COMP 250

Lecture 32

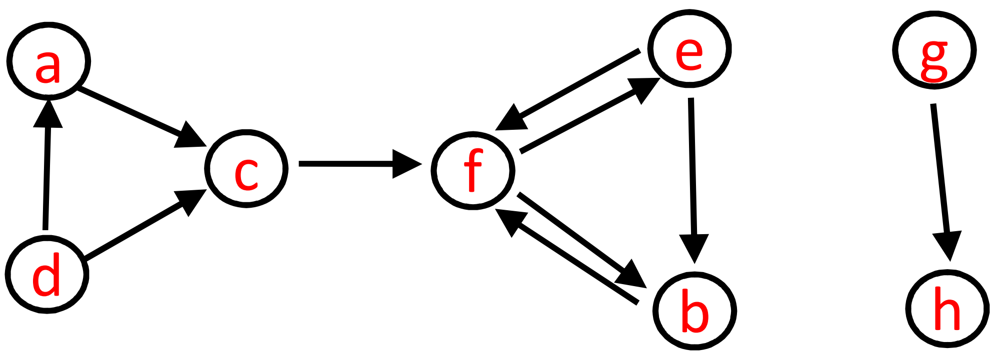
graph traversal

March. 28, 2022

Today

- Recursive graph traversal
  - depth first

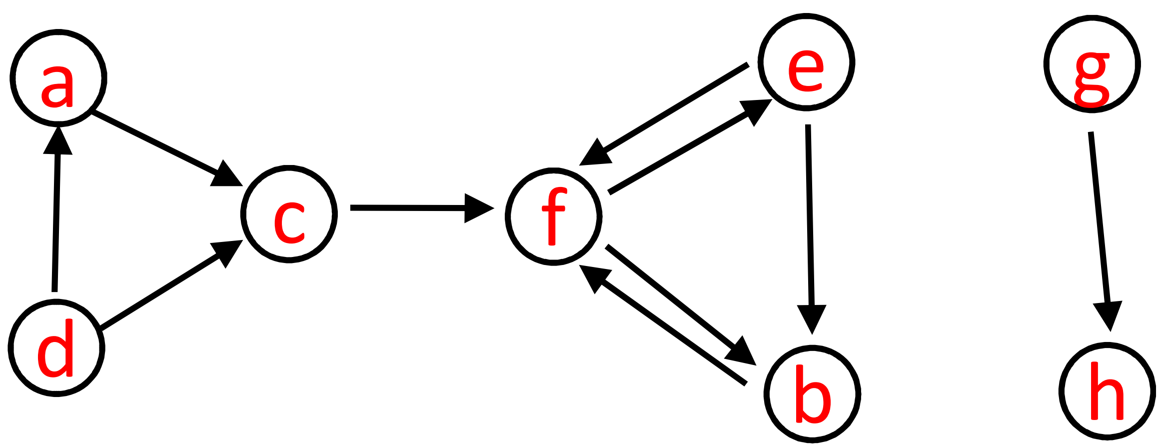
- Non-recursive graph traversal
  - depth first
  - breadth first



Graph traversal (recursive)

Specify a starting vertex.

Visit all nodes that are “reachable” by a path from a starting vertex. Today we will say “reaching” is the same as “visiting”.



Recall: Tree traversal (recursive)

```
depthfirst_Tree (root){
      root.visited = true                 //      "preorder"
      for each child of root
            depthfirst_Tree( child )
   }
}
```

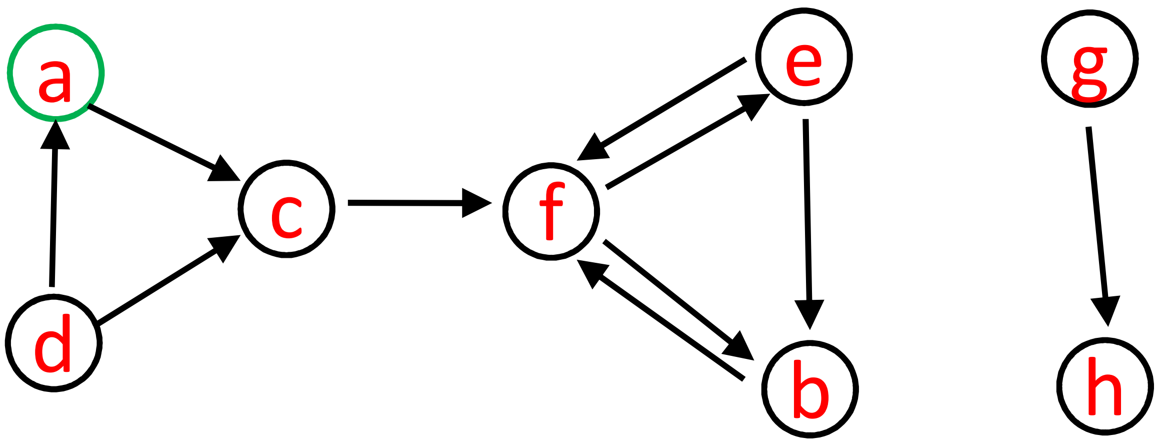
Graph traversal (recursive)

```
depthfirst_Graph(v){
    v.visited = true
    for each w such that (v,w) is in E   //  w in v.adjList
        ________?____________
}
```

Graph traversal (recursive)

```
depthfirst_Graph(v){
    v.visited = true
    for each w such that (v,w) is in E   //  w in v.adjList
        if   ! (w.visited)                        //  avoids cycles
            depthfirst_Graph(w)
}
```

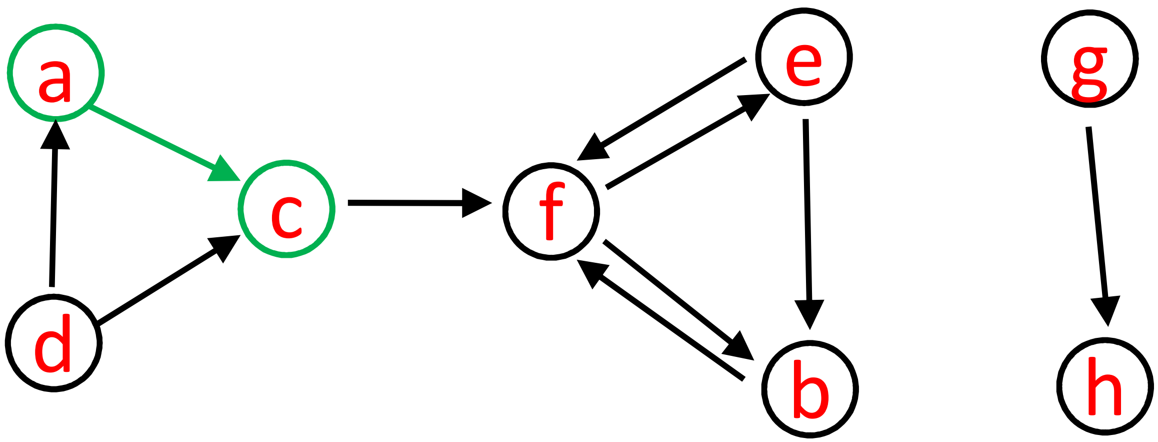
Call Stack for depthFirst_Graph(a)




```
depthFirst_Graph(v){
  v.visited = true
  for each w such that (v,w) is in E
     if  ! (w.visited)
        depthFirst_Graph(w)
}
```

a

Call Stack for depthFirst_Graph(a)

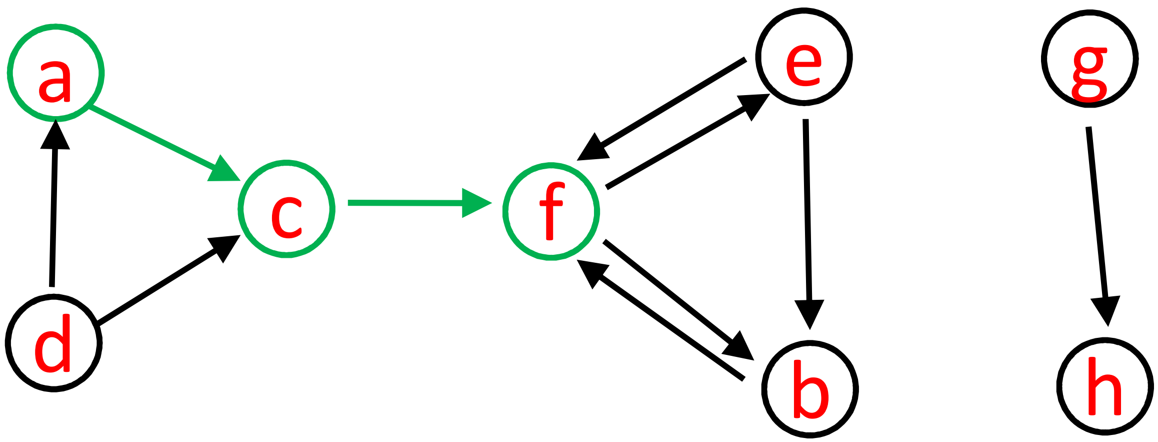


c

a a

```
depthFirst_Graph(v){
  v.visited = true
  for each w such that (v,w) is in E
      if   ! (w.visited)
          depthFirst_Graph(w)
}
```

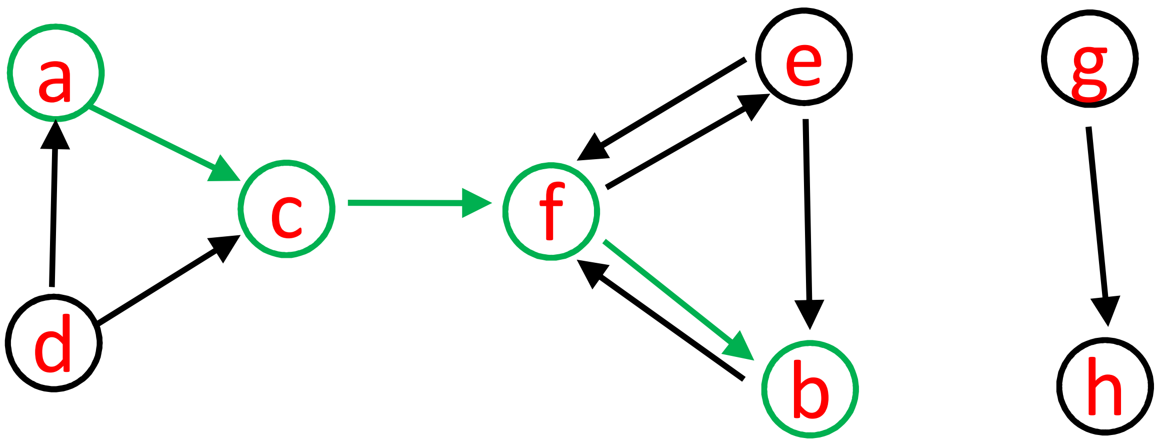
Call Stack for depthFirst_Graph(a)

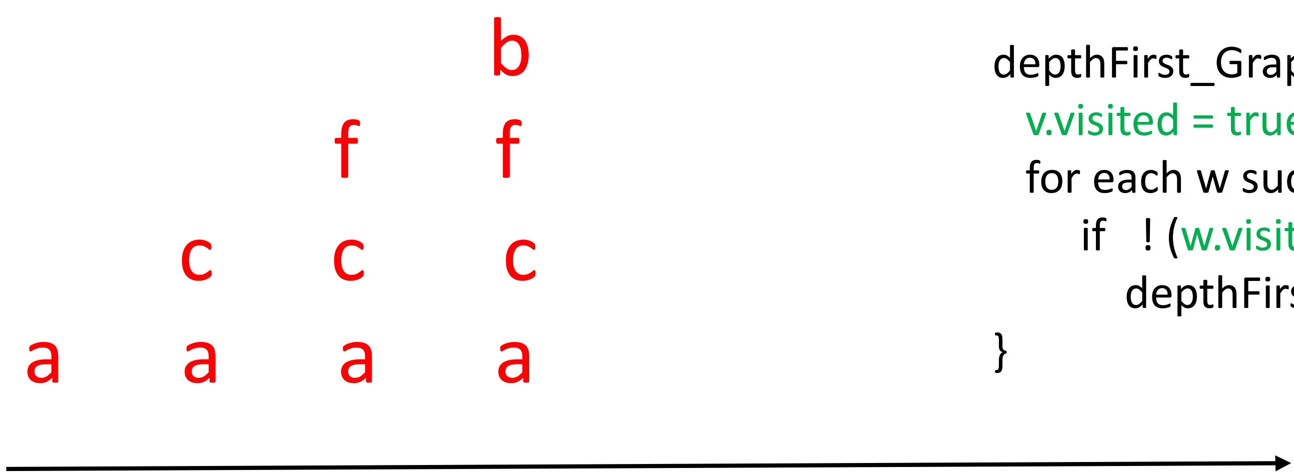


		f
	c	c
a	a	a

```
depthFirst_Graph(v){
  v.visited = true
  for each w such that (v,w) is in E
     if  ! (w.visited)
        depthFirst_Graph(w)
}
```

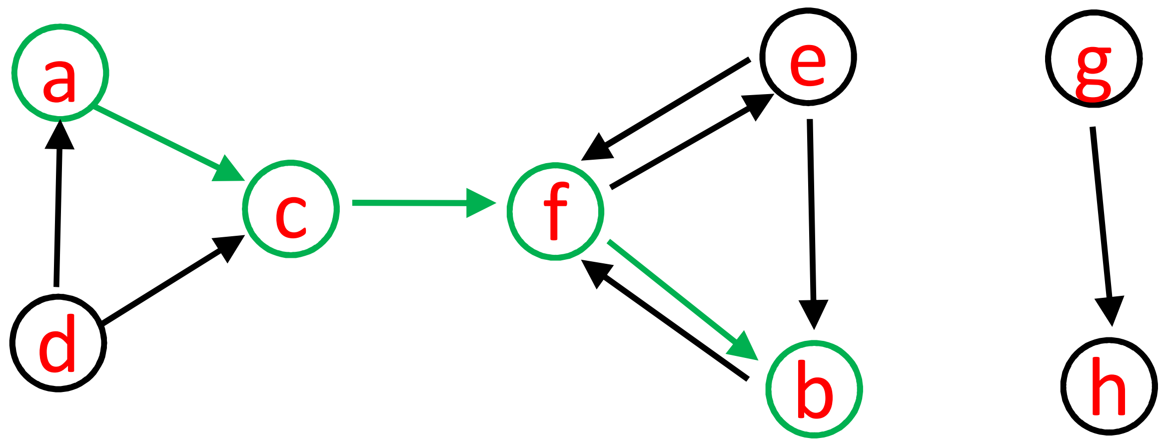
Call Stack for depthFirst_Graph(a)

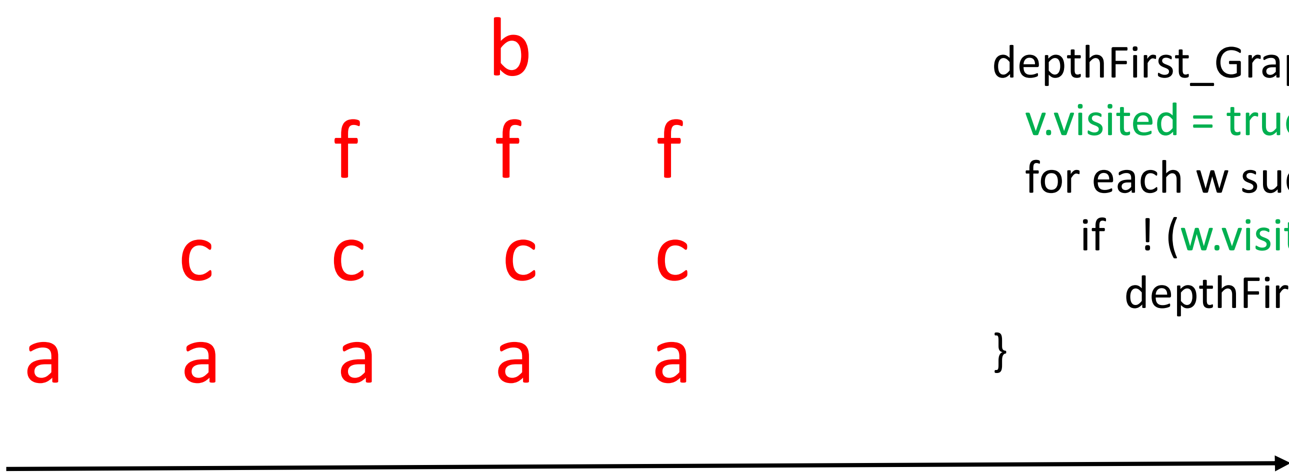





```
depthFirst_Graph(v){
  v.visited = true
  for each w such that (v,w) is in E
     if  ! (w.visited)
        depthFirst_Graph(w)
}
```

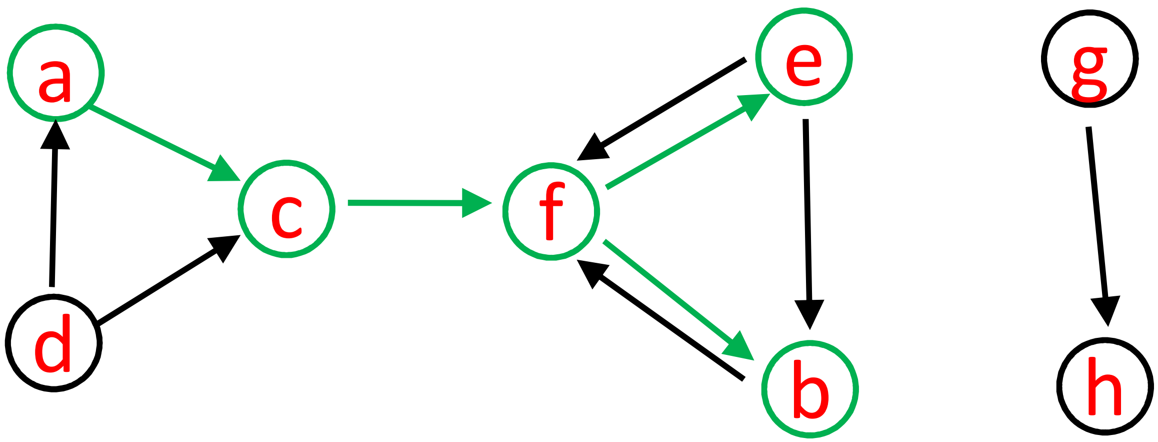
Call Stack for depthFirst_Graph(a)






```
depthFirst_Graph(v){
  v.visited = true
  for each w such that (v,w) is in E
      if  ! (w.visited)
          depthFirst_Graph(w)
}
```

Call Stack for depthFirst_Graph(a)

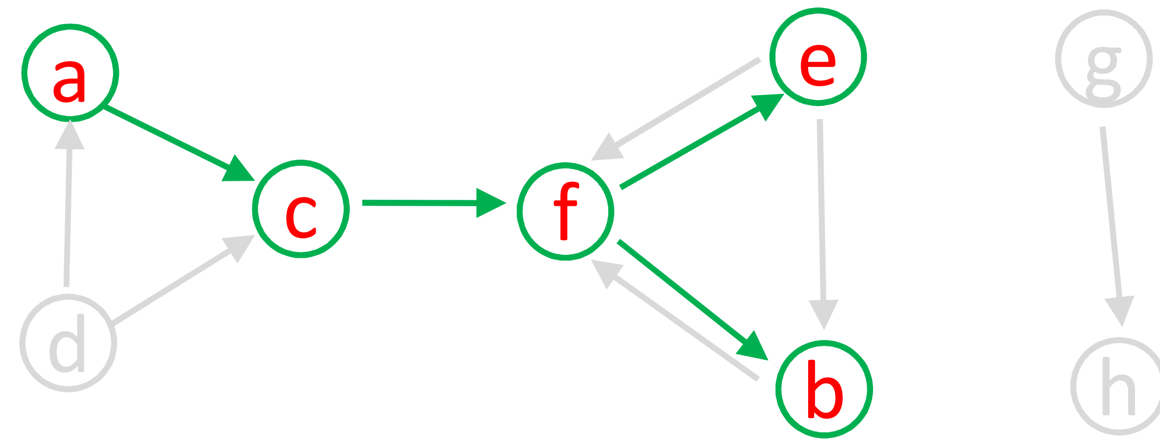


			b		e
		f	f	f	f
	c	c	c	c	c
a	a	a	a	a	a

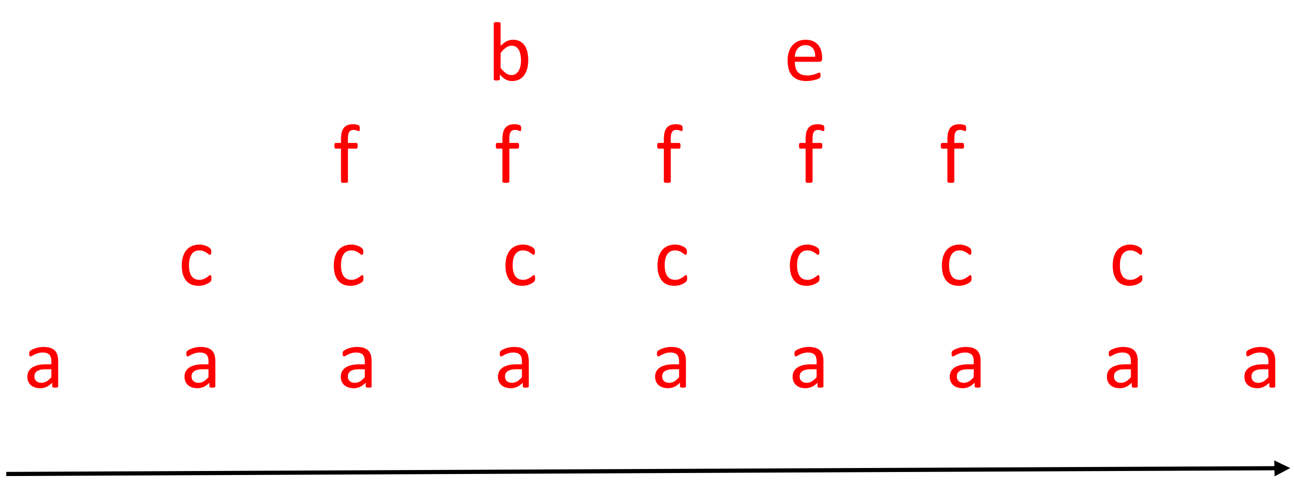
```
depthFirst_Graph(v){
  v.visited = true
  for each w such that (v,w) is in E
     if  ! (w.visited)
        depthFirst_Graph(w)
}
```

“Call Tree” for depthFirst_Graph(a)

root



In a running program, the call stack actually exists but the call tree does not exist. The call tree is only a way to visualize what the recursive calls are.

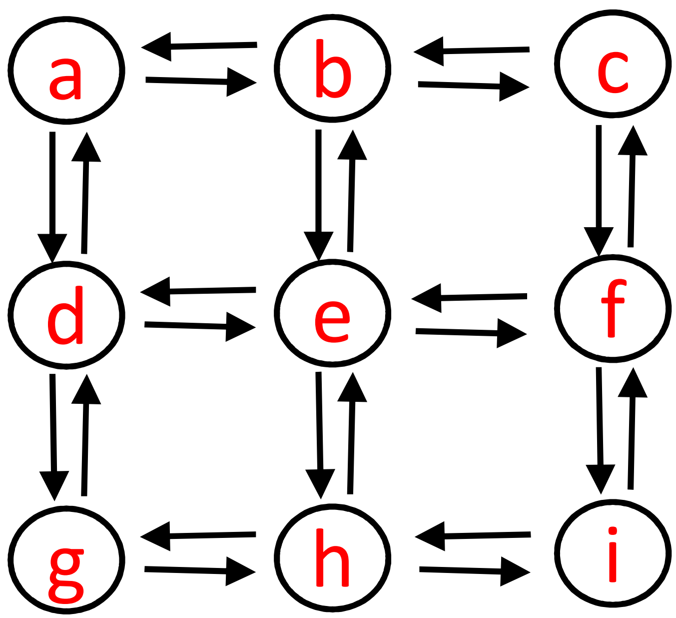


Graph Connectivity

Unlike tree traversal for rooted tree, a graph traversal started from some arbitrary vertex does not necessarily reach all other vertices.

Knowing which vertices can be reached by a path from some starting vertex is itself an important problem. You will learn about such ***graph `connectivity'*** *problems in COMP 251.*

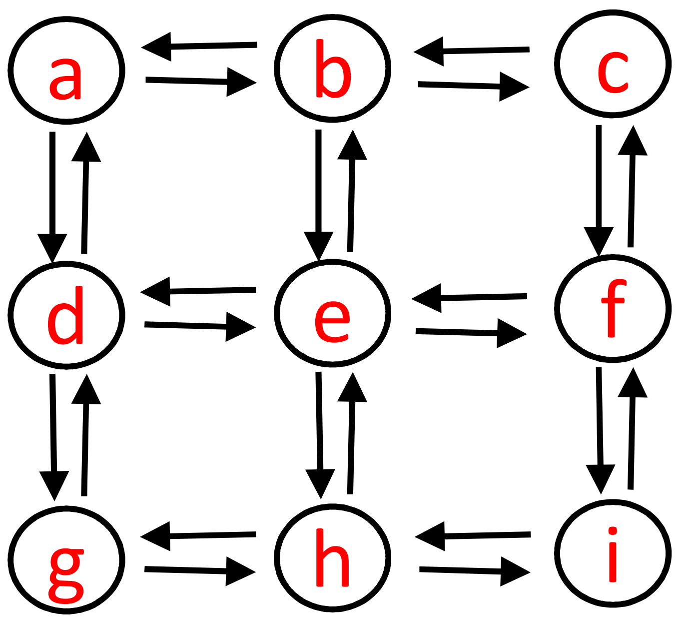
Example 2



Adjacency List

a - (b,d)
b - (a,c,e)
c - (b,f)
d - (a,e,g)
e - (b,d,f,h)
f - (c,e,i)
g - (d,h)
h - (e,g,i)
i - (f,h)

Example 2

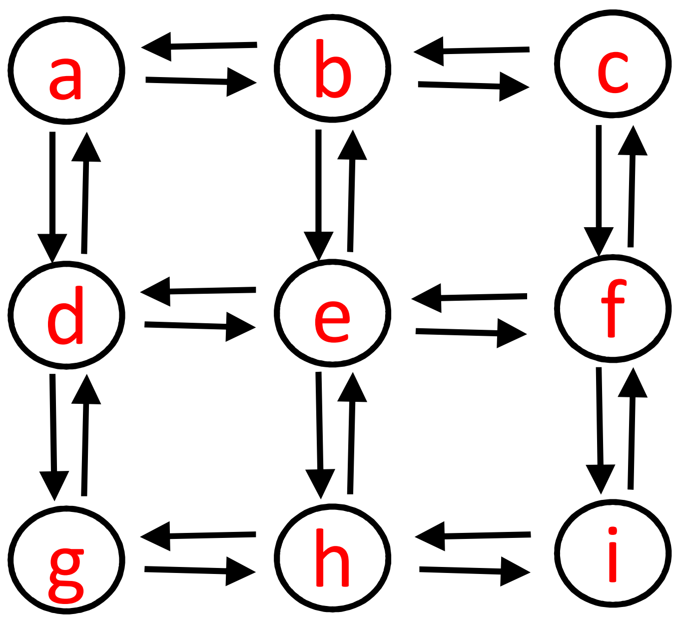


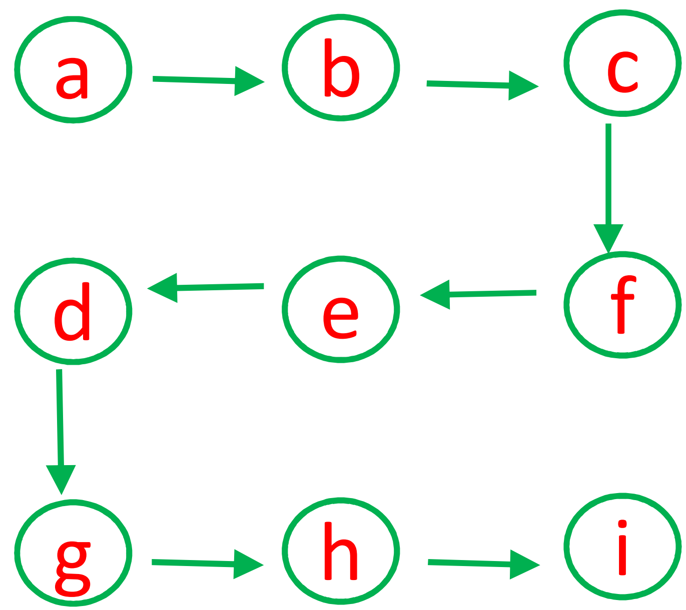
What is the call tree
for depthFirst_Graph(a) *?*

(Do it in your head.)

Example 2

call tree for depthFirst_Graph (a)





Heads up -- Initialization

```
depthfirstWithReset(v){
    for each vertex w in graph              //  reset vertices
        w.visited = false
    depthfirst_Graph (v){
}

depthfirst_Graph(v){    //  helper method
    v.visited = true
    for each w such that (v,w) is in E
        if  ! (w.visited)
            depthfirst_Graph(w)
}
```

Heads up – Initialization (Java)

```
class Graph<T> {
    HashMap< String, Vertex<T> >   vertexMap;

   class Vertex<T>  {
        ArrayList<Vertex>     adjList;
        T                     element;
        boolean               visited;
    }

    void resetVisited() {
         for  ( Vertex<T>   v  :    vertexMap.values() ){
              v.visited = false;
     }

    //    Implementation of pseudocode on previous slide
}
```

ASIDE: Graph Traversal Example A3 part 2

Recursive depth first graph traversal and visiting can have many forms, e.g.

```
solveMazeUtil(char maze[][], boolean found, int x, int y) {

     :
    if (solveMazeUtil(maze, found, x + 1, y)) {
        return true;
    } else if (solveMazeUtil(maze, found, x - 1, y)) {
        return true;
    } else if (solveMazeUtil(maze, found, x, y + 1)) {
        return true;
    } else if (solveMazeUtil(maze, found, x, y - 1)) {
        return true;
    } else {  // backtrack
          :
}
```

Today

- Recursive graph traversal
  - depth first

- Non-recursive graph traversal
  - depth first (using stack)
  - breadth first (using queue)

Recall: depth first tree traversal
(non-recursive, using stack)

```
treeTraversalUsingStack(root){
    initialize empty stack s
    s.push(root)
    while  s is not empty {
        cur = s.pop()
        visit cur
        for each child of cur
            s.push(child)
    }
}
```

Slight variation on depth first <u>tree</u> traversal (using stack)

```
treeTraversalUsingStack(root){
    visit root                  //  visit before push
    initialize empty stack s
    s.push(root)
    while  s is not empty {
       cur = s.pop()
       for each child of cur
              visit child       //  visit at 'same time' as push
              s.push(child)
    }
}
```

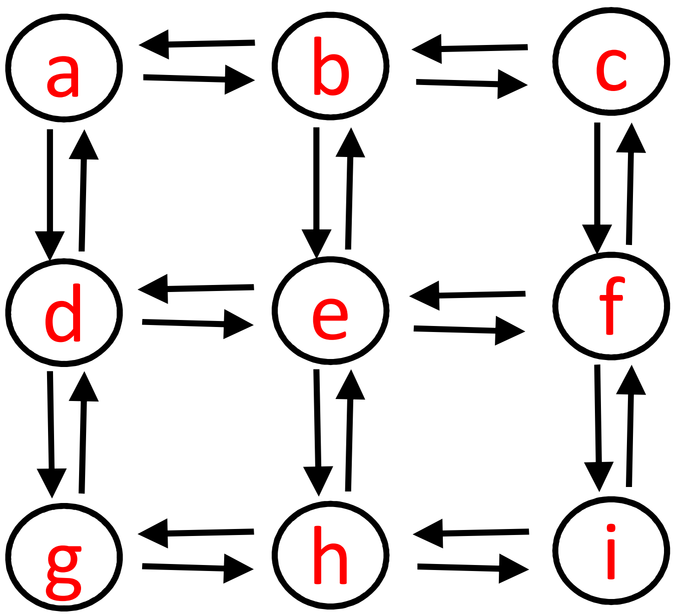
We are still visiting each node before its children (but visit order is different).

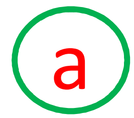
Depth first graph traversal (using stack)

```
graphTraversalUsingStack( v ){
   visit  v      //  this can be done after push below
   initialize empty stack s
   s.push(v)
   while (s is not empty) {
      u =  s.pop()
      for each w in u.adjList{          //  new part
         if (!w.visited){
            visit w             //  these two instruction can be done
            s.push(w)           //  in either order   ('same time')
         }
      }
   }
}
```

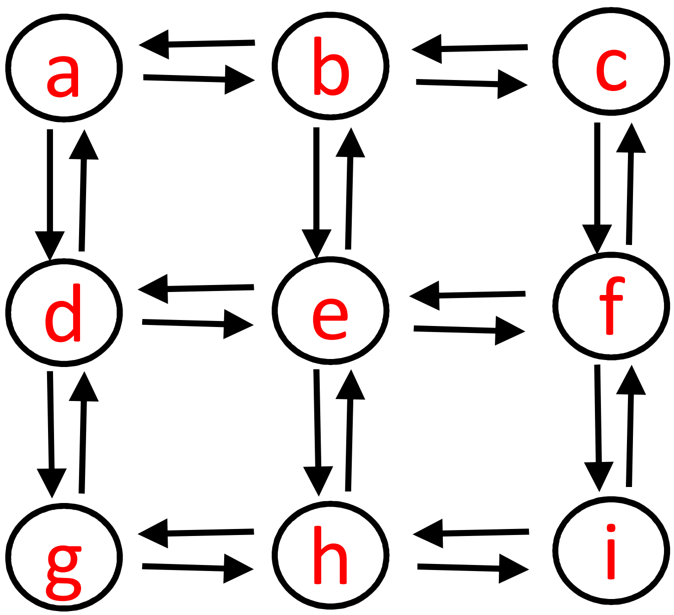
Updated after lecture:
see Exercises 12 (graphs) Question 6.

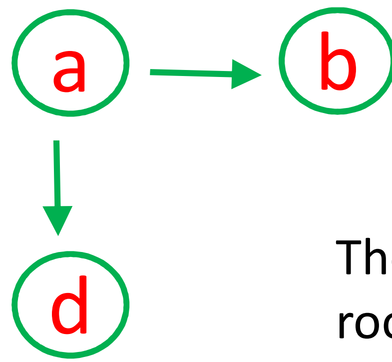
Example: graphTraversalUsingStack(a)





Example: graphTraversalUsingStack(a)

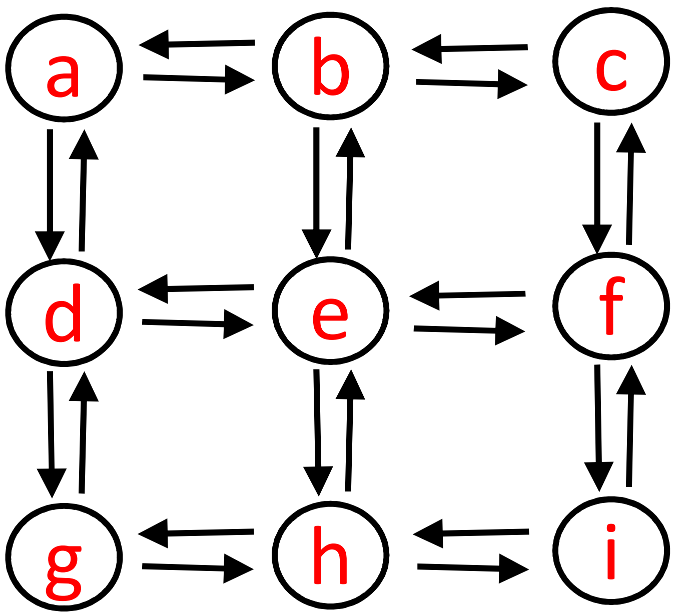


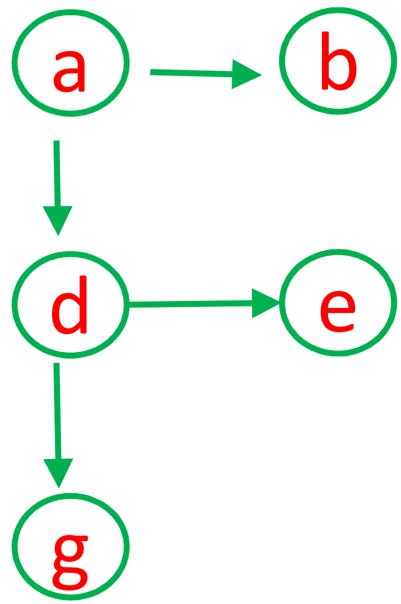


The traversal defines a rooted tree, but it is not a "call tree".
(The algorithm is not recursive.)

d
a b 'a' was popped. 'b' and 'd' were pushed.

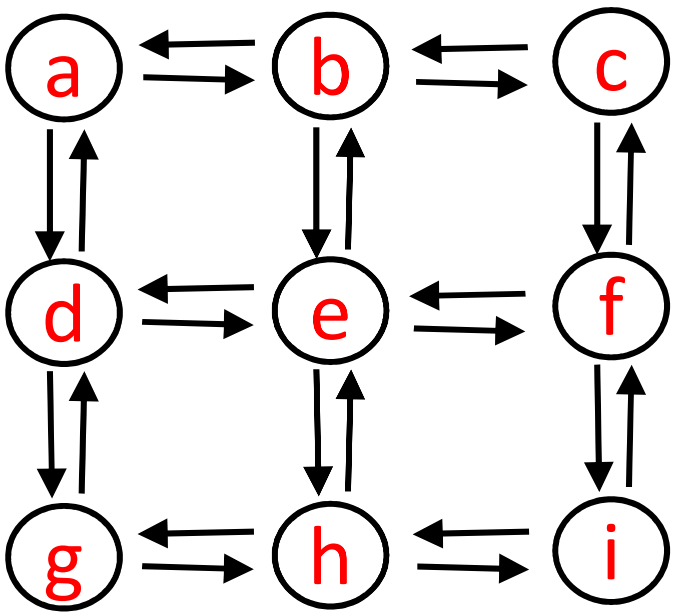
Example: graphTraversalUsingStack(a)

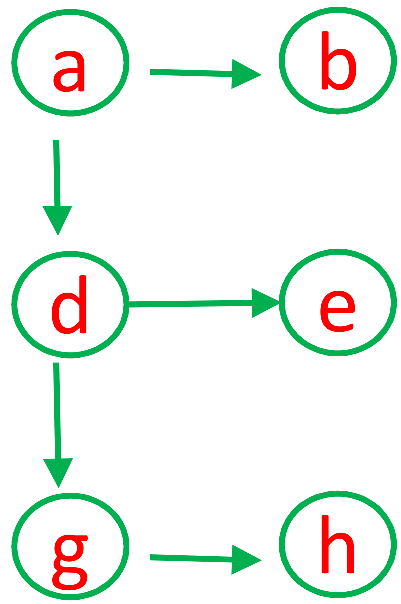




'd' was popped. 'e' and 'g' were pushed.

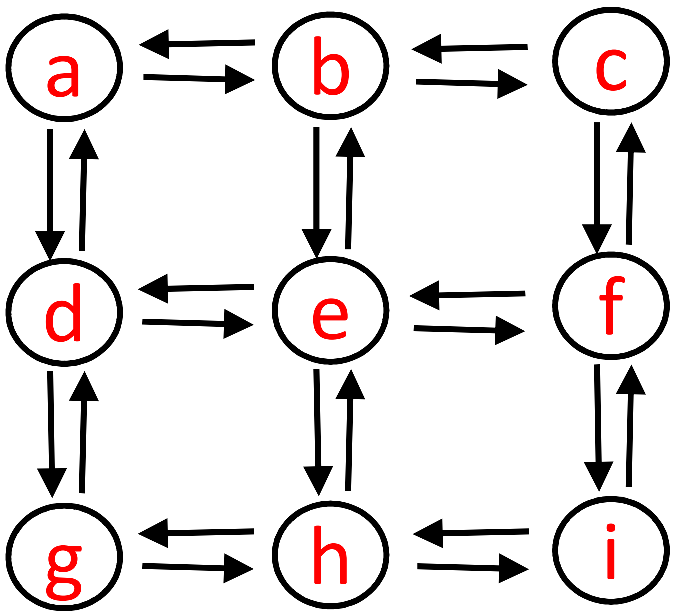
Example: graphTraversalUsingStack(a)

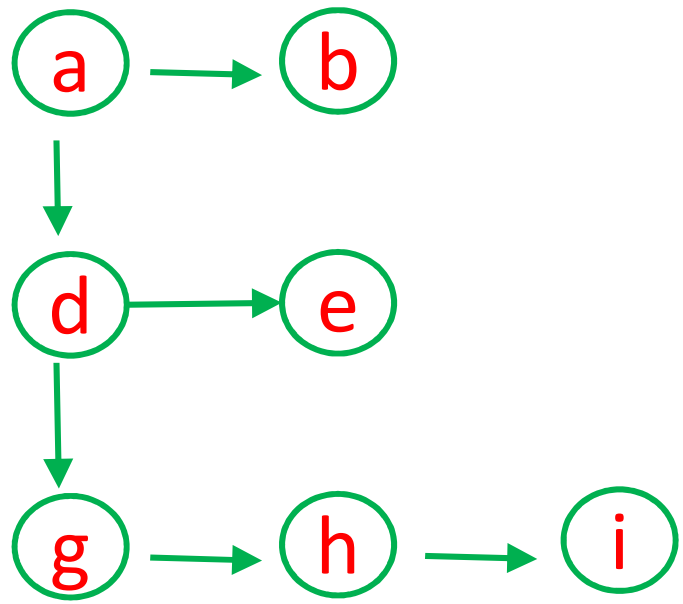




‘g’ was popped. ‘h’ was pushed.

Example: graphTraversalUsingStack(a)

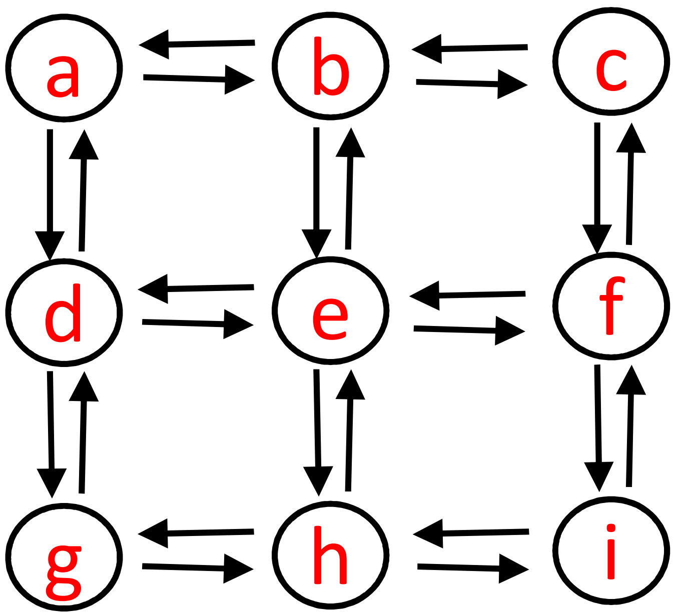


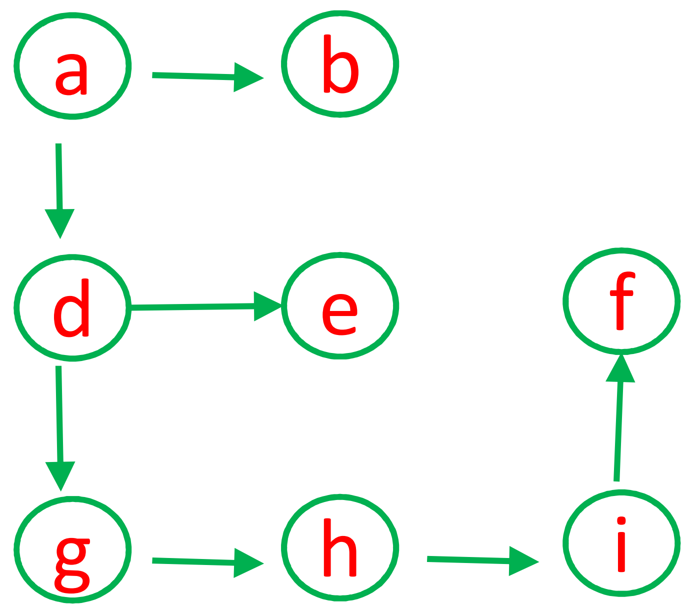


		g	h	i
	d	e	e	e
a	b	b	b	b

‘h’ was popped. ‘i’ was pushed.

Example: graphTraversalUsingStack(a)

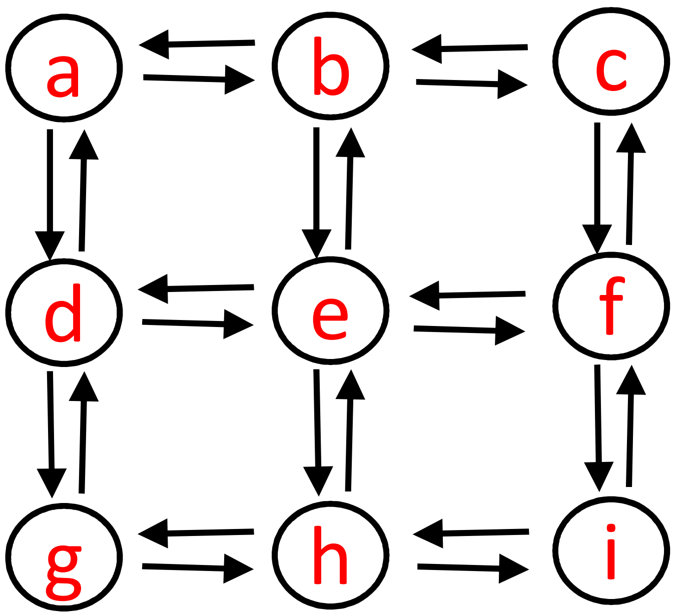


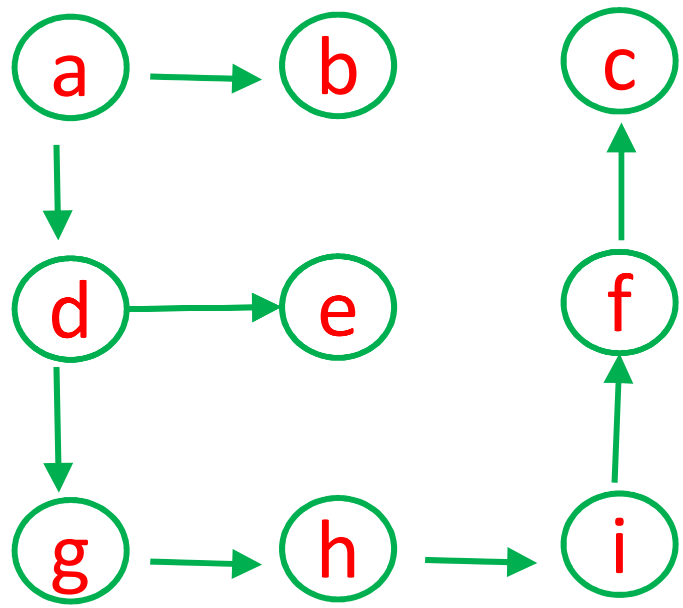


		g	h	i	f
	d	e	e	e	e
a	b	b	b	b	b

‘i’ was popped. ‘f’ was pushed.

Example: graphTraversalUsingStack(a)

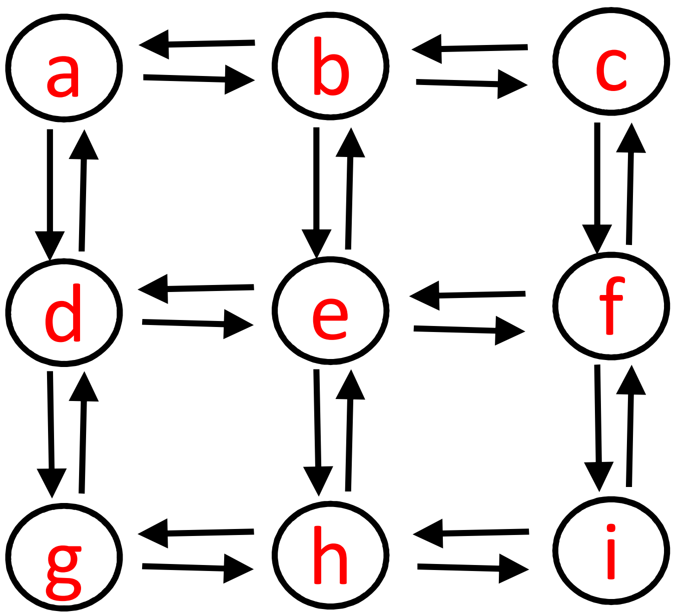


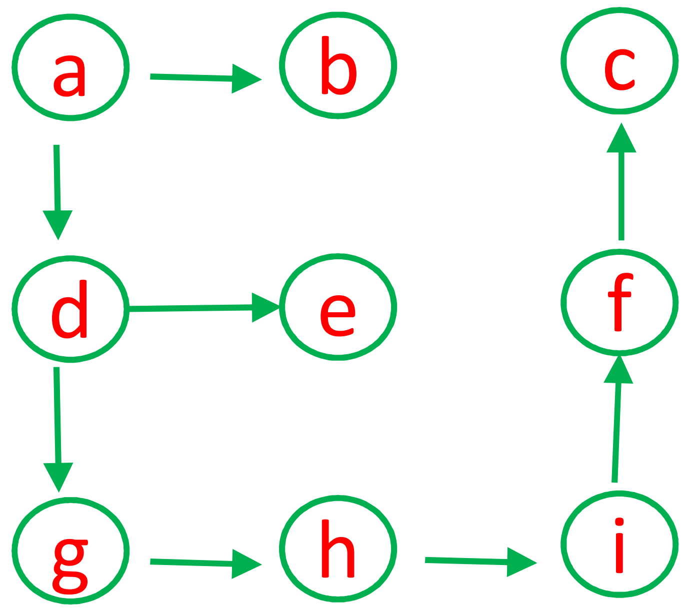


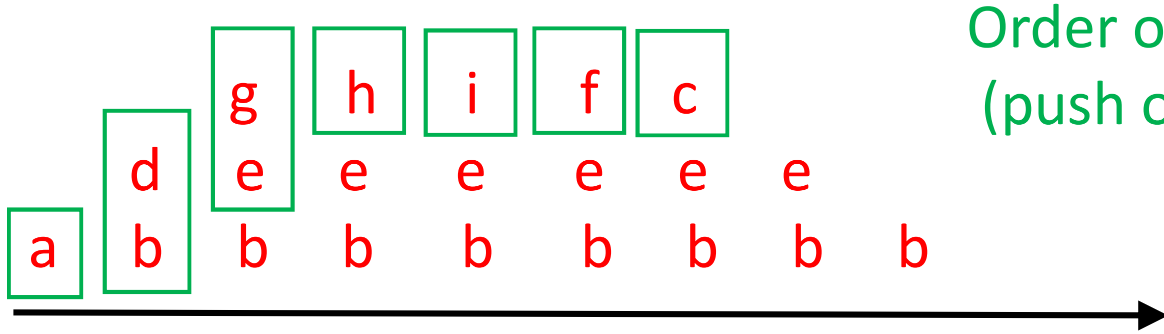
		g	h	i	f	c
	d	e	e	e	e	e
a	b	b	b	b	b	b

'f' was popped. 'c' was pushed.

Example: graphTraversalUsingStack(a)



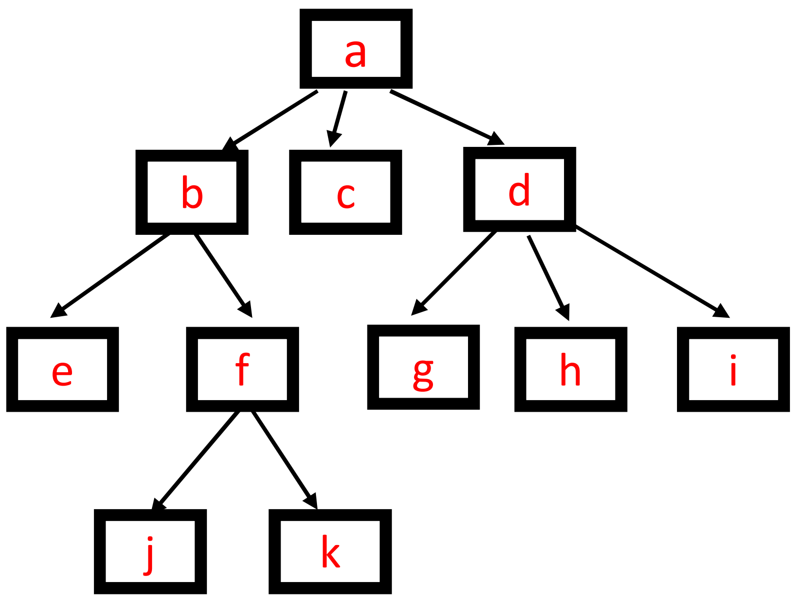




Order of nodes visited
(push order) : abdeghifc

Recall: breadth first tree traversal

for each level i
    visit all nodes at level i




```
treeTraversalUsingQueue(root){
    initialize empty queue  q
    q.enqueue(root)
    while q is not empty {
       cur = q.dequeue()
       visit cur
       for each child of cur
            q.enqueue(child)
     }
}

//  visit after dequeue
```

Breadth first graph traversal

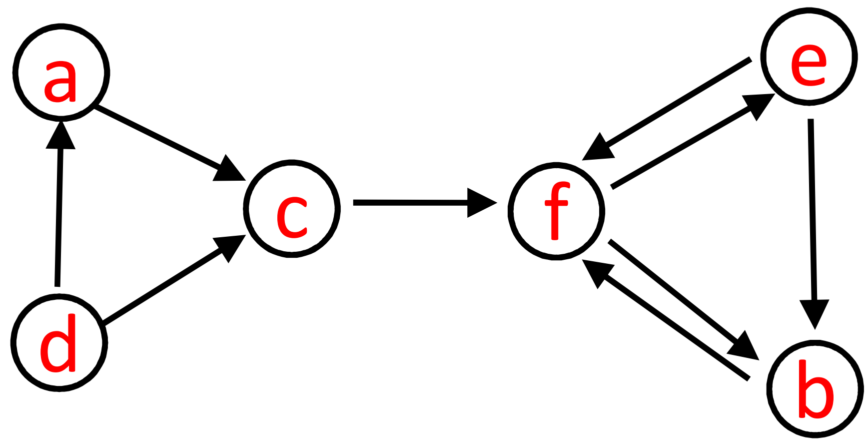
```
graphTraversalUsingQueue(v){
   visit v
   initialize empty queue q
   q.enqueue(v)
   while (q is not empty) {
      u =  q.dequeue()
      for each w in u.adjList{
         if (!w.visited){
            visit  w              //  visit at 'same time' as enqueue
            q.enqueue(w)
         }
      }
   }
}
```

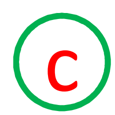
Example

graphTraversalUsingQueue(c)

<u>queue</u>

c





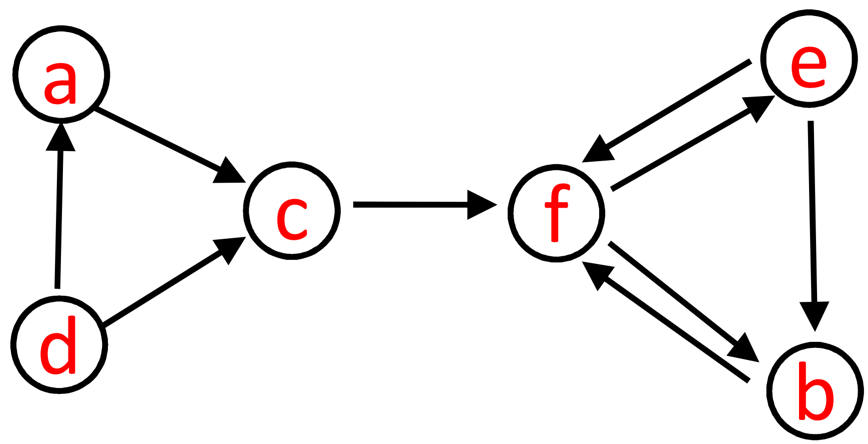
Example

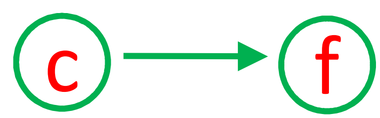
graphTraversalUsingQueue(c)

<u>queue</u>

c

f





Example

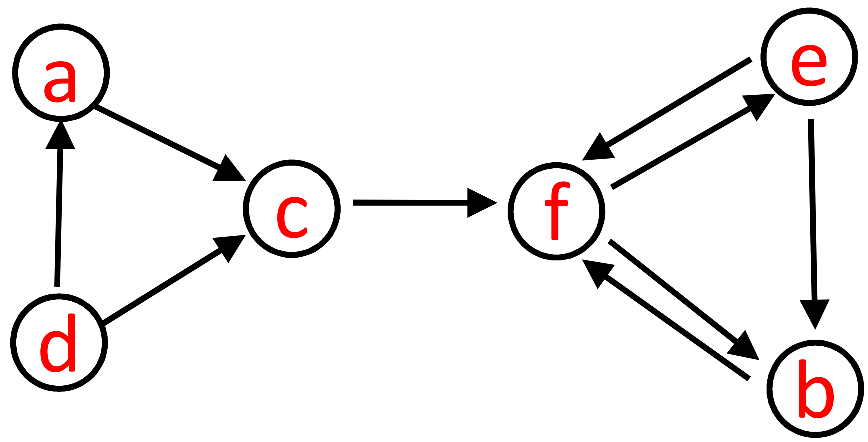
graphTraversalUsingQueue(c)

<u>queue</u>

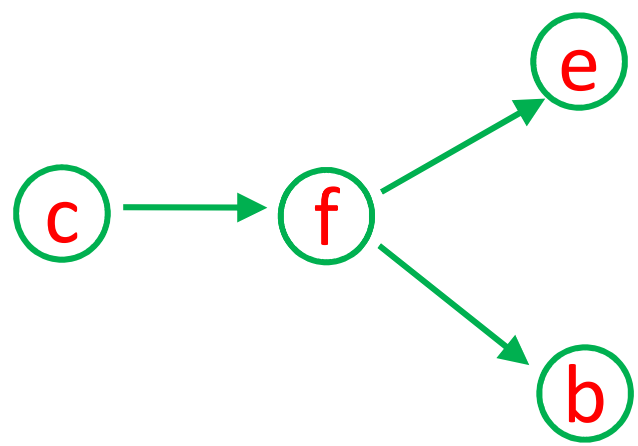
c

f

be



Using alphabetical
order for adjacency list.



Example

graphTraversalUsingQueue(c)

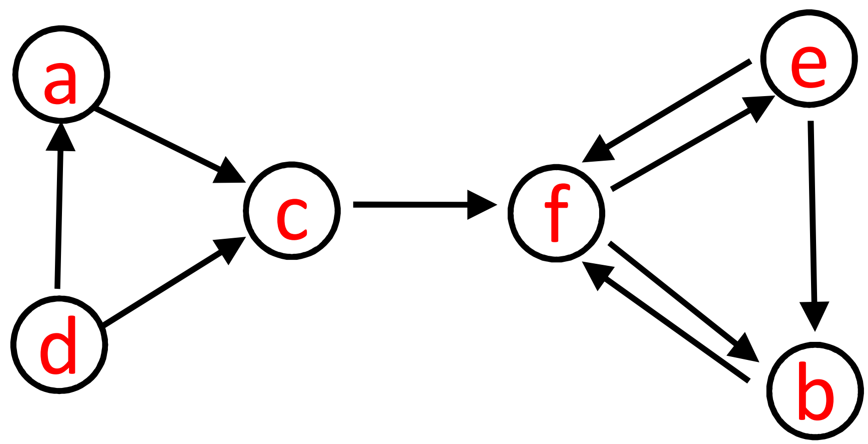
<u>queue</u>

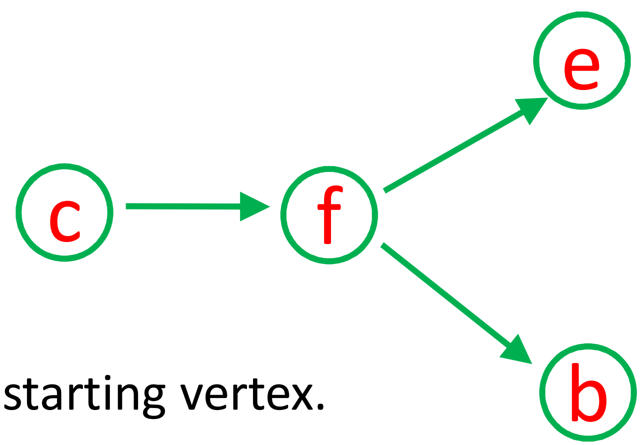
c

f

be

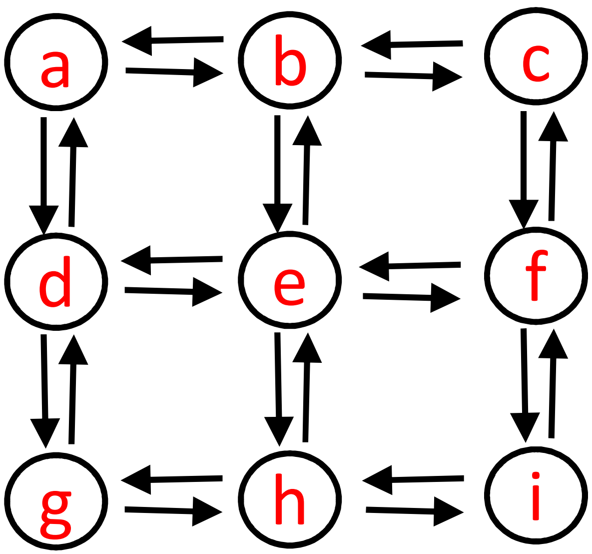
e

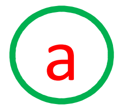


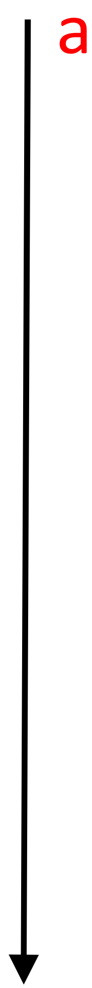


Traversal defines a tree whose root is the starting vertex.

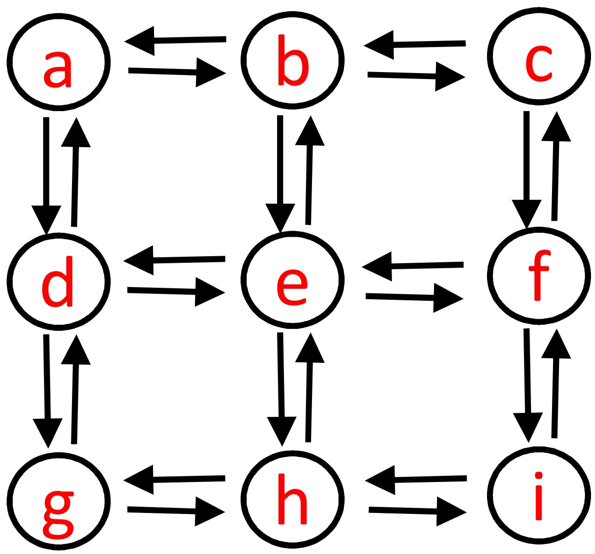
Example: graphTraversalUsingQueue(a)

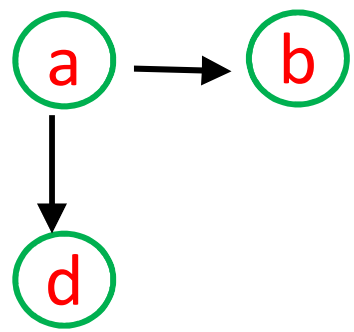


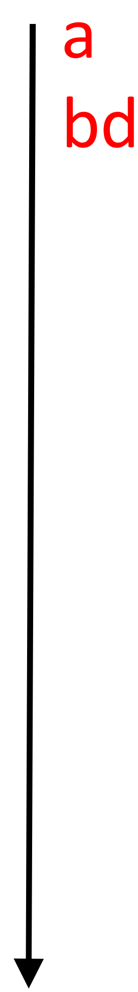




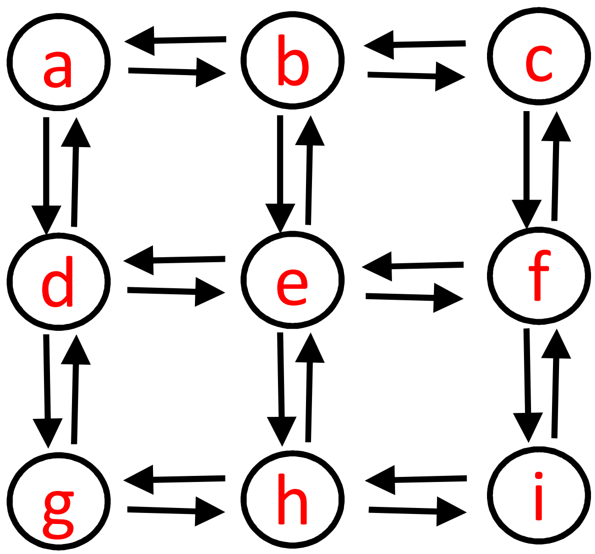
Example: graphTraversalUsingQueue(a)

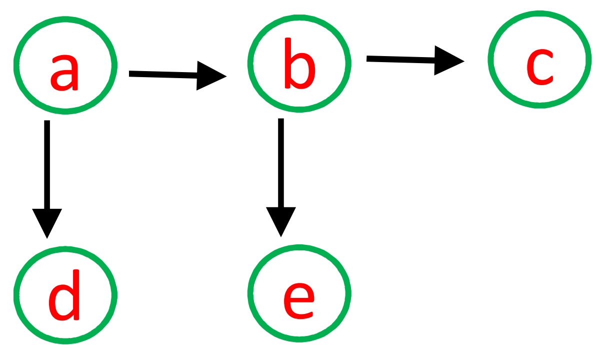


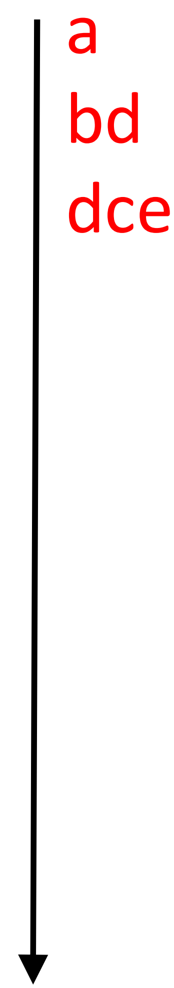




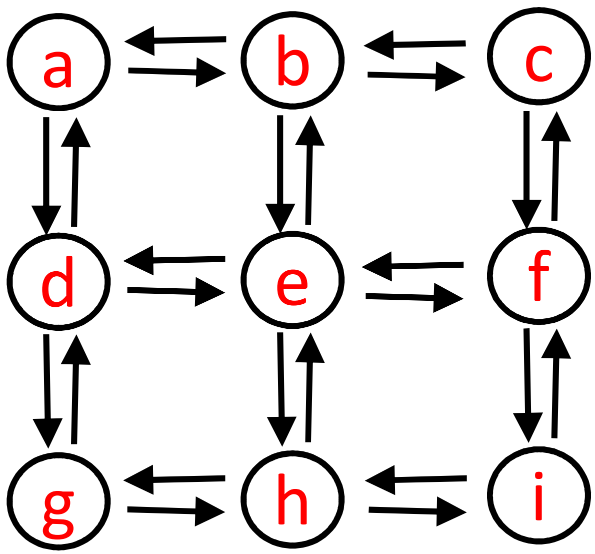
Example: graphTraversalUsingQueue(a)

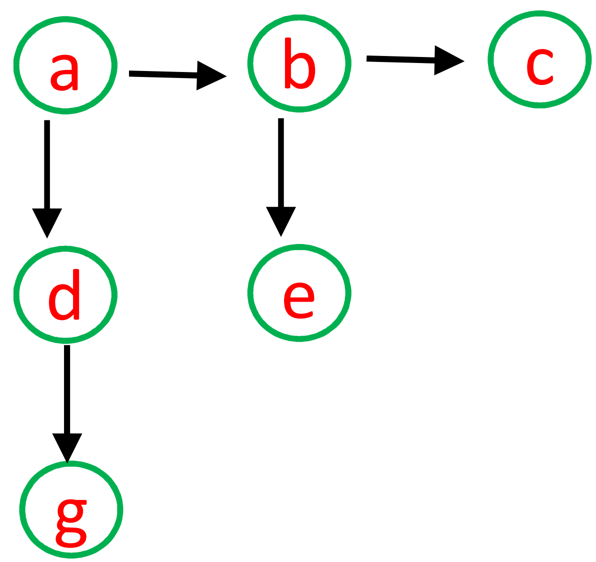






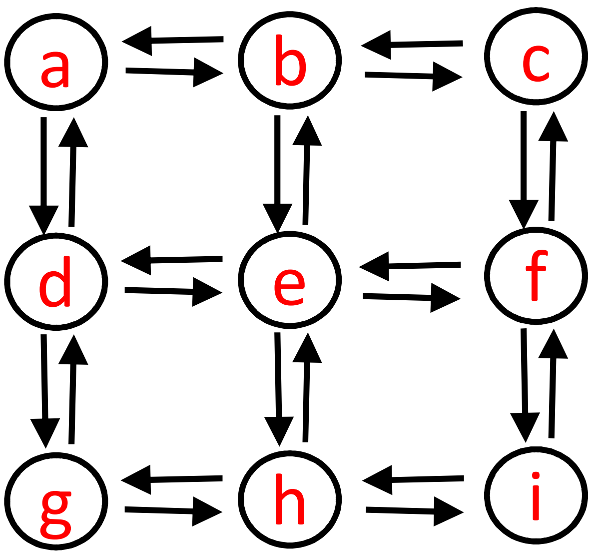
Example: graphTraversalUsingQueue(a)

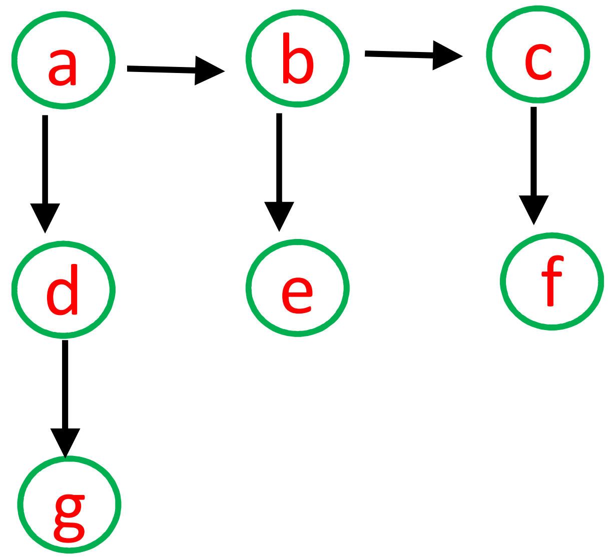




a
bd
dce
ceg

Example: graphTraversalUsingQueue(a)





a
bd
dce
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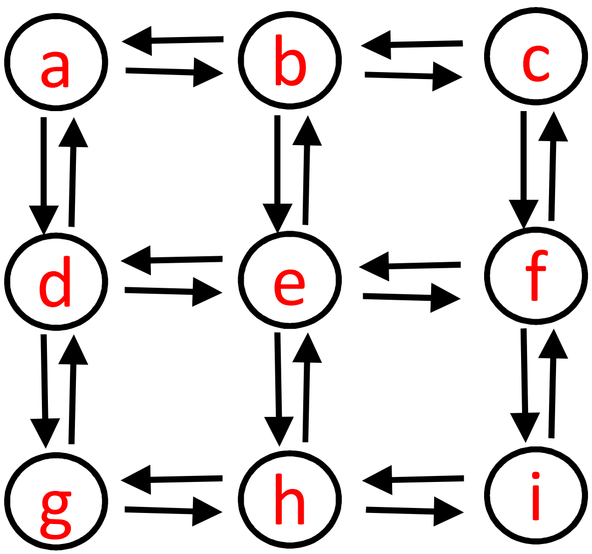
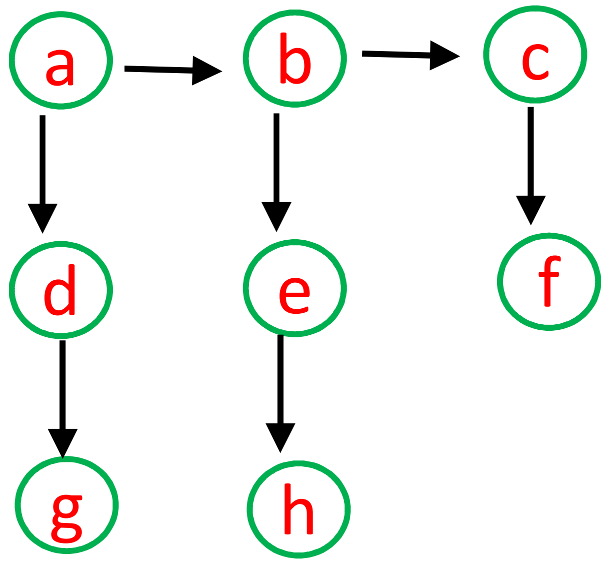
egf

Example: graphTraversalUsingQueue(a)





a
bd
dce
ceg
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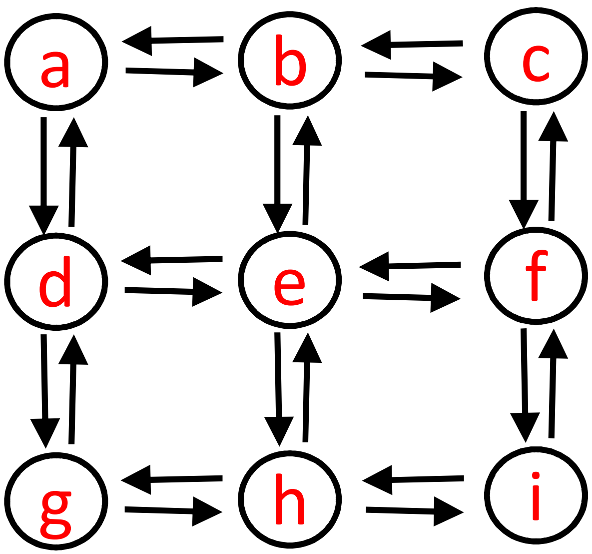
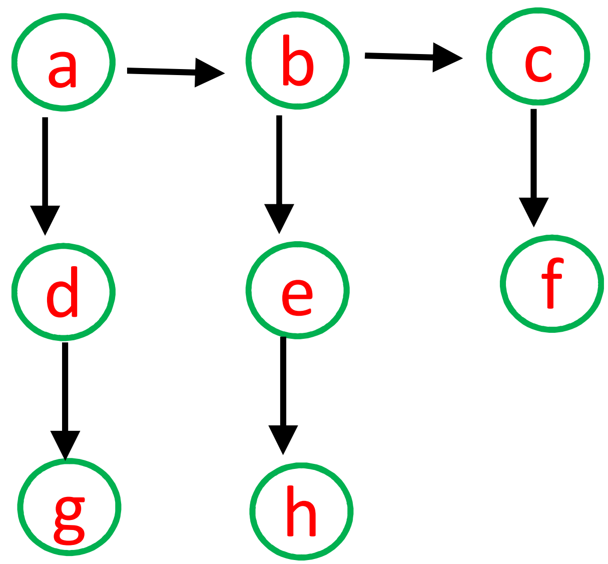
gfh

Example: graphTraversalUsingQueue(a)





a
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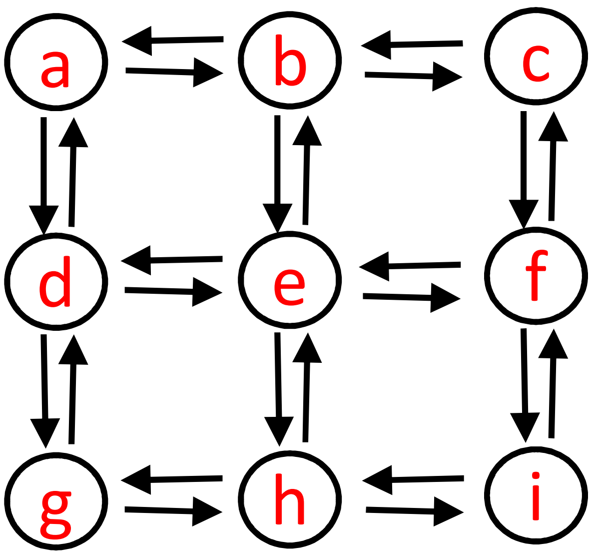
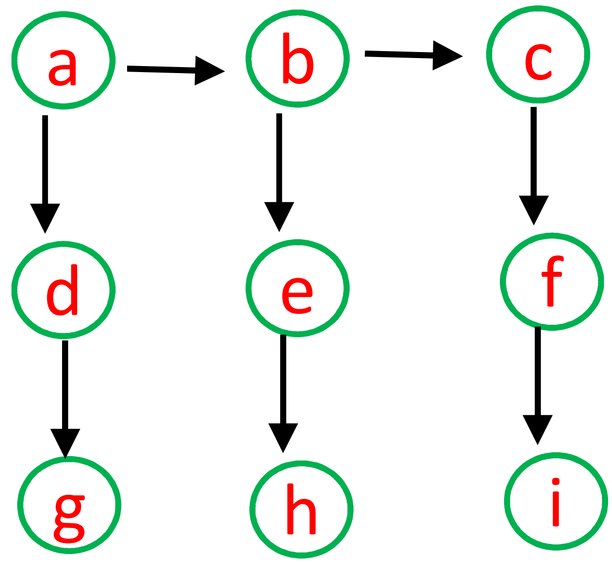
gfh
fh

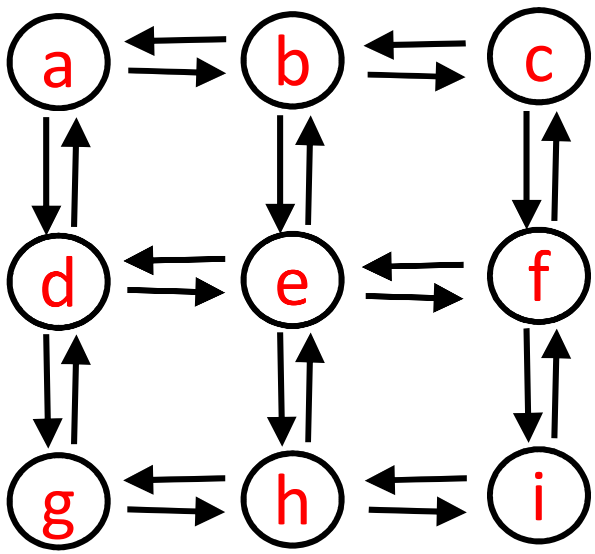
Example: graphTraversalUsingQueue(a)

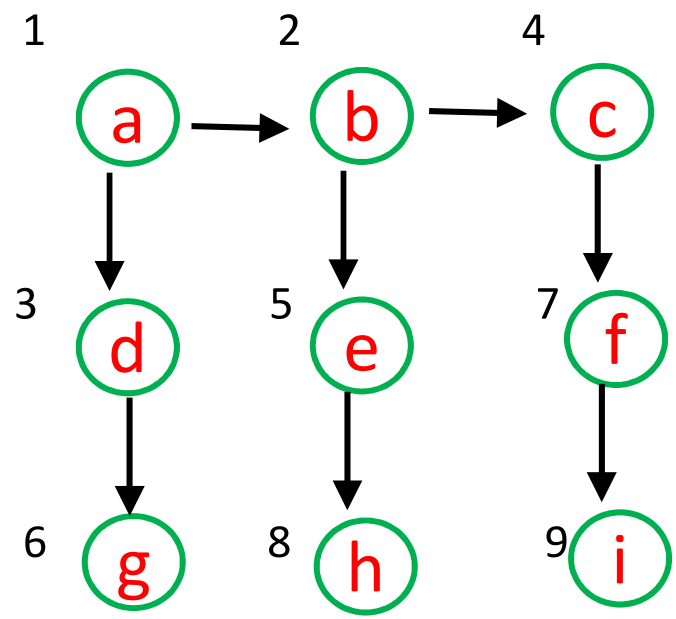


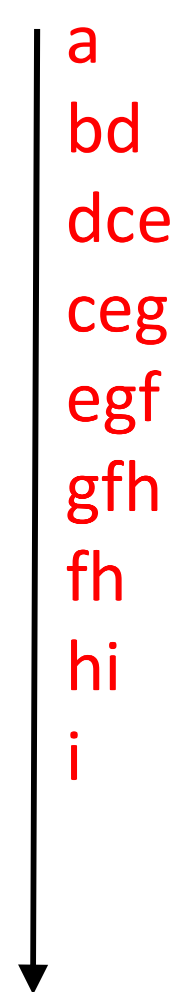


a
bd
dce
ceg
egf
gfh
fh
hi

Example: graphTraversalUsingQueue(a)







Note order of nodes visited: abdcegfhi.

Traversal defines a tree whose root is the starting vertex.

One can show in general that this traversal first reaches nodes whose shortest path is length 0, then 1, then 2, etc. i.e. *breadth first.*

Coming up...

Lectures

Wed & Fri,   March 30 & April 1

recurrences

Mon, Wed, Fri :   April 4, 6, 8

big O, ...

Assessments

Quiz 5 is in Mon. April 4

Assignment 4 due Wed. April 6.