Asymmetric Rendezvous Search at Sea

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***Abstract*— In this paper we address the rendezvous problem between an autonomous underwater vehicle (AUV) and a passively floating drifter on the sea surface. The AUV's mission is to keep an estimate of the floating drifter's position while exploring the underwater environment and periodically attempting to rendezvous with it. We are interested in the case where the AUV loses track of the drifter, predicts its location and searches for it in the vicinity of the predicted location. We parameterize this search problem with respect to both the uncertainty in the drifter's position estimate and the ratio between the drifter and the AUV speeds. We examine two search strategies for the AUV, an inward spiral and an outward spiral. We derive conditions under which these patterns are guaranteed to find a drifter, and we empirically analyze them with respect to different parameters in simulation. In addition, we present results from field trials in which an AUV successfully found a drifter after periods of communication loss during which the robot was exploring.**

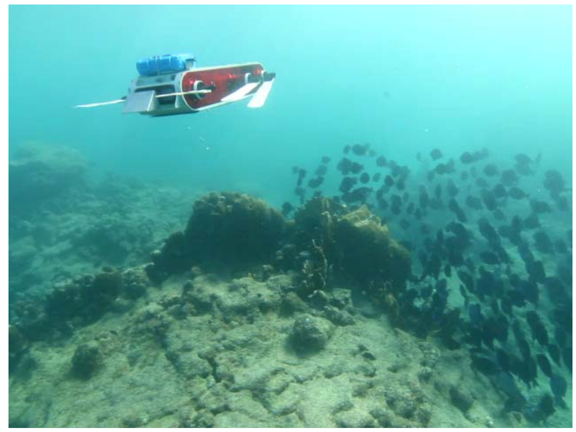

Fig. 1: The Aqua AUV performing a round of exploration

I. INTRODUCTION

This paper analyzes different search strategies for an asymmetric rendezvous at sea. Specifically, we consider an autonomous underwater vehicle (AUV) which explores the underwater environment in coordination with a free-floating drifter sensor at sea. Our work focuses on the search patterns that can be executed by an AUV to perform either a guaranteed search or a fast search to find the lost target.

A commonly known search strategy for lost targets is a spiral pattern, as discussed in [1], [2] and [3]. Spiral patterns can either be executed as inward spiral or outward spiral depending on if the search is initialized at the outer edge or at the center of the region of interest. We hypothesize that the inward spiral strategy performs a guaranteed search given an appropriate set of parameter values for the speed of the AUV and the drifter while, the outward spiral strategy will perform as a greedy search, optimizing the mean time to find the target.

We test the two search patterns to validate our hypotheses in simulation and in field experiments. We focus on the AUV's estimate of the drifter's position and on the search patterns that will find the drifter in the least amount of time. Specifically, we:

- identify conditions under which the patterns mentioned above have a guaranteed outcome
- empirically compare inward and outward spiral strategies in simulation, where the center is the estimated drifter position
- present results from field trials in which the AUV performed several rounds of rendezvous with a floating drifter

In our simulation, we represent the AUV and the drifter as point objects and reproduce all the sequence of steps executed at the sea. Our aim is to iteratively test the two search patterns given a set of input parameters and conclusively select one search strategy over the other.

In our field trials we exploit the motion and computation capabilities of a highly agile six-legged AUV (Fig. 1), whose objectives are: to perform underwater exploration, and occasionally to communicate with a single drifter. Given the relatively high cost and power requirements of long-range or satellite communications, we examine the scenario in which communication between the AUV and the drifter happens via low-cost XBee or WiFi, and thus has very limited range in the presence of waves. The latter goal requires the AUV to go within a certain distance of the drifter, known as the communication radius, so as to update the AUV's estimate of the drifter's position. We tested only the outward spiral pattern in our field experiment, as it is suggested by our simulations results to be the strategy which would take least amount of time to find the drifter.

We structure this paper beginning with our problem formulation in the Section III where we discuss the conditions for guaranteed search using spiral strategies. This is followed by an explanation of our evaluations, first in simulation in Section IV and then in the field in Section V.

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II. Related Work

Cooperative rendezvous during multi-robot exploration is an evolving research problem. Dudek et al. [4] formalize this problem as two mobile robots attempting to rendezvous while exploring a non-uniform environment. The robots collect and analyze sensor data to find landmarks in the environment which can be used for localization and as potential rendezvous locations [5]. The combined approach of exploration and rendezvous is extended for multiple robots in [6] and [7]. With the rise in popularity of Bayesian statistics, probabilistic rendezvous algorithms have appeared as well [8], which account for position uncertainty of the meeting agents. It is important to note that while a large amount of work on multi-robot exploration exists [9] [10] [11] [12] [13], the majority of it assumes constant communication between explorers, either directly or through a central command. In this paper, we do not make any such assumptions since, the radio signals get attenuated underwater, allowing only intermittent surface communication.

Our problem differs from the traditional rendezvous problem in two major respects. First, only one of the agents has control over its position and velocity (the AUV), while the other agent's movement is determined by currents and winds (the drifter). Second, the aquatic environment makes the use of landmarks infeasible. Therefore it may be more appropriate to think of the drifter as a target which the AUV must find. When formalized thus, this work has a lot in common with an autonomous search and tracking problem.

Search and tracking problem has been primarily focused on optimal search patterns for the searching robot, and on accurate position estimation of the target. Search patterns have been based on natural processes [3], geometric patterns [2] and boustrophedon coverage [12]. An example of the geometric search pattern is presented in [1] where the authors bound the estimate for the target's initial location within a circle. The probability distribution of the target's position was considered uniform inside the shape, and zero outside of it.

Bourgault et al. [14] and Furukawa et al. [15] represent the target's possible location using Bayesian statistics. Given that the state of a lost target is by definition unknown, it is unsurprising that a Bayesian approach is a suitable choice. Furukawa et al. [16] also summarize the mathematical basis of multi-robot, single-target search within a recursive Bayesian framework. Their goal is to unify search and tracking under a single objective function. This allowed them to retain the state estimation of the target when transitioning from tracking to search and vice-versa. Our work uses a recursive Bayesian framework as well, and we are able to retain our state estimation of the drifter through both of our states, search and exploration.

Recent work by Das et al. [17] has a very similar experimental, setup to that used in our field trials, however their goals are quite distinct from ours. Das et al. attempted to use a floating drifter to coordinate exploration of a moving ocean patch by an AUV. The drifter demarcates the center of the ocean patch, while the AUV moves in a box pattern around the perimeter, changing depth in a saw-tooth motion. Their work substantially differs from ours in that the drifter and AUV maintain near constant contact through the use of satellite communication.

III. Problem Formulation

Our aim in this paper is to find a drifter at the sea with a mobile robot. Let the position of the robot be x^r, and the position of the drifter, which is unknown, be x^d with an initial distance D_0 between them. Since, in our case, the robot operates in open water we assume that there are no obstacles in our world. If the robot is given no information about the state of the drifter, the best it can do is to search by performing a systematic coverage of the environment. We consider that the robot has found the drifter if it falls within the communication radius of the robot R_{comm}. For our problem, we specify that the robot and drifter are initially outside of communication range ($D_0 > R_{\text{comm}}$).

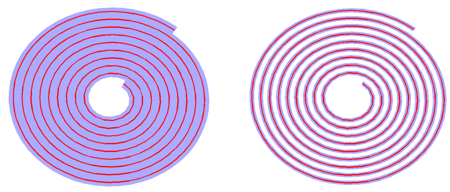
Our objective is to analyze the success of two search strategies for finding the drifter in the form of precomputed trajectories. These two trajectories are an inward spiral and an outward spiral to the estimated drifter location. The motivation for following a spiral trajectory is that it have been shown to be the optimal strategy for search in the plane with no information [18]. In [14], both types of spirals have been obtained by solving an optimal control problem, when the hypothesis of the drifter's location is unimodal and concentrated around the mode. In our case, we avoid solving the optimal control problem, and instead opt to precompute the trajectories and evaluate their success under variations of the position uncertainties, the communication range and the speed of the drifter.

A. Guaranteed search with spiral trajectories

In this section we derive conditions under which the spiral strategies have a guaranteed outcome i.e. either a guaranteed success or a time at which a decision can be made for a guaranteed failure. Consider first an outward spiral pattern. Without loss of generality, let the initial position of the robot be at the origin of our coordinate system. To guarantee that every given point in the space will eventually be explored, the trajectory of the robot is determined by an arithmetic spiral; i.e. the position of the robot, at time t, will be:

$$x_t^r = b\theta_t \begin{pmatrix} \cos(\theta_t) \\ \sin(\theta_t) \end{pmatrix} \tag{1}$$

where b is a parameter that determines the distance between points in consecutive turns, along a particular direction. Since the robot needs to explore all of the space, we set the parameter b to depend on the communication radius. If we assume that the robot does not know the direction in which the drifter moves, the separation between turns in the spiral pattern must be smaller than the communication range; i.e. $b \leq \frac{R_{\text{comm}}}{2\pi}$. If the distance between turns is greater then there is no guarantee that the robot will not miss the drifter, regardless of the robot's speed (see Fig. 2).



(a) Good Spiral (b) Bad Spiral

Fig. 2: When consecutive turns of the spiral are at a distance greater than the communication radius, the robot might miss the drifter in the gaps of the spiral on the right.

For the robot to miss the drifter, the drifter should travel a distance of $2\pi b$ along a line passing through the center of the spiral in less time than the time the robot needs to complete a full turn. Similarly, let $s(\theta_t)$ be the arc length traveled by the robot after traversing an angle of θ_t, i.e.

$$s(\theta_t) = \frac{b}{2}\left(\theta_t\sqrt{1+\theta_t^2} + \log\left(\theta_t + \sqrt{1+\theta_t^2}\right)\right) \qquad (2)$$

Thus the condition that the speed of the robot is greater than the radial speed of the drifter can be expressed as

$$\frac{s(\theta_t+\Delta\theta) - s(\theta_t)}{\|v_t^r\|} \leq \frac{(\theta_t+\Delta\theta)b - b\theta_t}{\|v_{\text{max}}^d\|} \qquad (3)$$

where v_t^r is the velocity of the robot *along the spiral path* and $\|v_{\text{max}}^d\|$ is the maximum speed at which the drifter can move. Letting $\Delta\theta \to 0$, the speed of the robot should satisfy

$$\|v_t^r\| > \|v_{\text{max}}^d\|\sqrt{1+\theta_t^2} \qquad (4)$$

where $ds/d\theta_t = b\sqrt{1+\theta_t^2}$. As θ_t grows, the arc length between consecutive turns in the spiral grows, therefore the robot must increase its speed over time to keep up with the drifter. Assuming the robot can keep increasing its speed until it finds the drifter, the time to find the drifter is given by the inequality $b\theta_t > D_0$. Since our robot's maximum speed is finite, after some time the robot will never find the drifter, assuming the drifter travels in the same direction. The time when it happens is given by the value of θ_t at which Eq. 4 does not hold anymore, i.e.

$$\theta_{t,\text{failure}} = \sqrt{\left(\frac{\|v_{\text{max}}^r\|}{\|v_{\text{max}}^d\|}\right)^2 - 1} \qquad (5)$$

If the robot always moves at its maximum speed, and $D_0 > R_{\text{comm}}$ the time when this happens is given by

$$t_{\text{failure}} = \frac{s(\theta_{t,\text{failure}})}{\|v_{\text{max}}^r\|} \qquad (6)$$

This means that the time to miss the drifter increases with the robot's speed, while it decreases with the drifter's speed, making guaranteed search very hard except for the cases in which the drifter is close to not moving.

When the robot knows the direction in which the drifter is moving, but not its initial location, a similar analysis can be performed. This scenario is akin to the drifter being dragged by the wind, the speed and direction of which the robot could measure. The trajectory of the robot would be given by

$$x_t^r = b\theta_t\begin{pmatrix}\cos(\theta_t)\\ \sin(\theta_t)\end{pmatrix} + v^{\text{wind}}t \qquad (7)$$

which will produce a moving spiral pattern as the one in Figure 3. In this case the analysis is done in a frame of reference moving with velocity v^{wind}. If $v_{\text{max}}^d = v^{\text{wind}}$ then there is no restriction on the robot's speed, other than $\|v_t^r\| \geq \|v^{\text{wind}}\|, \forall t$. In such case, the robot will find the drifter if it is given enough time, since the drifter will have zero speed in the moving frame of reference. The amount of time needed is given by the inequality $b\theta_t > D_0 + v^{\text{wind}}t$.

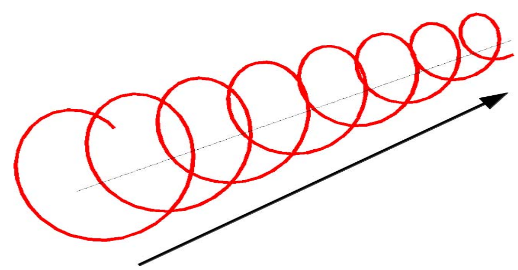


Fig. 3: A inward spiral pattern with a moving center

When $v_{\text{max}}^d \neq v^{\text{wind}}$, the same analysis of equations 4 to 6 is applicable by using relative velocities in the new frame of reference. The difference is that by using the relative velocities, the time until failure increases as the robot's estimate of v^{wind} approaches v_{max}^d.

For an inward spiral trajectory, the robot should start the trajectory at an initial radius of $R_{\text{in}} = b\theta_{t,\text{failure}}$ to guarantee that the drifter will be found. This will only work if the drifter is within the disk of radius $R_{\text{in}} - R_{\text{comm}}$, which makes such strategy less attractive than spiraling out.

IV. Controlled Simulation

A. Setup

We analyze the proposed search strategies from Section III by implementing a simulated robot-drifter pair. Specifically, we tested our two search patterns while varying the values of drifter's position uncertainty and speed. Parameters such as R_{comm}, the noise in drifter's velocity estimate, and the drifter's speed were all set to realistic values for the scenario of an AUV at sea. A snapshot of the simulation system for the robot-drifter interaction during the search phase is presented in Figure 4. We assumed a point robot with uniform speed which performed a sequence of motions similar to the ones that our AUV executed. The drifter was also simulated as a point object with random heading directions to simulate sea-like conditions. The communication range of the drifter was considered as the limiting factor for setting the width of the spiral. We tested two search patterns to empirically test the effect of randomly changing drifter's position uncertainty and systematically varying the drifter's speed.

During each run of the experiment a sequence of actions was simulated. First, the drifter received noisy position

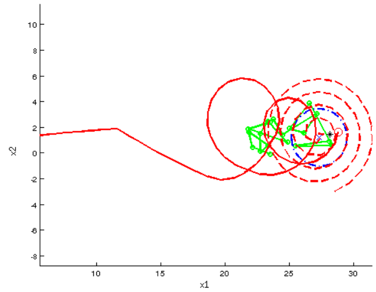


Fig. 4: Simulated paths for Aqua (solid red) and drifter (green circles). The dashed spirals represent the search pattern to be executed by Aqua.

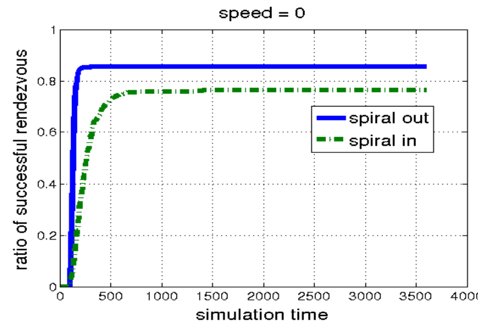
measurements which were used to fit a motion model, which similar to [14], consisted of a Beta distribution for its speed and a von Mises distribution for its heading. The motivation for using such motion model was that the Beta distribution has a bounded support. At the same time, the drifter also kept a hypothesis on its position by using a Kalman Filter. Second, the robot would receive the estimated motion model, and the position estimate from the drifter. After receiving the model, the robot would go out of the communication range of the drifter, to simulate the execution of an underwater task. During this time, the robot would update its hypothesis using the received motion model. Finally, after a fixed amount of time, it would use its hypothesis on the drifter's location to compute the waypoints of either an inward spiral or an outward spiral trajectory. The radius of these spiral trajectories would be computed according to the size of the 99 percent confidence ellipse around the robot's hypothesis on the drifter's location.

B. Results

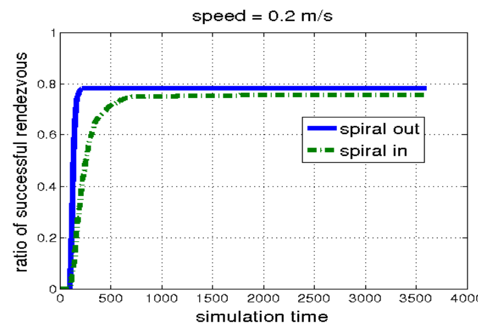
We tested the two search patterns using realistic values taken from our field experiment setup. Specifically, we tested with $R_{\text{comm}} = 5m$, with the position uncertainty varying between 2 meters to 8 meters, and with the drifter's speed being 0 and 0.2 m/s. Each parameter combination was averaged over 100 trials. The results with these input parameters are presented in Figures 5 and 6.

In Figure 5, we plotted the cumulative number of successful rendezvous trials between the robot and the drifter at any given simulated time. It can be observed that the outward spiral pattern does consistently better than inward spiral for all the simulation steps and different drifter speeds. In the case where the drifter is stationary, the outward spiral pattern clearly outperforms the inward spiral. However, at drifter speed of 0.2 m/s, the outward spiral pattern does not outperform the inward spiral pattern by a significant amount. This implies that when the drifter is in motion both outward spiral and inward spiral will perform equally well.

In addition, we analyzed the effect of the drifter's position uncertainty on the mean time to find, in Figure 6. These results suggest that the outward spiral found the drifter faster



(a)



(b)

Fig. 5: Cumulative ratio of successful rendezvous attempts vs. simulation time for the two strategies

than the inward spiral with shorter mean time to find for both the speed cases. This outcome can be explained due to the unimodal nature of the gaussian estimate around the estimated mean of the drifter's location. The outward spiral visits the mean first, while the inward spiral starts searching at the 3-sigma border of the distribution.

We also observed that inward spiral performs better than outward spiral for certain input set of parameters. This result is presented in Figure 7 where inward spiral has marginally higher number of successes than outward spiral with position uncertainty of 4 meters and 6 meters for static drifter and drifter with speed 0.2 m/s respectively.

V. Sea Trials

A. Setup

In addition to our controlled simulation, we implemented our system and tested it on a real robot-drifter pair in the sea. We performed trials that involved the Aqua underwater robot and a passive drifter. Our drifter was a free-floating, compact sensor box containing a GPS receiver, XBee and WiFi communications, and a Raspberry Pi for on-board recording and processing of the data. The average floating speed that we observed was 0.4m/s, however this speed was dependent on the many factors related to currents, waves, and weather conditions. The drifter filtered the incoming GPS data, and continuously updated and transmitted its own location and velocity estimate at a rate of 1Hz. This is a notable difference between existing drifters that are designed

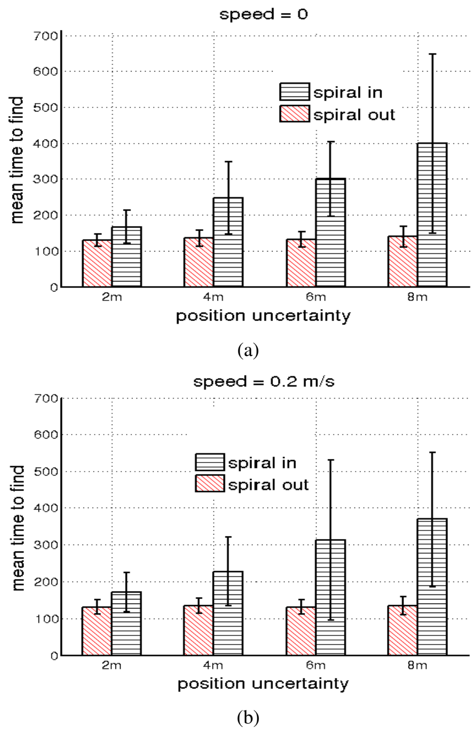


Fig. 6: Mean time to find vs. the increasing uncertainty in the drifter's position. The error bars correspond to standard deviation.

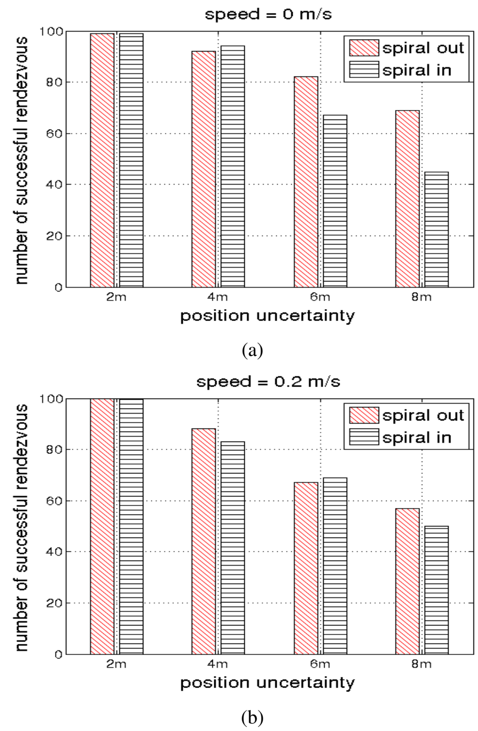


Fig. 7: Number of successful rendezvous attempts vs. the increasing uncertainty in the drifter's position

to float for months or years in the ocean (e.g. [19]), and thus need to conserve power and minimize communication costs. In most cases these design constraints allow them to only emit their GPS position every hour. In our case the drifter needs to emit its location frequently enough to allow the AUV searcher to acknowledge and find it whenever they are within communication range.

The Aqua AUV is equipped with its own GPS receiver, XBee and WiFi antennas. It is capable of forward swimming speeds of 1m/s. It navigates to the given target GPS waypoint by means of a *porpoising* motion, by which the robot periodically surfaces to get a GPS fix, heaves down to a certain depth, and then travels on a straight line towards the target. These three components are better illustrated in Fig. 8 and were implemented in our previous work [20]. This navigation method is used to avoid the need for acoustic localization with fixed beacons.

B. Results

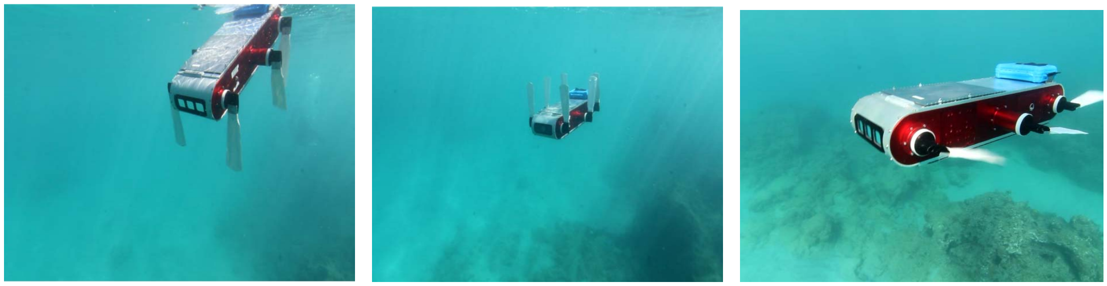
We performed five trials[1] of a field experiment in which the AUV and the drifter initially start near each other, within WiFi communication range, so that the AUV can form an estimate of the drifter's position and velocity. Once this happens the AUV starts the process of underwater exploration. The time spent exploring is initially set to two minutes and is doubled at the beginning of each new round, every time the AUV finds the drifter. Immediately after each exploration phase the AUV surfaces to communicate with the drifter. If the two are within communication radius then the AUV incorporates this new measurement into its estimate, navigates towards the drifter, and starts exploring anew. If not, then it predicts the drifter's location, navigates towards it, and once there, it begins the search process. In our experiment we only used outward square spirals as our search strategy since they performed best in our simulation results.

Fig. 9 shows the search path of the AUV, which consists of full-speed navigation through a series of GPS target waypoints that form a square outward spiral shape. In this particular round, the drifter was found at the end of the eighth target waypoint, after approximately 30 minutes of searching. Performing the search via surface swimming was an option that we did not opt for due to the difficulties presented by the wave action, as well as by the non-holonomicity of the vehicle. Another potential option for the spiral trajectory execution would be to base it solely on IMU corrections, however, the drift in the IMU yaw would be prohibitve for any long-term execution of such trajectories.

VI. Discussion and Conclusion

We presented an analysis of search strategies for rendezvous between an active underwater robot and a passively floating drifter on the ocean surface. Specifically,

[1] By "trial" we mean a part of the experiment from the time the AUV explores to the time it finds the drifter



(a) (b) (c)

Fig. 8: (a) Heave up motion at 45 degree pitch to raise the GPS box (blue) over the surface. (b) Heave down motion to reach an operating depth. (c) Straight line in the direction of the target waypoint. Yaw corrections are IMU-based.

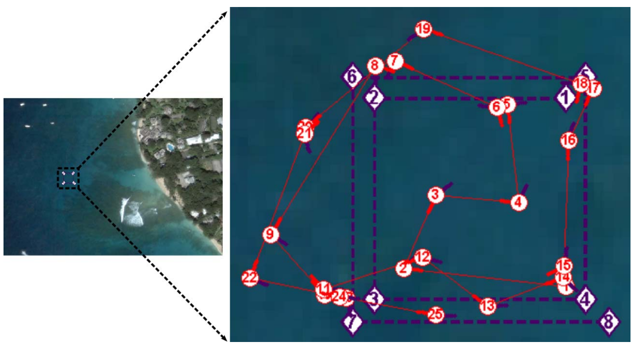


Fig. 9: Aqua's path for searching the drifter (in red). The square outward spirals are also shown (purple).

we examined inward and outward spiral strategies and the conditions under which their search outcome is guaranteed. We also presented results from a field trial in which an AUV searched for a drifter after it had lost communication during its exploration phase.

VII. Acknowledgement

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References

[1] L.C. Thomas and PB Hulme. Searching for targets who want to be found. *Journal of the Operational Research Society*, pages 44–50, 1997.

[2] S. Burlington and G. Dudek. Spiral search as an efficient mobile robotic search technique. In *Proceedings of the 16th National Conf. on AI, Orlando Fl*. Citeseer, 1999.

[3] E. Gelenbe, N. Schmajuk, J. Staddon, and J. Reif. Autonomous search by robots and animals: A survey. *Robotics and Autonomous Systems*, 22(1):23–34, 1997.

[4] G. Dudek and N. Roy. Multi-robot rendezvous in unknown environments, or, what to do when you're lost at the zoo. In *Association for the Advancement of Artificial Intelligence (AAAI), On-Line Search AAAI Technical Report WS-97-10*, pages 22–29. AAAI, 1997.

[5] N. Roy and G. Dudek. Collaborative robot exploration and rendezvous: Algorithms, performance bounds and observations. *Autonomous Robots*, 11(2):117–136, 2001.

[6] J. de Hoog, S. Cameron, and A. Visser. Selection of rendezvous points for multi-robot exploration in dynamic environments. In *International Conference on Autonomous Agents and Multi-Agent Systems*, 2010.

[7] M. Meghjani and G. Dudek. Combining multi-robot exploration and rendezvous. In *CRV '11: Proceedings of the 2011 Canadian Conference on Computer and Robot Vision*, pages 80–85. IEEE Computer Society, May 2011.

[8] P. CHEN, J. GU, X. LIN, and R. TAN. A probabilistic approach for rendezvous decisions with uncertain data. *Journal of Computational Information Systems*, 7(13):4668–4677, 2011.

[9] I. Rekleitis, G. Dudek, and E. Milios. Multi-robot collaboration for robust exploration. In *Proceedings of International Conference in Robotics and Automation*, pages 3164–3169, San Francisco, USA, April 2000.

[10] W. Burgard, M. Moors, D. Fox, R. Simmons, and S. Thrun. Collaborative multi-robot exploration. In *IEEE International Conference on Robotics and Automation*, volume 1, pages 476–481, 2000.

[11] M.A. Batalin and G.S. Sukhatme. Coverage, exploration and deployment by a mobile robot and communication network. *Telecommunication Systems*, 26(2):181–196, 2004.

[12] I. Rekleitis, A.P. New, E.S. Rankin, and H. Choset. Efficient boustrophedon multi-robot coverage: an algorithmic approach. *Annals of Mathematics and Artificial Intelligence*, 52(2):109–142, 2008.

[13] R. Vincent, D. Fox, J. Ko, K. Konolige, B. Limketkai, B. Morisset, C. Ortiz, D. Schulz, and B. Stewart. Distributed multirobot exploration, mapping, and task allocation. *Annals of Mathematics and Artificial Intelligence*, 52(2):229–255, 2008.

[14] Frédéric Bourgault, Tomonari Furukawa, and Hugh F Durrant-Whyte. Optimal search for a lost target in a bayesian world. In *Field and service robotics*, pages 209–222. Springer, 2006.

[15] Tomonari Furukawa, Frederic Bourgault, Benjamin Lavis, and Hugh F Durrant-Whyte. Recursive bayesian search-and-tracking using coordinated uavs for lost targets. In *Robotics and Automation, 2006. ICRA 2006. Proceedings 2006 IEEE International Conference on*, pages 2521–2526. IEEE, 2006.

[16] Tomonari Furukawa, Lin Chi Mak, Hugh Durrant-Whyte, and Rajmohan Madhavan. Autonomous bayesian search and tracking, and its experimental validation. *Advanced Robotics*, 26(5-6):461–485, 2012.

[17] Jnaneshwar Das, Frédéric Py, Thom Maughan, Tom OReilly, Monique Messié, John Ryan, Gaurav S Sukhatme, and Kanna Rajan. Coordinated sampling of dynamic oceanographic features with underwater vehicles and drifters. *The International Journal of Robotics Research*, 31(5):626–646, 2012.

[18] S. Alpern and S. Gal. The theory of search games and rendezvous. pages 165–178, 2003.

[19] Dean Roemmich, Gregory C Johnson, Stephen Riser, Russ Davis, John Gilson, W Brechner Owens, Silvia L Garzoli, Claudia Schmid, and Mark Ignaszewski. The argo program: Observing the global ocean with profiling floats. *Oceanography*, 22, 2009.

[20] F. Shkurti, A. Xu, M. Meghjani, J.C.G. Higuera, Y. Girdhar, P. Giguere, B.B. Dey, J. Li, A. Kalmbach, C. Prahacs, et al. Multi-domain monitoring of marine environments using a heterogeneous robot team. In *Intelligent Robots and Systems (IROS), 2012 IEEE/RSJ International Conference on*, pages –. IEEE, 2012.