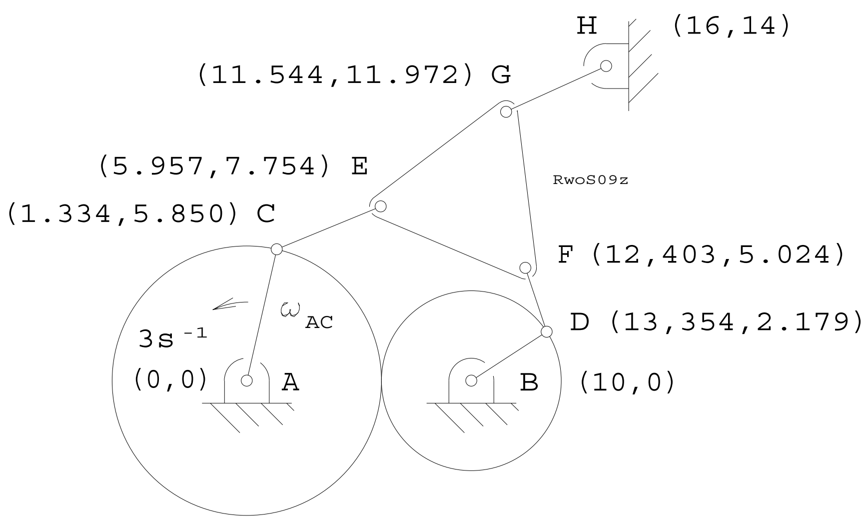

Figure 1: A 1dof Geared 7-Bar Mechanism

2 Principles of Planar Statics

It is recommended that chapter 13 in [1] be studied carefully. Note that some topics, like buckling of slender links under compression, will not be of immediate concern.

2.1 Joints

A prismatic or P-joint removes one translational dof and also the single rotational dof relevant to planar kinematics. It supplies reaction force in the direction *normal* to the direction of the sliding motion that it permits, It supplies reaction torque as well. These reactions are always sufficient to inhibit any such motions. A revolute or R-joint removes both translational dofs, *i.e.*, it inhibits *all* relative translation between the pair of rigid body links that it connects.

2.2 Static Equilibrium of Rigid Bodies or Links

Static analysis of planar mechanisms and machines, composed of rigid bodies, includes all conditions of motion where all links are at rest with respect to some inertial frame or all are in motion such that the centre of mass of every link is moving at constant velocity and every link turns with constant angular velocity. Of course these velocities may differ from link to link as prescribed by velocity analysis. For the moment we shall consider only *concentrated* forces, those represented by a vector of finite magnitude and acting along a line, called its *line of action* (LoA), attached to the rigid body or link in question. As we saw in MECH 210 planar static equilibrium is governed by two key principles that are expressed by the equations

$$\Sigma\mathbf{f} = \mathbf{0}, \ \ \Sigma\mathbf{M} = \mathbf{0}$$

These state that

- The sum of all force vectors, including reactions, must add up to zero
- As must the sum of all moments $\mathbf{M}_i = \mathbf{r}_i \times \mathbf{f}_i$ of forces $\mathbf{f}_i$ taken about any convenient, arbitrary point –where $\mathbf{r}_i$ is any vector extending from that point to *any* point on its LoA–
- plus any torques $\mathbf{\Gamma}_i$ injected by, say, the shaft of an R-joint that would otherwise be free to rotate
- Plus any couples furnished by a pair of forces of equal magnitude and opposite direction acting along parallel, finitely separated LoAs.

2.3 Links under Load

To do a static analysis it is simpler and more convenient to break the mechanism into its rigid body elements and apply the key equations of static equilibrium or their equivalent graphical constructions to each separately or, sometimes, to a minimal combination of links or a subassembly. The notion of –force instead of velocity this time– vector triad (triangle) solutions with one known vector and two others where only their directions are known will be seen again. Recall how this worked when seen previously in graphical velocity analysis. The following types of link equilibrium and conditions will be encountered in the example that follows.

- A two-force axially loaded link like CE or DF has no moments applied to it. Equal and opposite forces acting along the link may be applied to R-joints at either end,
- A similar link, like GH, where, in addition a torque like $\mathbf{\Gamma}_{GH}$ is externally imposed on an R-joint at one of the link's ends so as to require an output torque,
- A rigid body like EFG that sustains three forces applied along lines on three known points so that the three LoAs must intersect on a common point in order to leave no unequilibated moment on the body,
- A gear like BD that reduces to a body like EFG because the LoA of the force transmitted by its partner AC via the rolling-without-slipping contact or *pitch-point* at TT' must be tangential to both pitch circles while the reaction on an LoA on B must concur with this LoA and the LoA on DF
- And finally a gear like AC whose force equilibrium results in a residual moment requiring an input torque like $\mathbf{\Gamma}_{AC}$ that, in this case is the answer sought.

3 The Example

Imagine there exists some mechanical *output* device attached to a shaft at H so as to impose a passive –like friction– torque $\mathbf{\Gamma}_{GH} = 10$Nm CCW. This is sustained by an *input* torque $\mathbf{\Gamma}_{AC}$ applied by a motor turning the shaft at A at a constant angular velocity. The problem at hand is to find the magnitude and sense of this moment and to find the forces acting at all R-joints and at the pitch-point TT'.

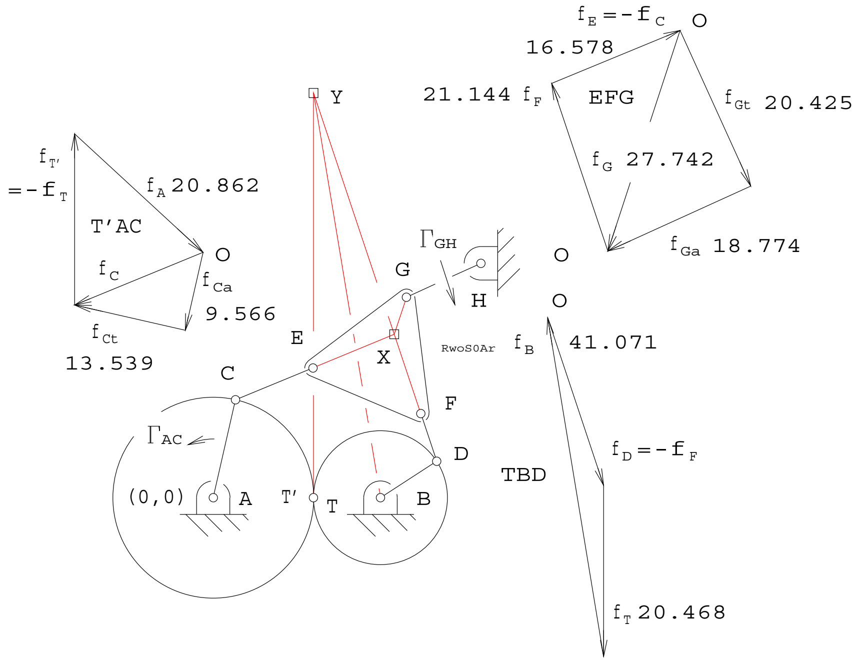

Figure 2: A Graphical Analysis with Three Force Vector Polygons

3.1 Diagram $G - EFG$

We start by observing that the output torque $\mathbf{\Gamma}_{GH} = 10$Nm CCW imposes a force component f_{Gt}, t =tangential, of 4.896N at G normal to $\mathbf{r}_{HG}$ but we do not know the component $\mathbf{f}_{Ga}$, a =axial, of the total force $\mathbf{f}_G$ applied at G so consider the equilibrium of link EFG. Since links CE and DF have no intermediate forces or moments applied to them the directions of forces $\mathbf{f}_E$ and $\mathbf{f}_F$ are known to be along CE and DF. These two lines intersect within the triangle ERG at point X and the LoA of $\mathbf{f}_G$ must go from G to X in order to maintain moment equilibrium of EFG. Since the direction of $\mathbf{f}_G$ and $\mathbf{f}_{Ga}$ are now both available the force triangle

$$\mathbf{f}_{Gt} + \mathbf{f}_{Ga} - \mathbf{f}_G = \mathbf{0}$$

can be completed. Due to static equilibrium of link EFG –there are now no unknown forces or moments acting on triangle ERG– the force triangle EFG can be constructed with known directions of $\mathbf{f}_E$ and $\mathbf{f}_F$ as

$$\mathbf{f}_G + \mathbf{f}_F + \mathbf{f}_E = \mathbf{0}$$

and the magnitudes of all forces on EFG can be measured and recorded as shown on Fig. 2.

3.2 Diagram TBD

Equilibrium of the gear TBD occurs under the influence of the three forces $\mathbf{f}_D$, $\mathbf{f}_T$ and $\mathbf{f}_B$, the one with unknown direction but whose LoA must be on B. $\mathbf{f}_T$ must be vertical, the direction of the common tangent at the contact between the two gears. $\mathbf{f}_D = -\mathbf{f}_F$ to keep link DF in equilibrium. Therefore the LoA of $\mathbf{f}_B$ must be on B and the intersection Y of a vertical on T and DF and the force triangle at lower-right can be drawn and the force magnitudes recorded.

3.3 Diagram $T'AC$

The only forces acting on gear $T'AC$ are $\mathbf{f}_{T'}$, $\mathbf{f}_C$ and $\mathbf{f}_A$. These must satisfy

$$\mathbf{f}_{T'} + \mathbf{f}_C + \mathbf{f}_D = \mathbf{0}$$

$\mathbf{f}_{T'} = -\mathbf{f}_T$ and $\mathbf{f}_C = -\mathbf{f}_E$ so $\mathbf{f}_A$ must close the triangle. But that is not the whole story. The LoA of $\mathbf{f}_A$ is on A so this force can exert no moment about that point. Breaking up $\mathbf{f}_C = \mathbf{f}_{Ca} + \mathbf{f}_{Ct}$ –notice that $\mathbf{f}_{Ca}$ is not required– gives us a quick method to calculate the moment about A as

$$r_{AC} f_{Ct} + r_{AT'} f_{T'} = 0.6(13.539) + 0.6(20.468) = 20.404\text{Nm CCW}$$

so the motor on the shaft at A must furnish a torque in the opposite sense.

$$\mathbf{\Gamma}_{AC} = -20.404\text{Nm CW}$$

4 Conclusion

Does $\mathbf{\Gamma}_{AC}$ look familiar? It should. It is –almost exactly; within 21/20.425=1%– equal but opposite to the output torque $\mathbf{\Gamma}_{GH}$. This is not surprising. The entire mechanism must be under static equilibrium so one might have just applied $\mathbf{\Gamma}_{AC} = -\mathbf{\Gamma}_{GH}$ right off the bat and saved all that work. However, as is often the case in statics, this provides a closure check on the rather lengthy chain of calculation. Such a check does not eliminate self-canceling error pairs but it will pick up many common instances of "finger-trouble" and will provide an estimate of the numerical accuracy of the overall process. Furthermore, in machine design one often needs a complete picture of forces and moments acting on each component –certainly at critical poses of the mechanism– in order to do a proper stress and distortion analysis.

References

[1] J.J. Uicker, Jr., G.R. Pennock and J.E. Shigley (2011) *Theory of Mechanisms and Machines, 4th ed.*, Oxford, ISBN 9780-19-537123-9.