

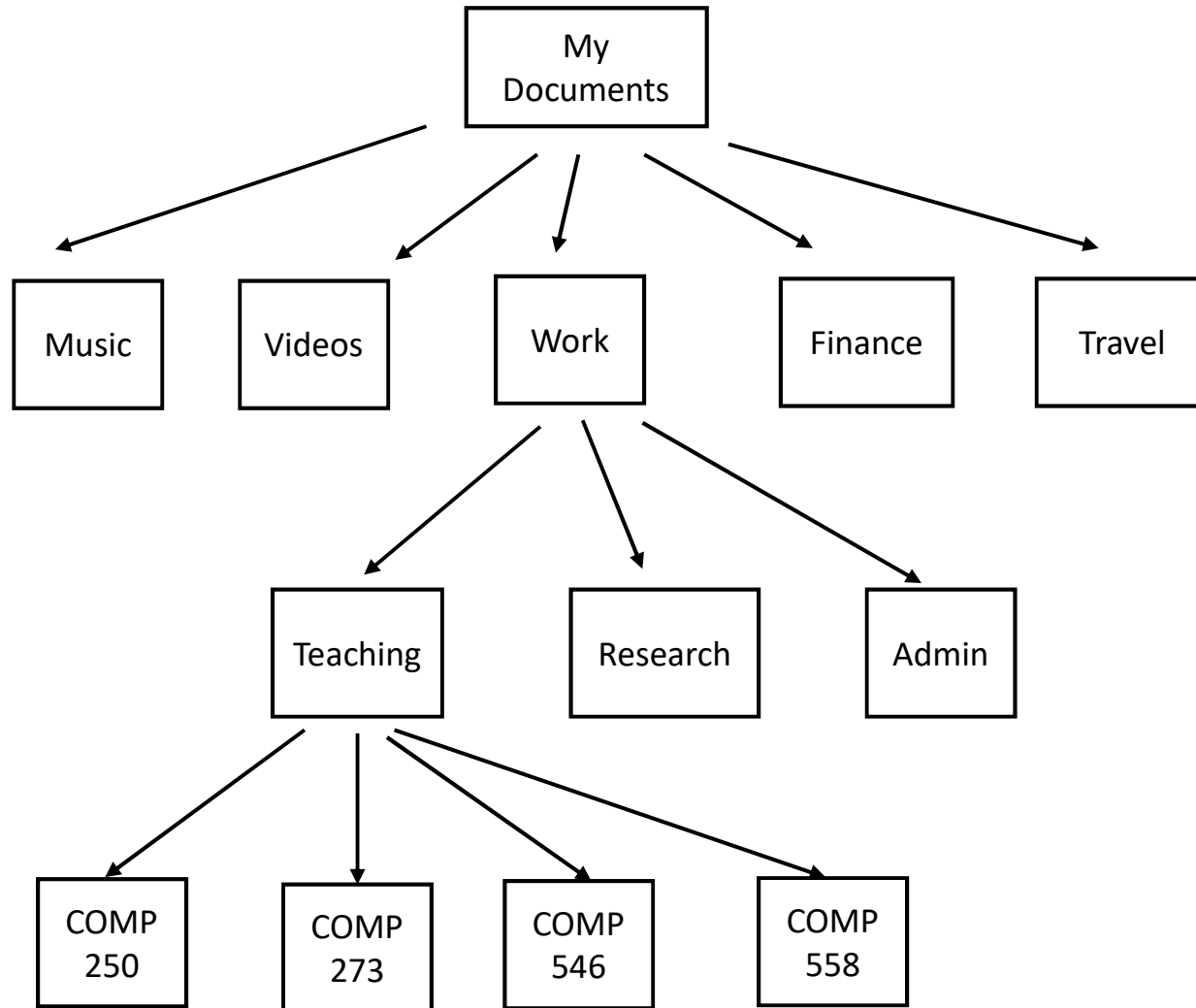
COMP 250

Lecture 24

tree traversal

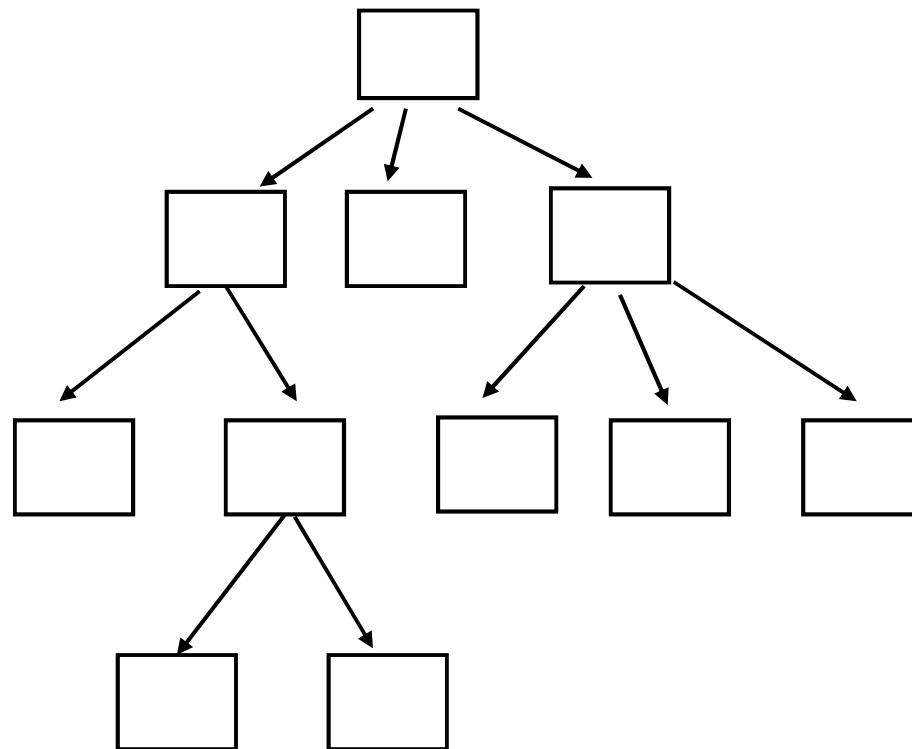
March 9, 2022

# Tree: Example



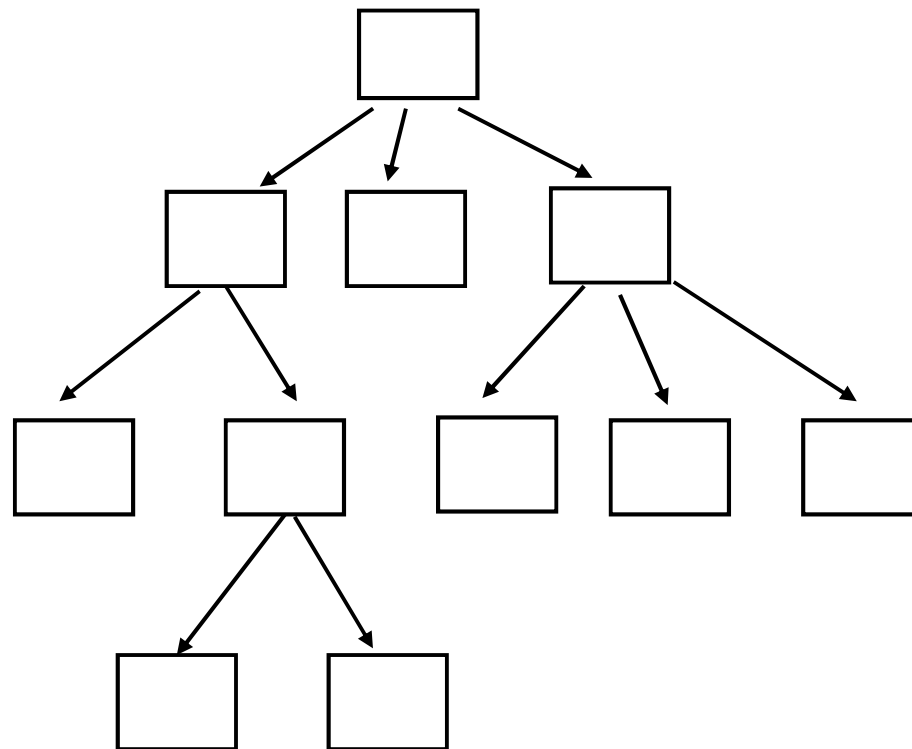
# Tree Traversal

How to “visit” / “traverse” / “iterate through” the nodes in a tree ?



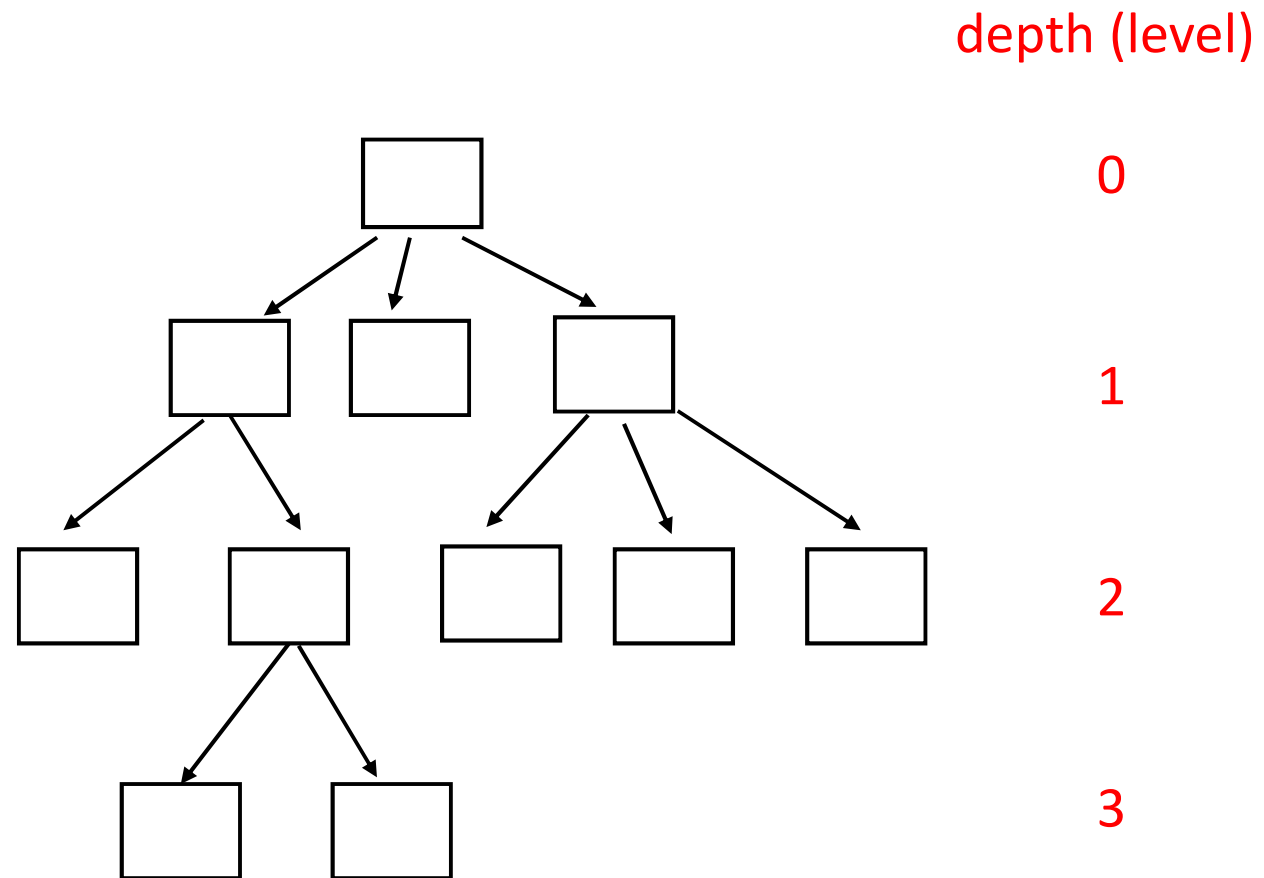
# Tree Traversal

How to “visit” / “traverse” / “iterate through” the nodes in a tree ?



# Recall – depth of a node

The **depth** or **level** of a node is the length of the path *from the root to the node*.



# Tree Traversal

- Depth first traversal (also called depth first *search*)

*Start from root and follow paths to the leaves, backtracking only when a leaf is found*

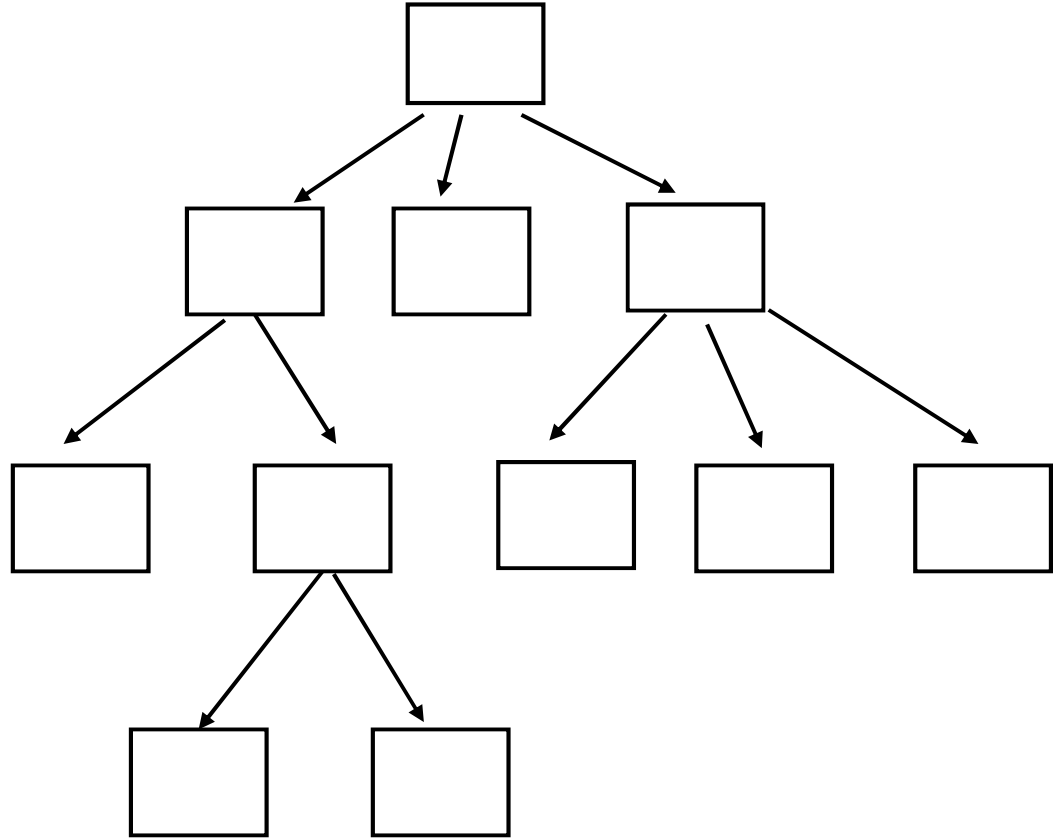
- Breadth first traversal (also called breadth first *search*)

*Start from root and visit all nodes at depth  $k$  before any nodes at depth  $k+1$ .*

“Visit” a node implies that you do something at that node.

We will see some examples later.

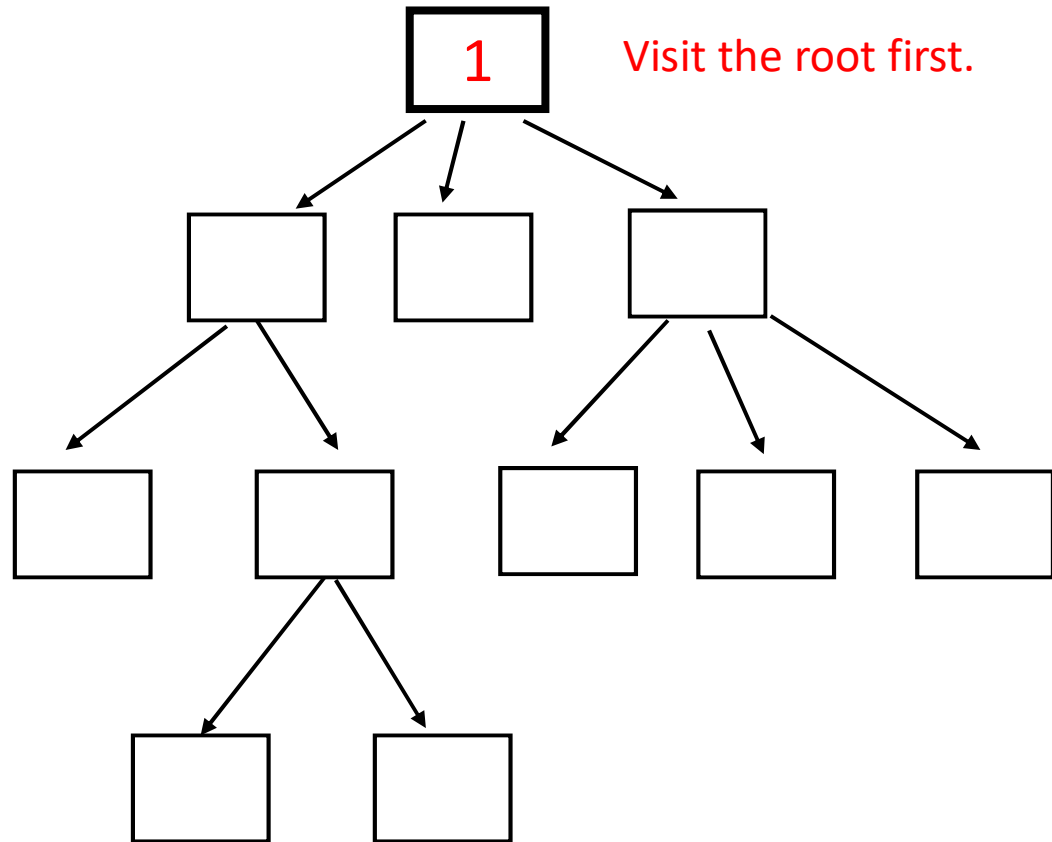
```
depthfirst (root){  
  visit root  
  for each child of root  
    depthfirst( child )  
}
```





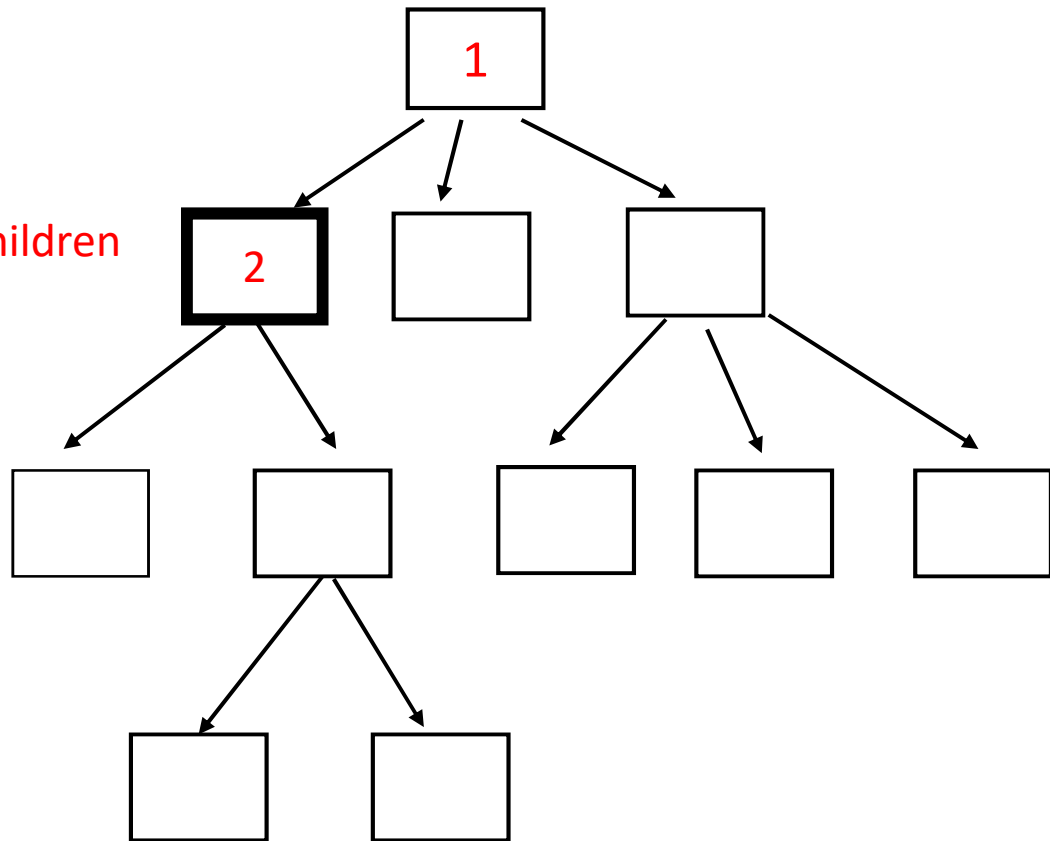
```
depthfirst (root){  
  visit root  
  for each child of root  
    depthfirst( child )  
}
```

“preorder” traversal:  
visit the root before  
the children

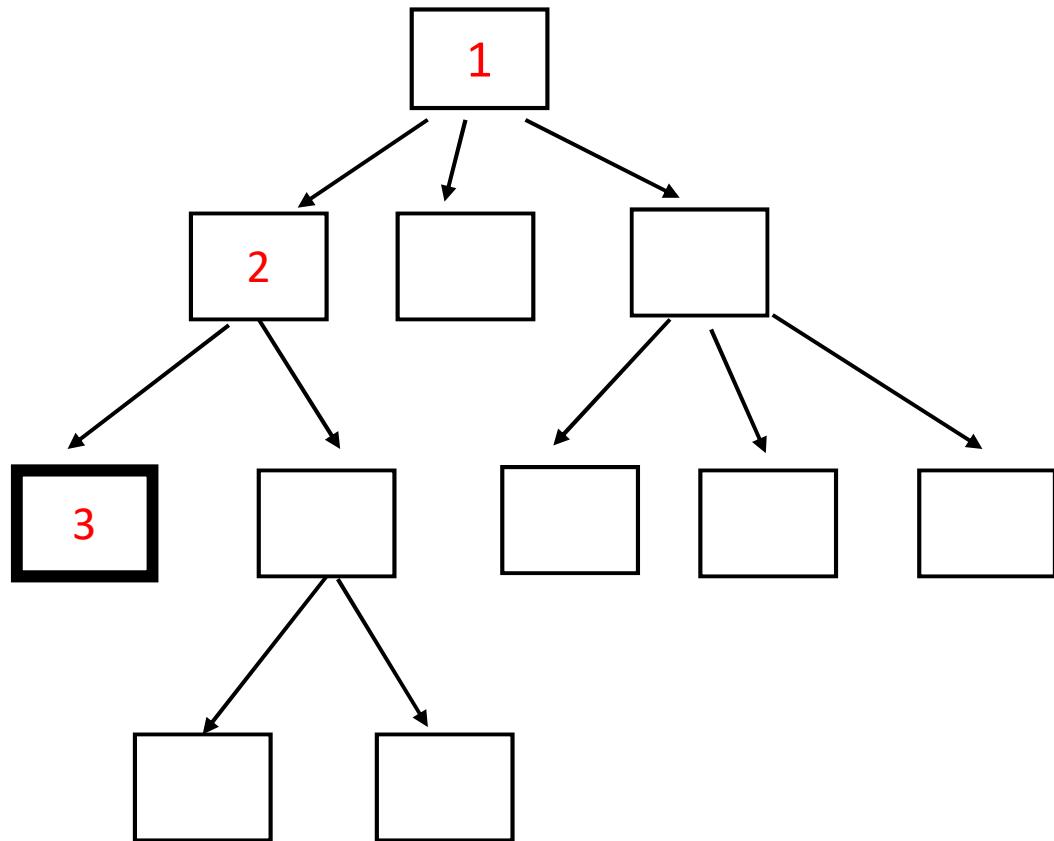


```
depthfirst (root){  
  visit root  
  for each child of root  
    depthfirst( child )  
}
```

Assume we visit children  
from left to right.

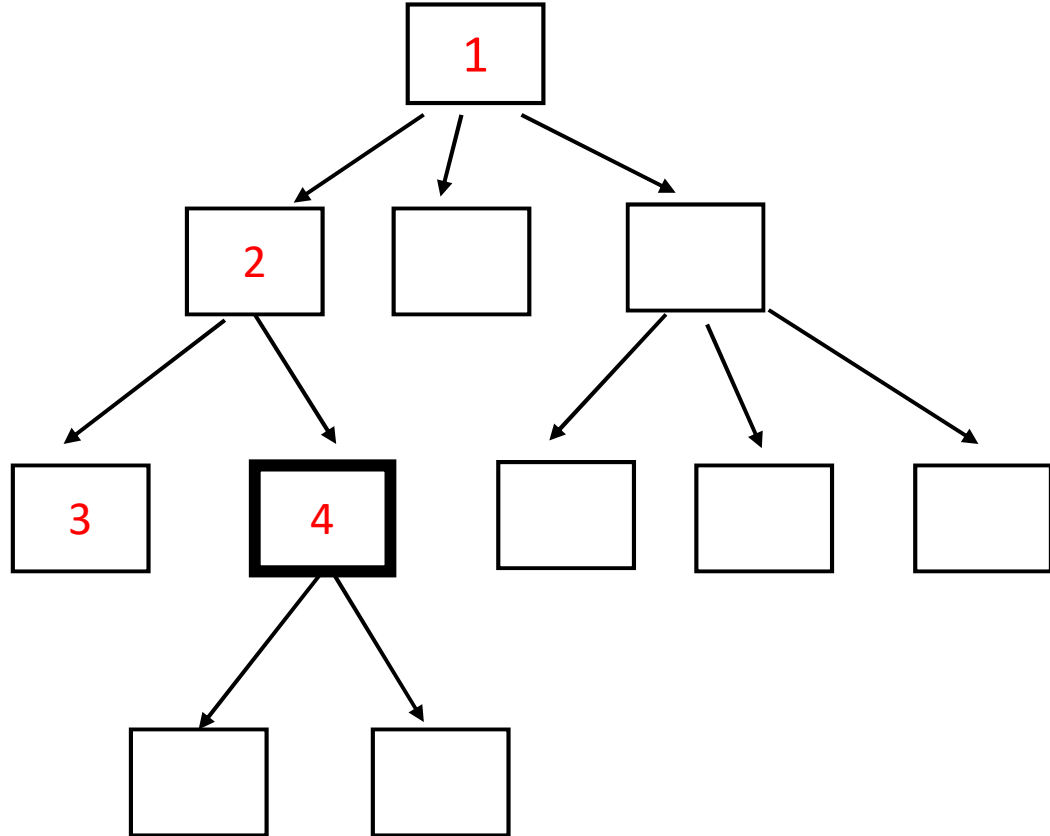


```
depthfirst (root){  
    visit root  
    for each child of root  
        depthfirst( child )  
}
```

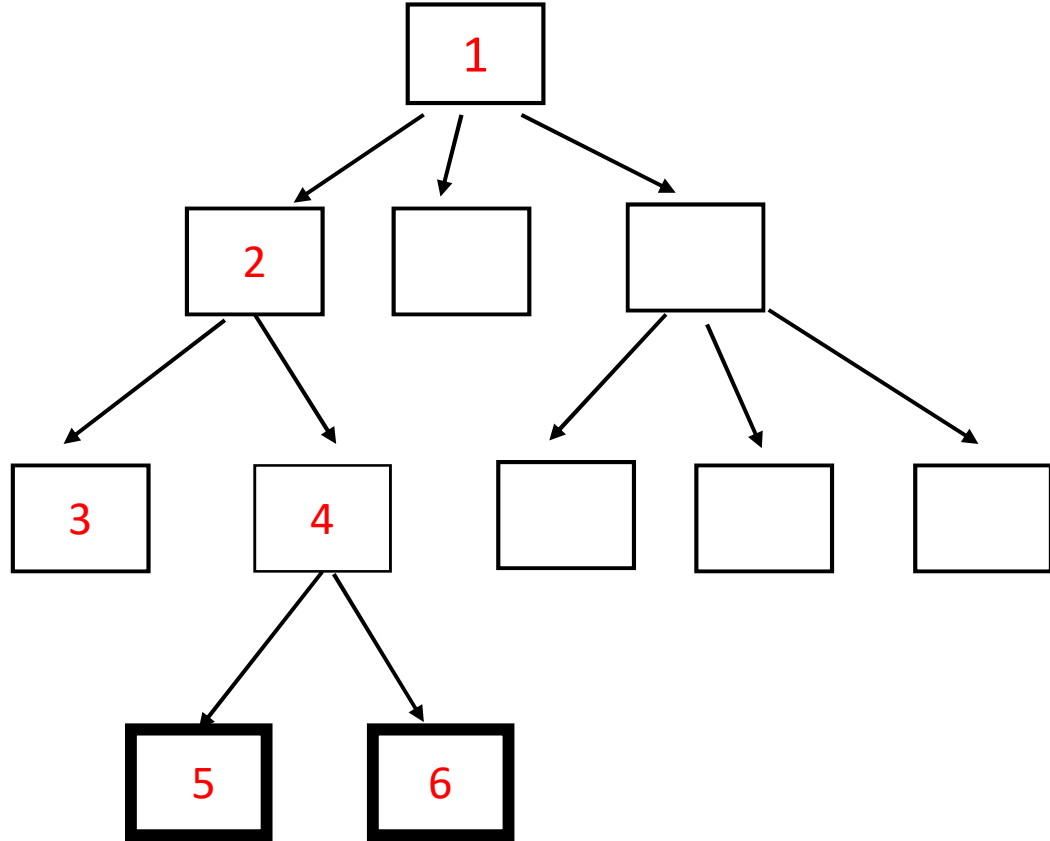


This node has no children.

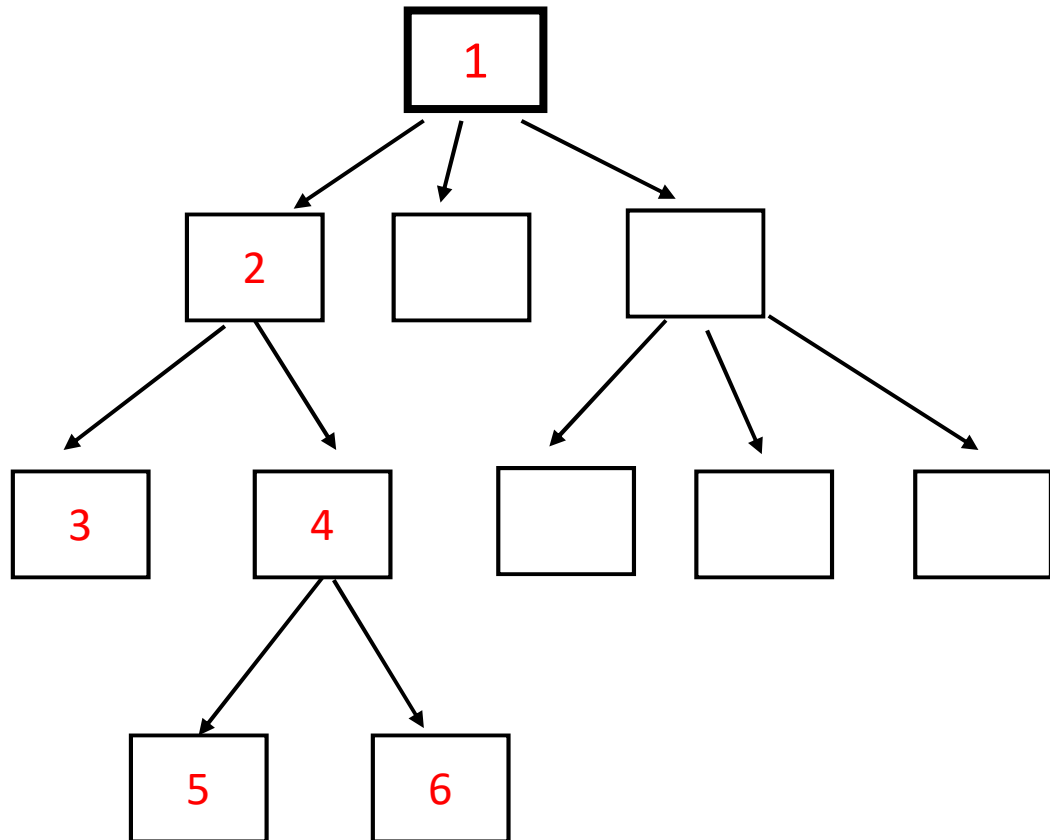
```
depthfirst (root){  
    visit root  
    for each child of root  
        depthfirst( child )  
}
```



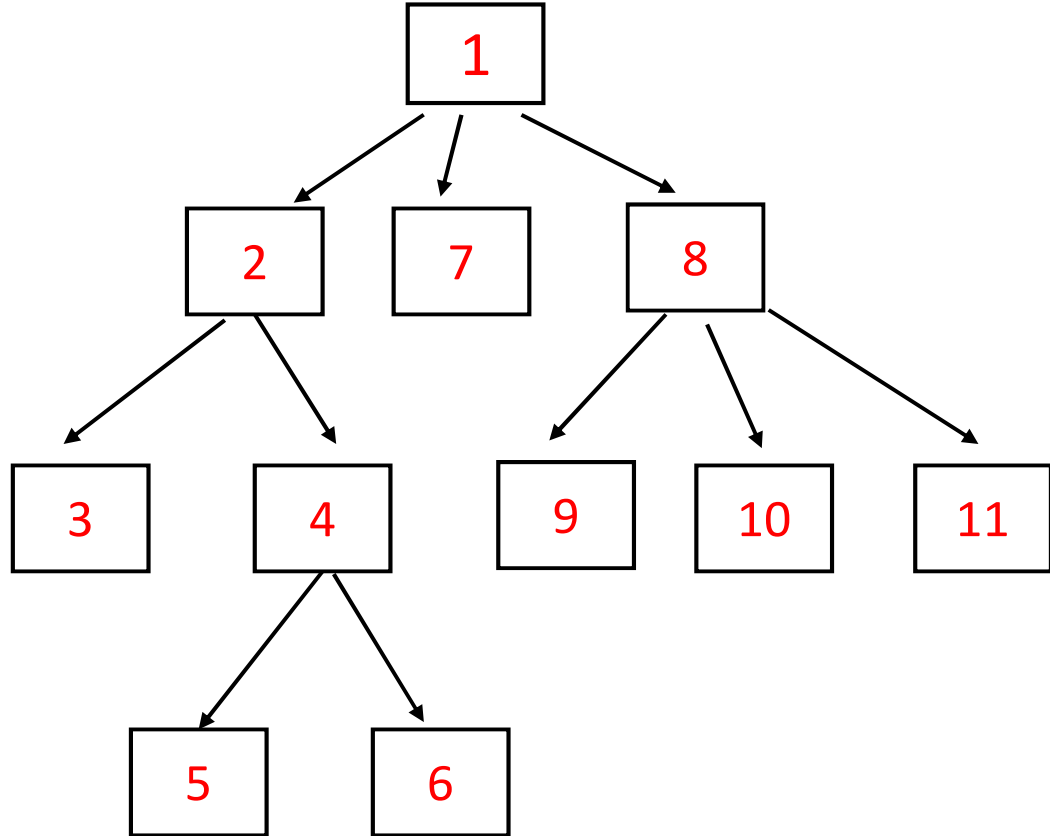
```
depthfirst (root){  
    visit root  
    for each child of root  
        depthfirst( child )  
}
```



```
depthfirst (root){  
    visit root  
    for each child of root  
        depthfirst( child )  
}
```



```
depthfirst (root){  
  visit root  
  for each child of root  
    depthfirst( child )  
}
```



# Implementation details

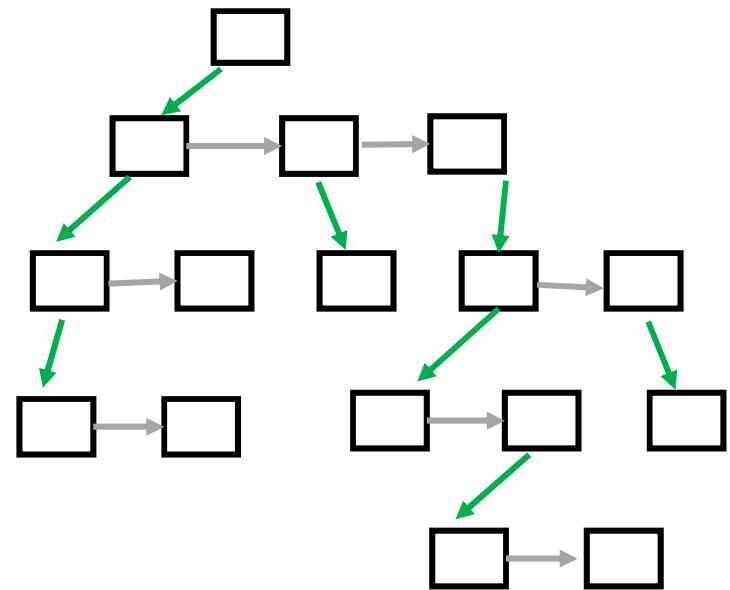
Recall the “**first child**, next sibling” implementation

Then when we write

```
for each child of root {  
    :  
}
```

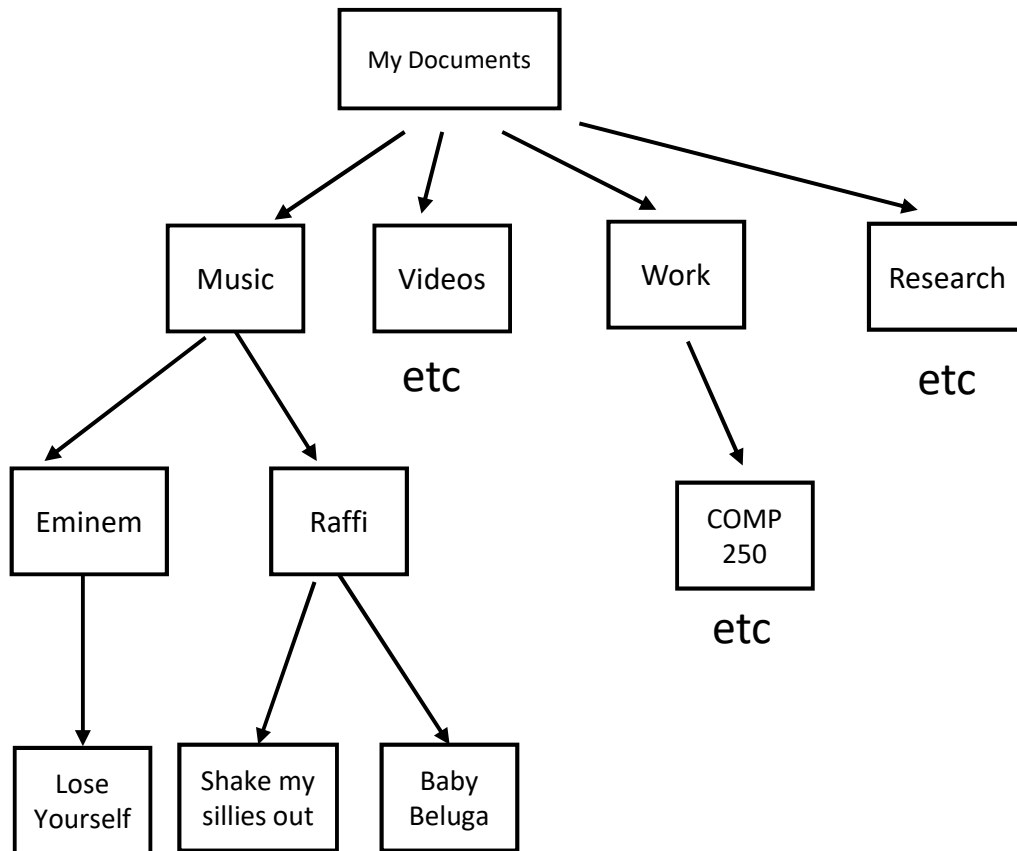
*we would mean:*

```
child = root.firstChild  
while (child != null) {  
    :  
    child = child.nextSibling  
}
```





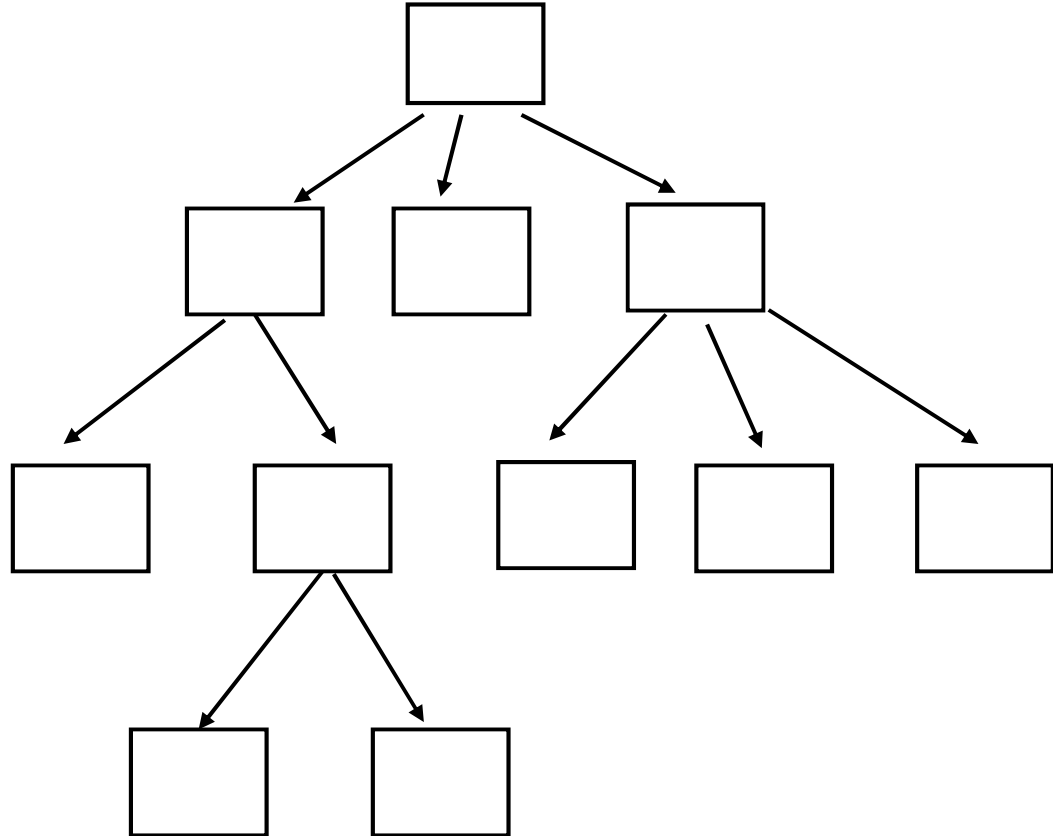
# Example of Preorder Traversal: printing a hierarchical file system (visit = print directory or file name)



```
My Documents      (directory)
Music             (directory)
  Eminem          (directory)
    Lose Yourself (file)
  Raffi           (directory)
    Shake My Sillies Out (file)
    Baby Beluga   (file)
Videos            (directory)
  :              (file)
Work              (directory)
  COMP250        (directory)
  :
Research         (directory)
  :
```

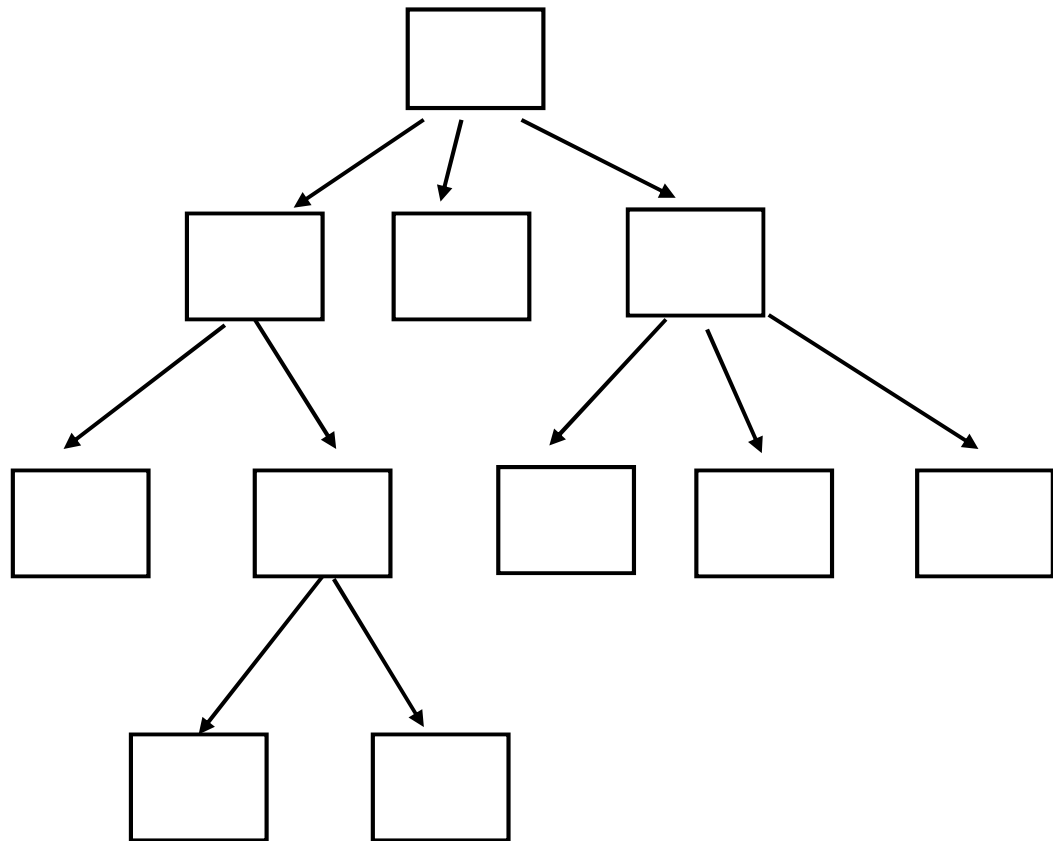
```
depthfirst (root){  
  for each child of root  
    depthfirst( child )  
  visit root  
}
```

“postorder” traversal:  
visit the root *after* the  
children



```
depthfirst (root){  
  for each child of root  
    depthfirst( child )  
  visit root  
}
```

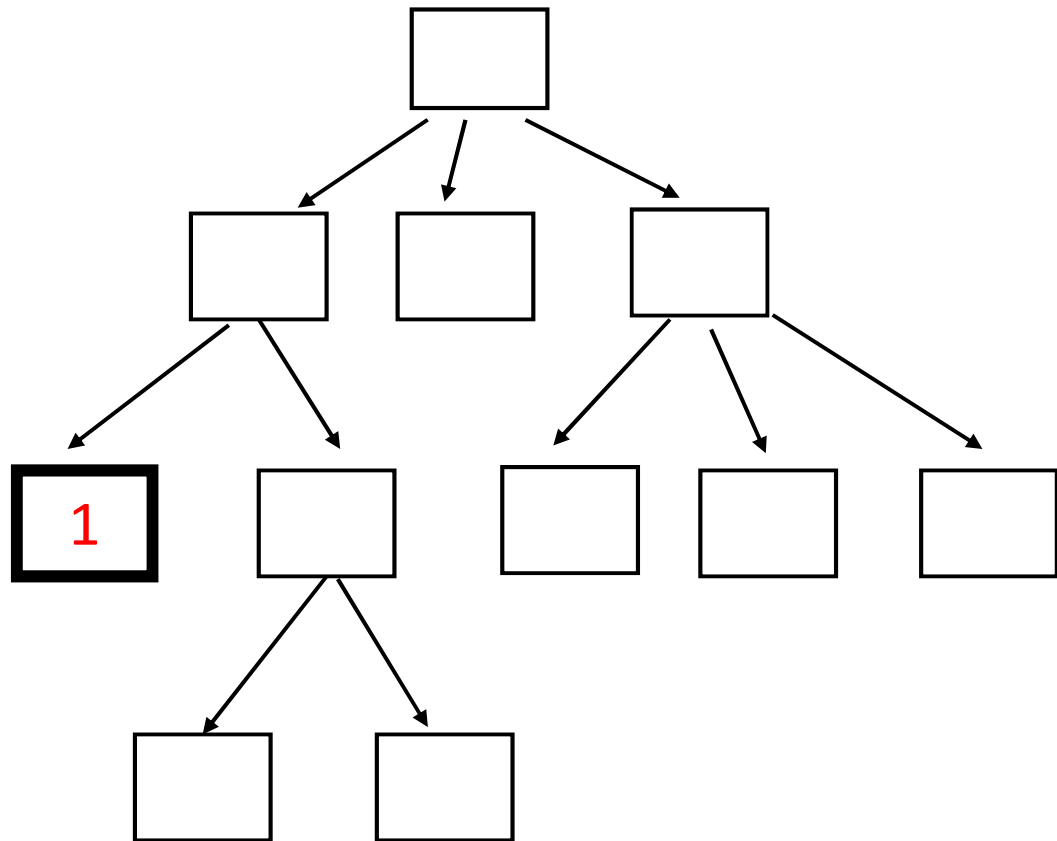
“postorder” traversal:  
visit the root after the  
children



Q: Which node  
is visited first?

```
depthfirst (root){  
  for each child of root  
    depthfirst( child )  
  visit root  
}
```

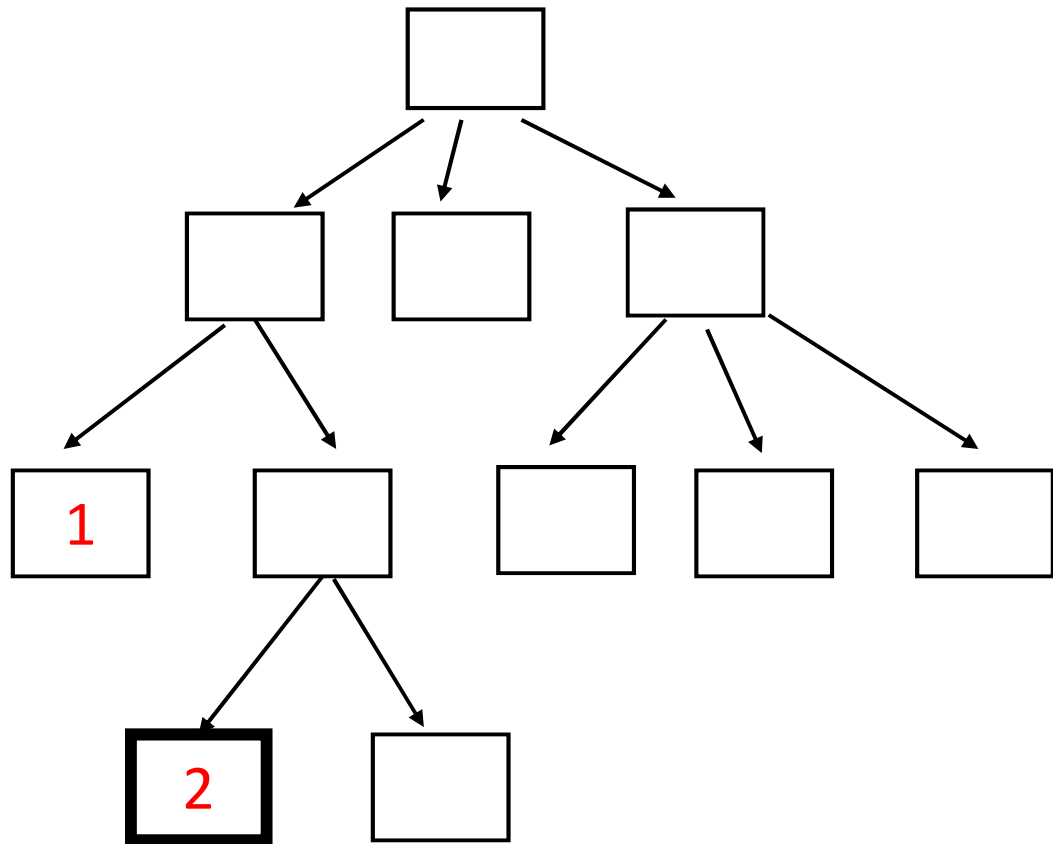
“postorder” traversal:  
visit the root after the  
children



Q: Which node is  
visited second?

```
depthfirst (root){  
  for each child of root  
    depthfirst( child )  
  visit root  
}
```

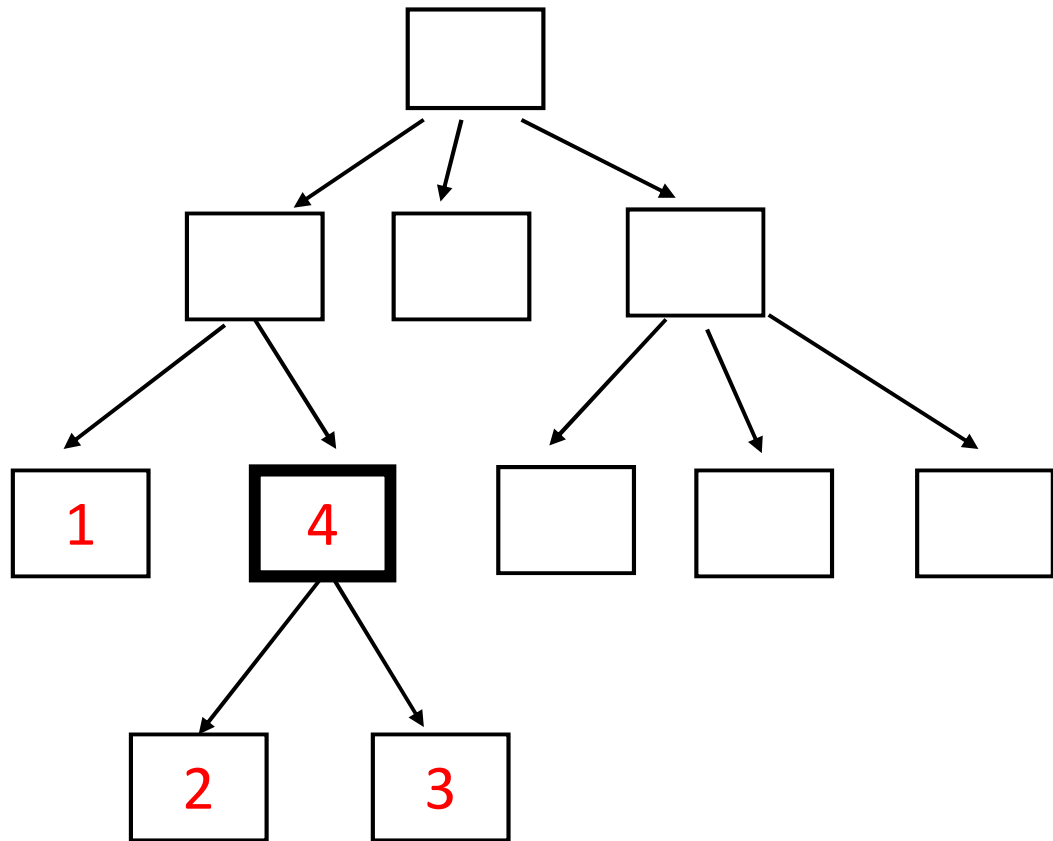
“postorder” traversal:  
visit the root after the  
children



Q: Which node is  
visited 3<sup>rd</sup> and 4<sup>th</sup> ?

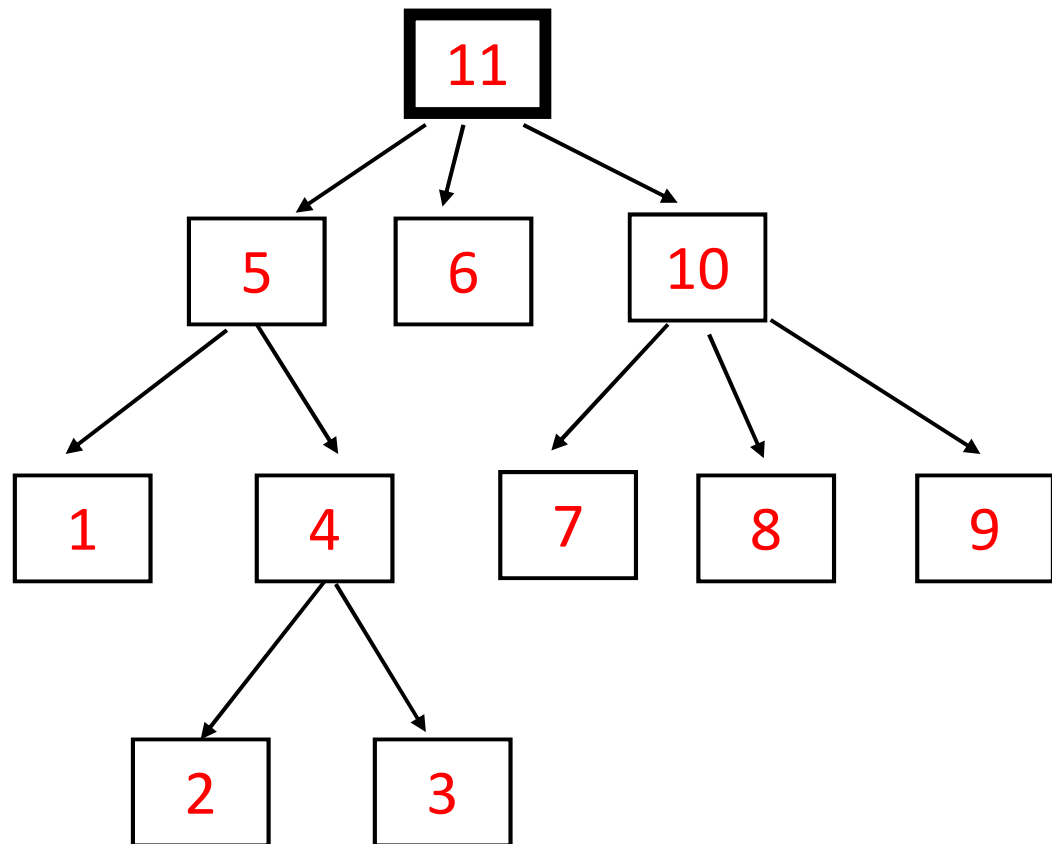
```
depthfirst (root){  
  if (root is not empty){  
    for each child of root  
      depthfirst( child )  
  }  
  visit root  
}
```

“postorder” traversal:  
visit the root after the  
children

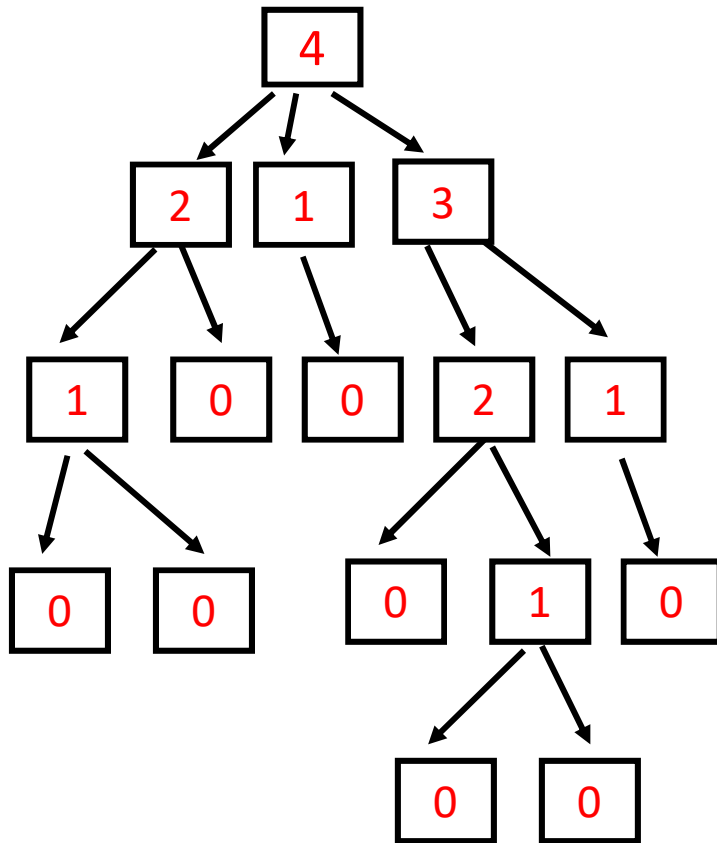


```
depthfirst (root){  
  for each child of root  
    depthfirst( child )  
  visit root  
}
```

“postorder” traversal:  
visit the root after the  
children



# Example 1 postorder: height

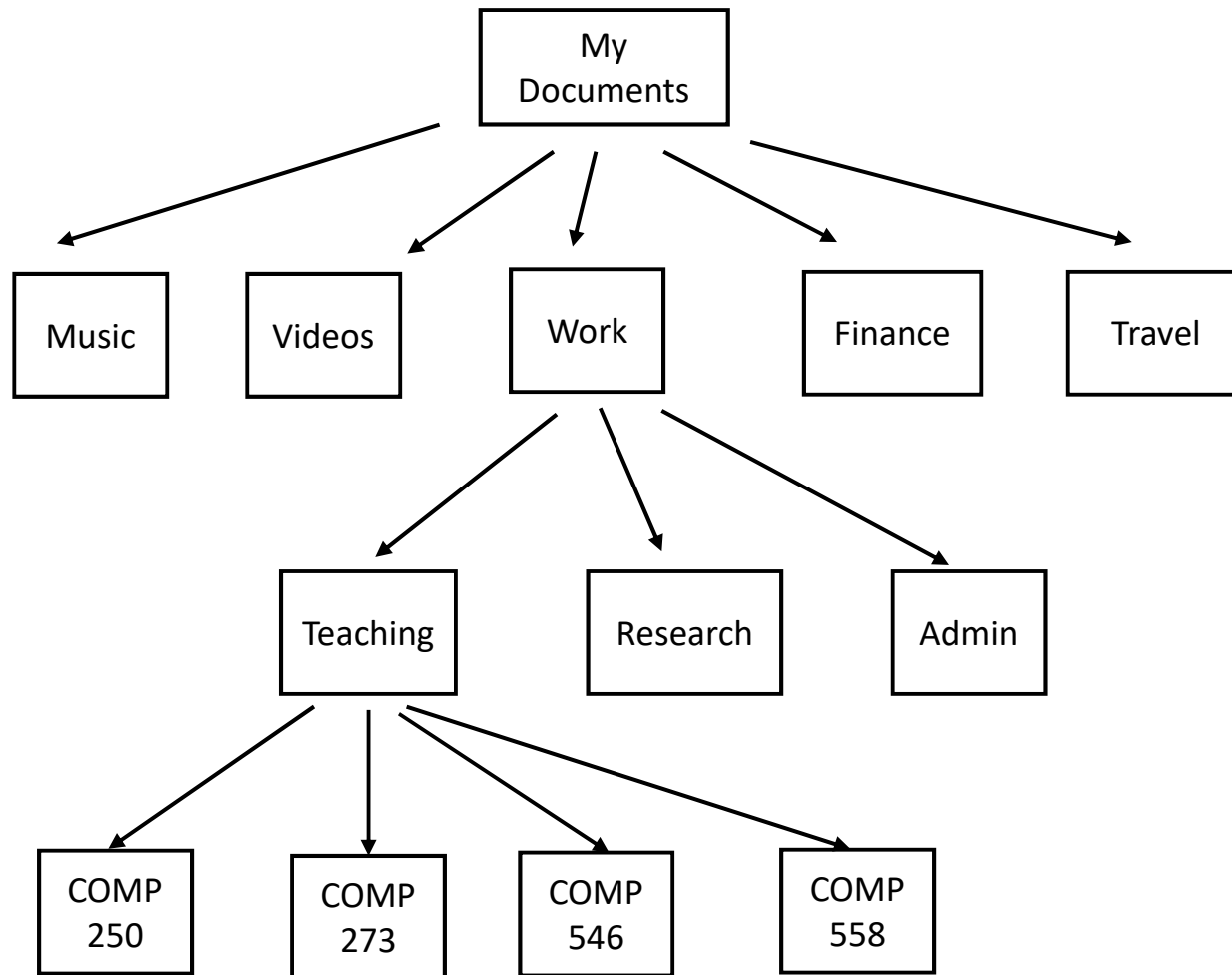


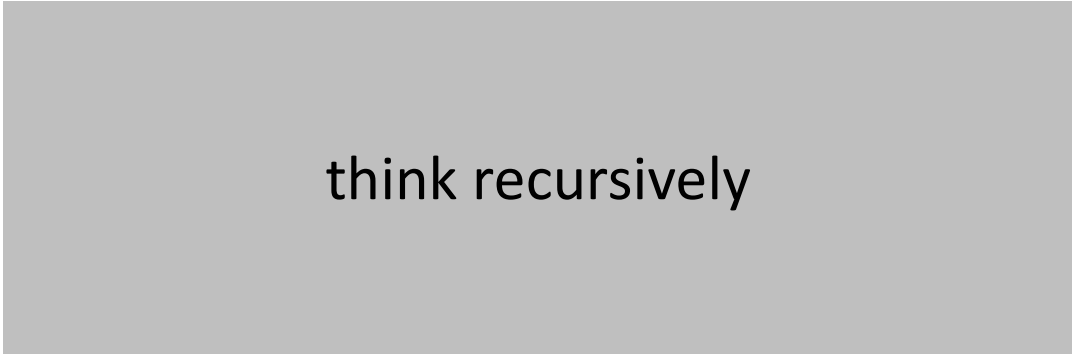
```
height(v){  
  if (v has no children)  
    return 0  
  else{  
    h = 0  
    for each child w of v  
      h = max(h, height(w))  
    return 1 + h  
  }  
}
```

‘visit’ a node here means ‘determine height of’ the node



## Example 2 Postorder: total number of bytes (in a file or in some directory including subdirectories)



```
numBytes(root){  
  if root has no children  
    return number of bytes at root // 0, if root is directory  
  else {  
      
  }  
}
```

**'visit' here means determine the number of bytes in subtree that is rooted at that node.**

“preorder” traversal

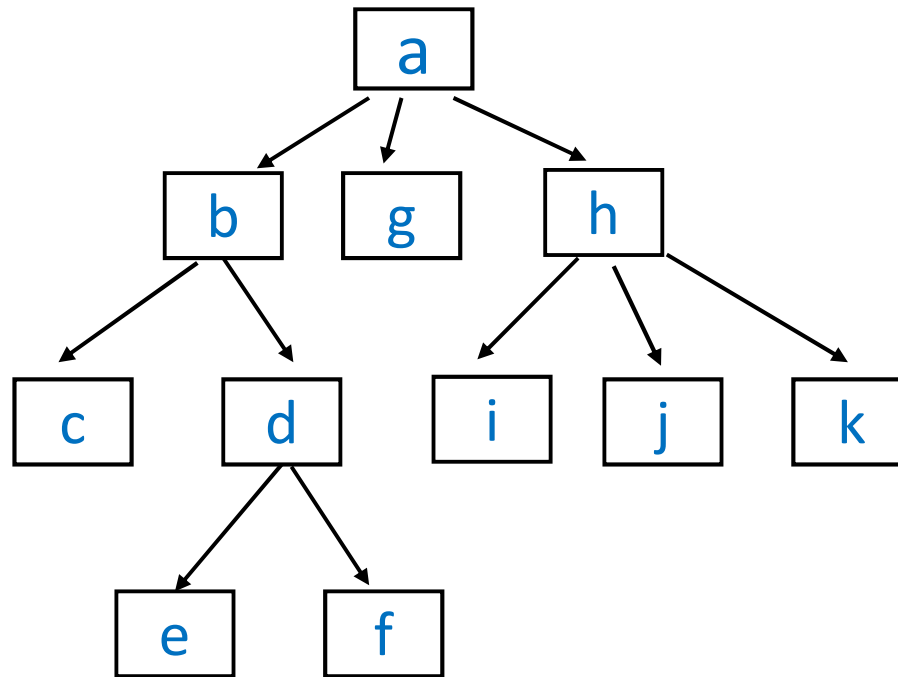
```
depthfirst (root){  
    visit root  
    for each child of root  
        depthfirst( child )  
}
```

“postorder” traversal

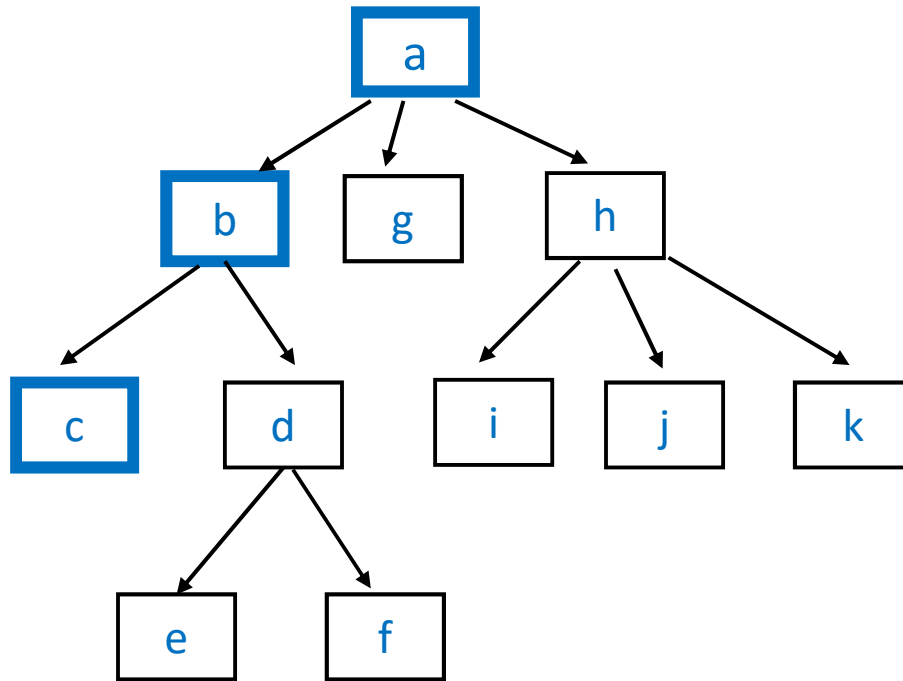
```
depthfirst (root){  
    for each child of root  
        depthfirst( child )  
    visit root  
}
```

The same `depthfirst(root)` *call sequence* occurs for preorder and postorder. Only the visiting order changes.

In example below, the letter order corresponds to the `depthfirst(root)` *call order*. Let's next examine the call stack.



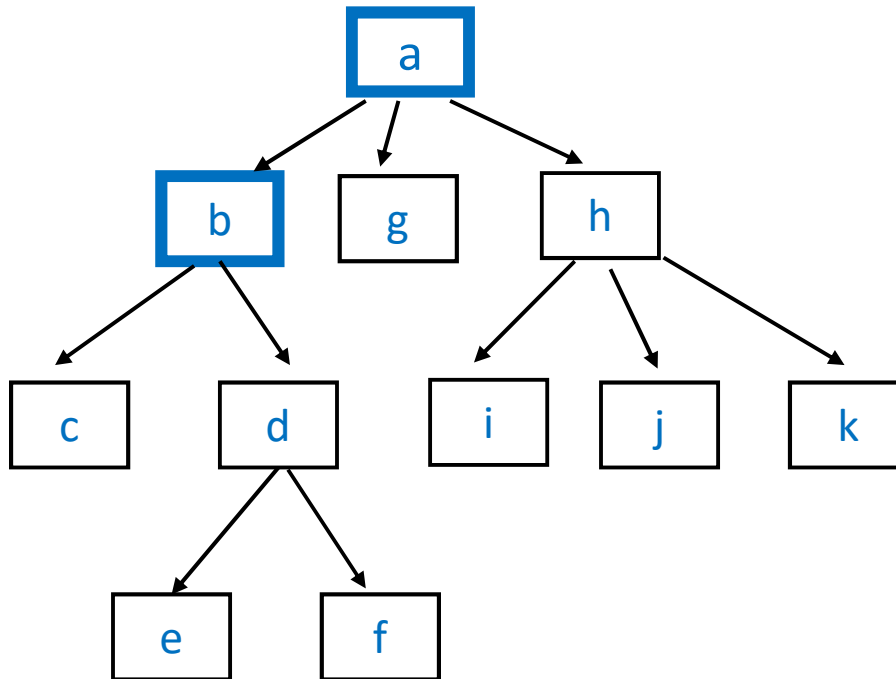
# Call stack for `depthfirst(root)`



The “call stack” figure below only shows the `root` parameter.

a a a  
b b  
c

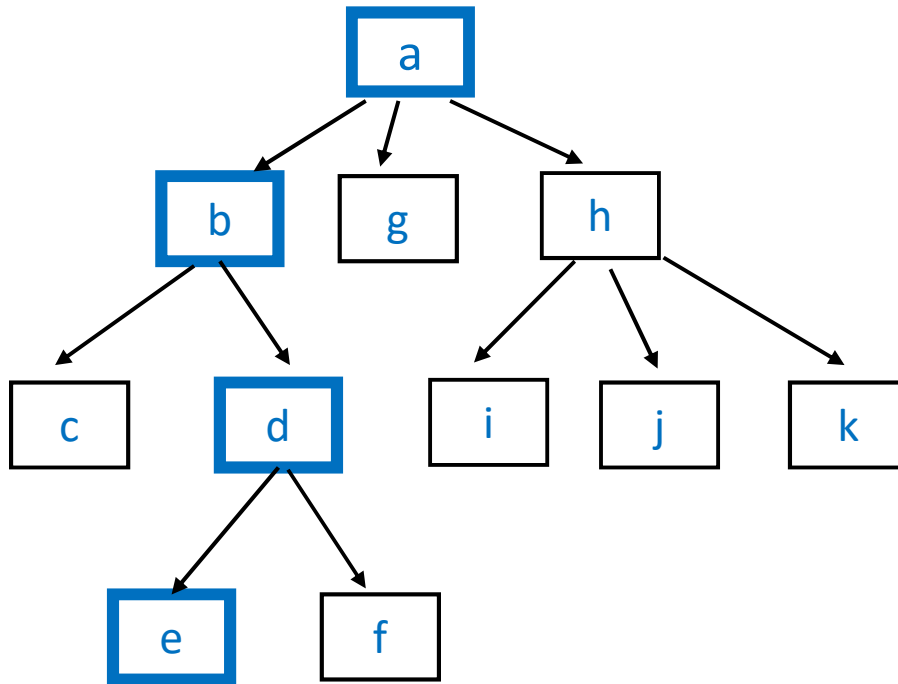
# Call stack for `depthfirst(root)`



The “call stack” figure below only shows the `root` parameter.

c  
b b b  
a a a a

# Call stack for `depthfirst(root)`

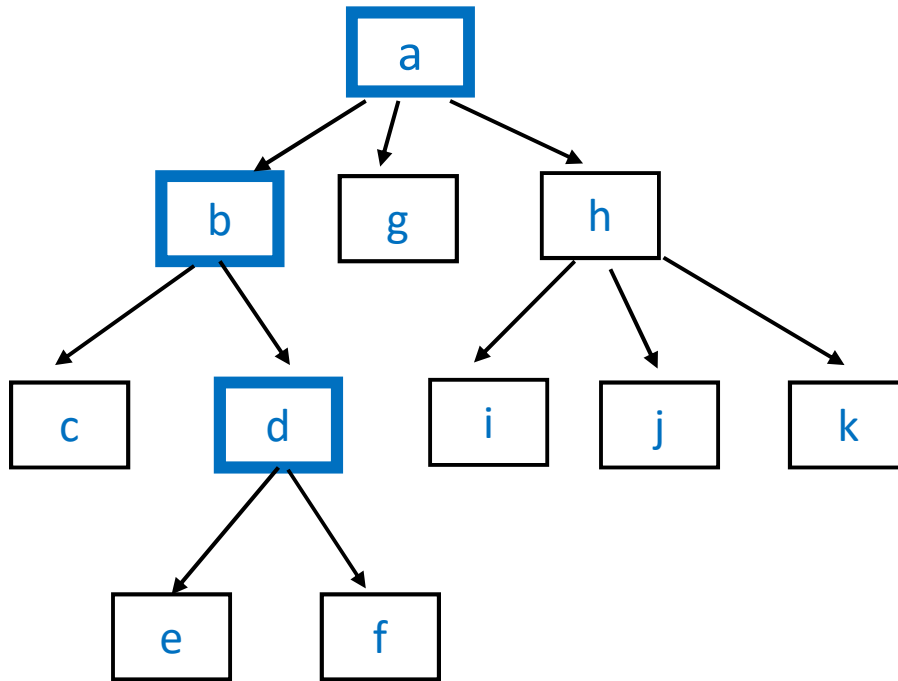


The “call stack” figure below only shows the `root` parameter.

```
          e
        c  d  d
       b  b  b  b  b
      a  a  a  a  a  a
```



# Call stack for `depthfirst(root)`

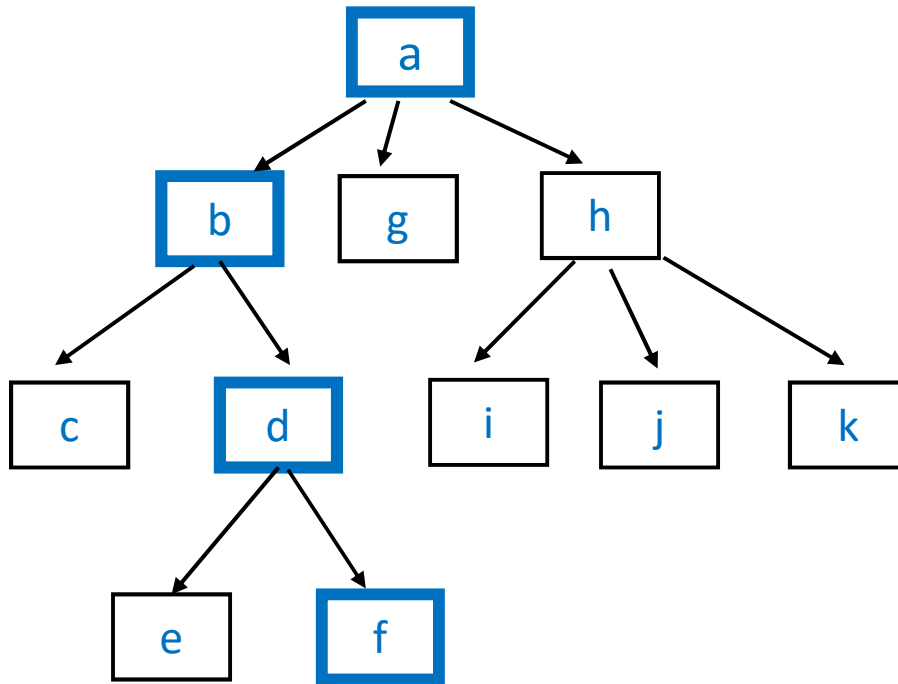


a a a a a a a  
b b b b b b  
c d d d  
e





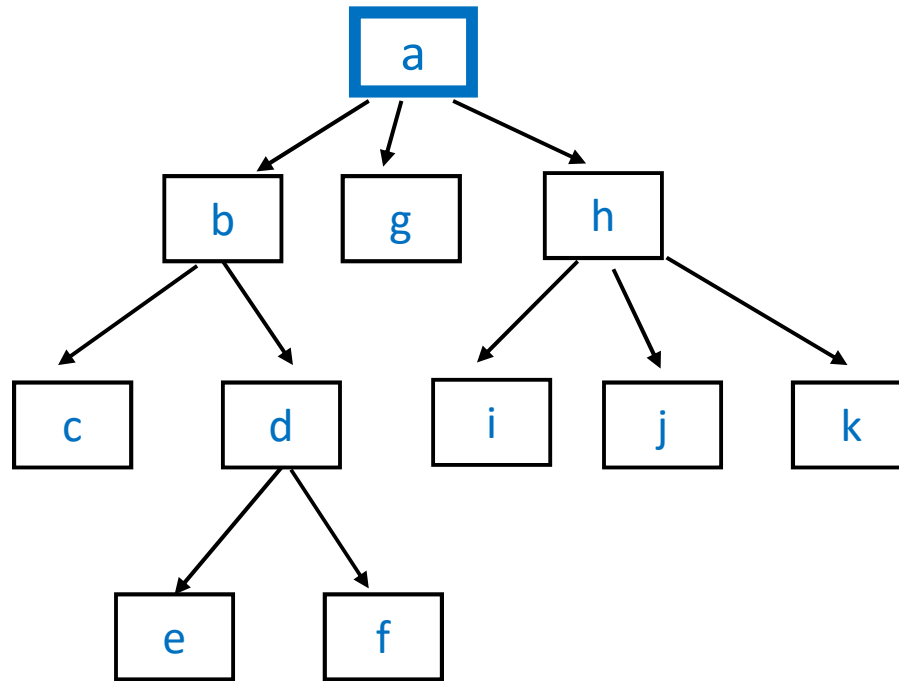
# Call stack for `depthfirst(root)`



				e	f	
	c		d	d	d	d
b	b	b	b	b	b	b
a	a	a	a	a	a	a



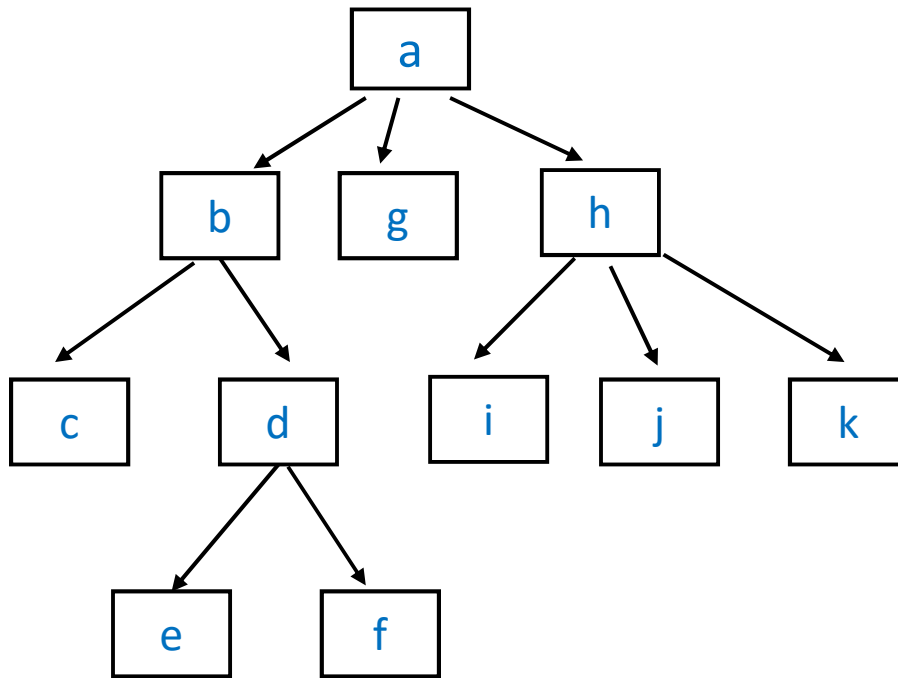
# Call stack for `depthfirst(root)`



a a a a a a a a a a  
b b b b b b b b b  
c d d d d d  
e f



# Call stack for `depthfirst(root)`



                  e      f  
          c      d  d  d  d  d          i      j      k  
      b  b  b  b  b  b  b  b          g      h  h  h  h  h  h  
a  a  a  a  a  a  a  a  a  a  a  a  a  a  a  a  a  a  a  a

# Tree traversal

## Recursive


- depth first (pre- versus post-order)

## Non-Recursive

- using a stack
- using a queue

```
treeTraversalUsingStack(root){  
  initialize empty stack s  
  s.push(root)
```

```
}
```

```
treeTraversalUsingStack(root){  
  initialize empty stack s  
  s.push(root)  
  while s is not empty {  
    cur = s.pop()  
    visit cur  
      
  }  
}
```

```
treeTraversalUsingStack(root){  
    initialize empty stack s  
    s.push(root)  
    while s is not empty {  
        cur = s.pop()  
        visit cur  
        for each child of cur  
            s.push(child)  
    }  
}
```

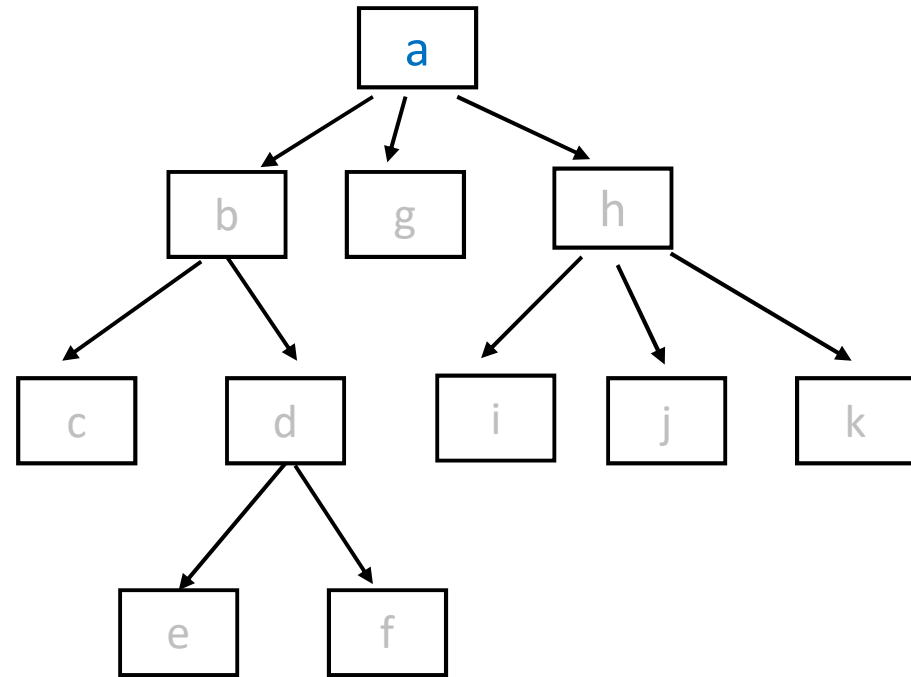
What is the order of nodes **visited** by `treeTraversalUsingStack()` ?



```

treeTraversalUsingStack(root){
  initialize empty stack s
  s.push(root)
  ● while s is not empty {
    cur = s.pop()
    visit cur
    for each child of cur
      s.push(child)
  }
}

```

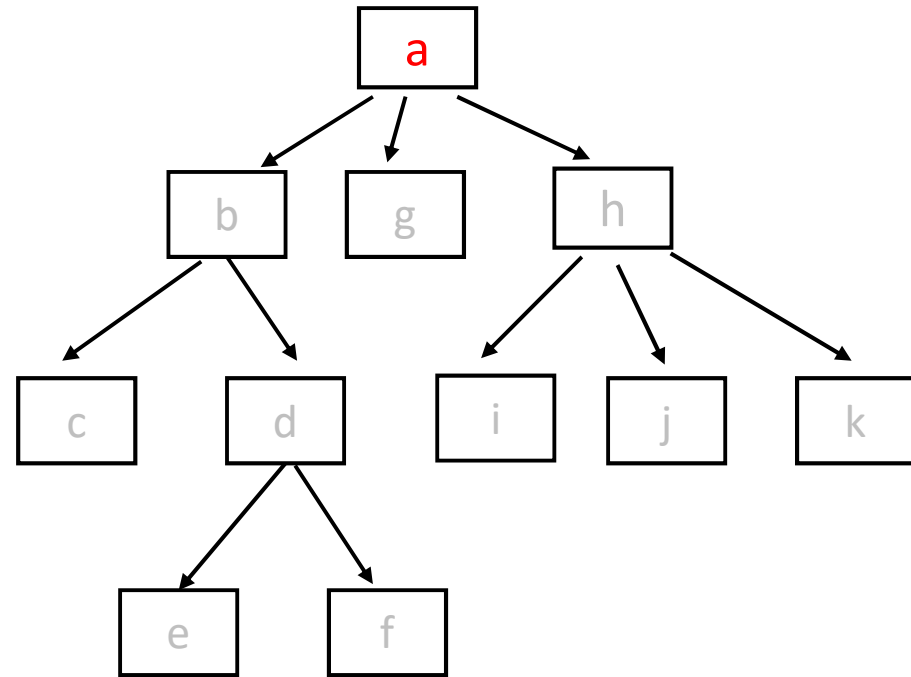


a

```

treeTraversalUsingStack(root){
  initialize empty stack s
  s.push(root)
  while s is not empty {
    cur = s.pop()
    visit cur
    for each child of cur
      s.push(child)
  }
}

```

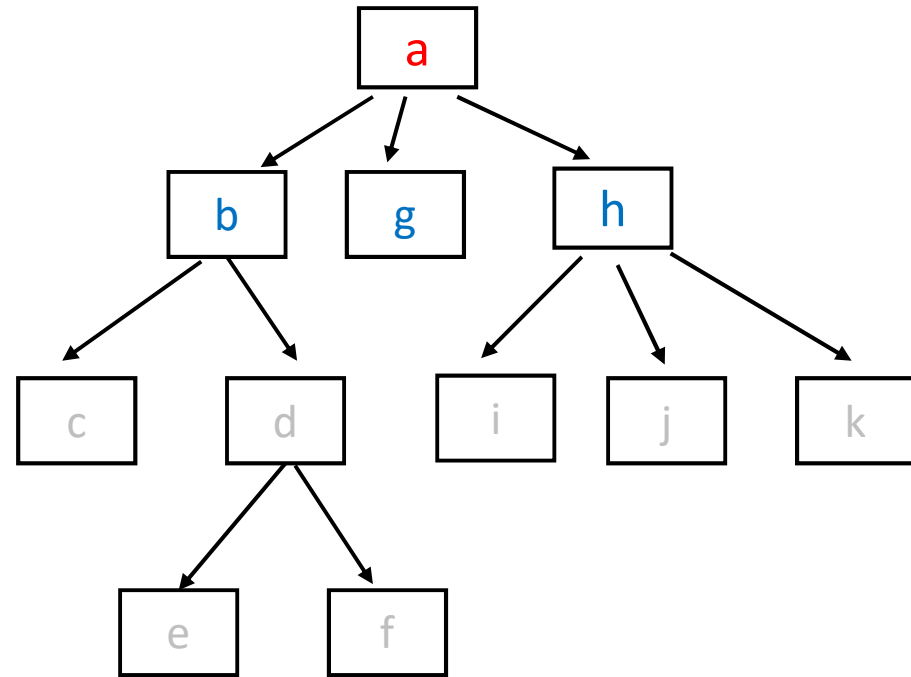


a \_

```

treeTraversalUsingStack(root){
  initialize empty stack s
  s.push(root)
  ● while s is not empty {
    cur = s.pop()
    visit cur
    for each child of cur
      s.push(child)
  }
}

```



```

a _ b b b
      g g
      h

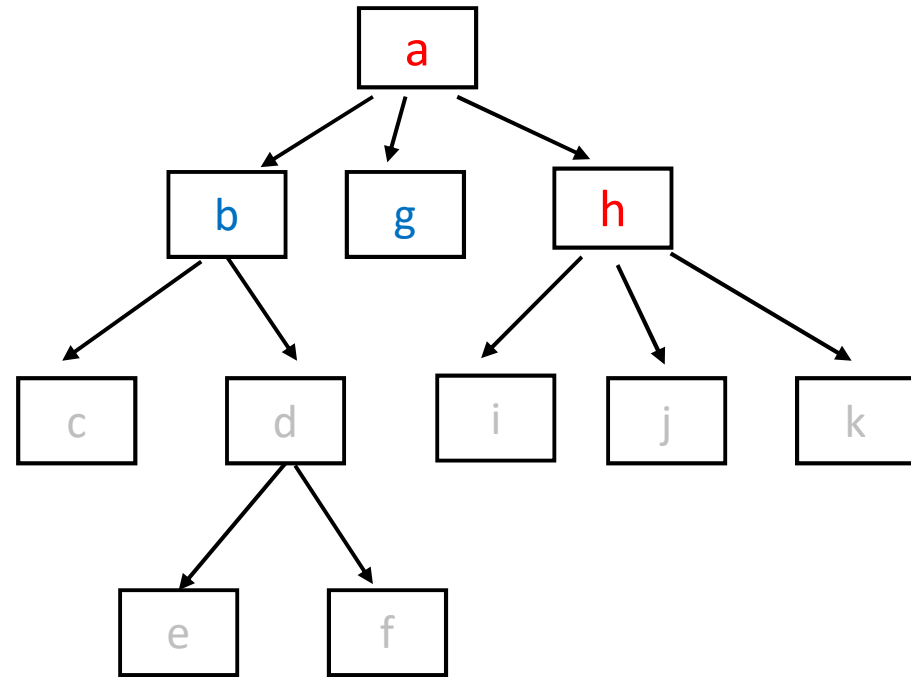
```



```

treeTraversalUsingStack(root){
  initialize empty stack s
  s.push(root)
  while s is not empty {
    cur = s.pop()
    visit cur
    for each child of cur
      s.push(child)
  }
}

```



```

      h
    g g g
  a _ b b b

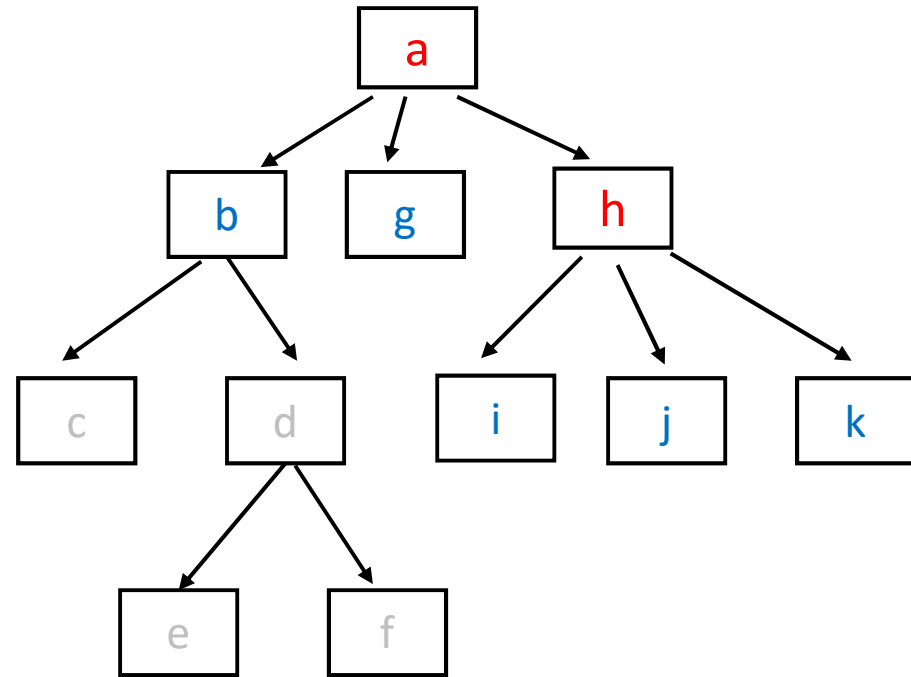
```



```

treeTraversalUsingStack(root){
  initialize empty stack s
  s.push(root)
  while s is not empty {
    cur = s.pop()
    visit cur
    for each child of cur
      s.push(child)
  }
}

```



```

a _ b b b b b b
      g g g g g g
        h i i i
          j j
            k

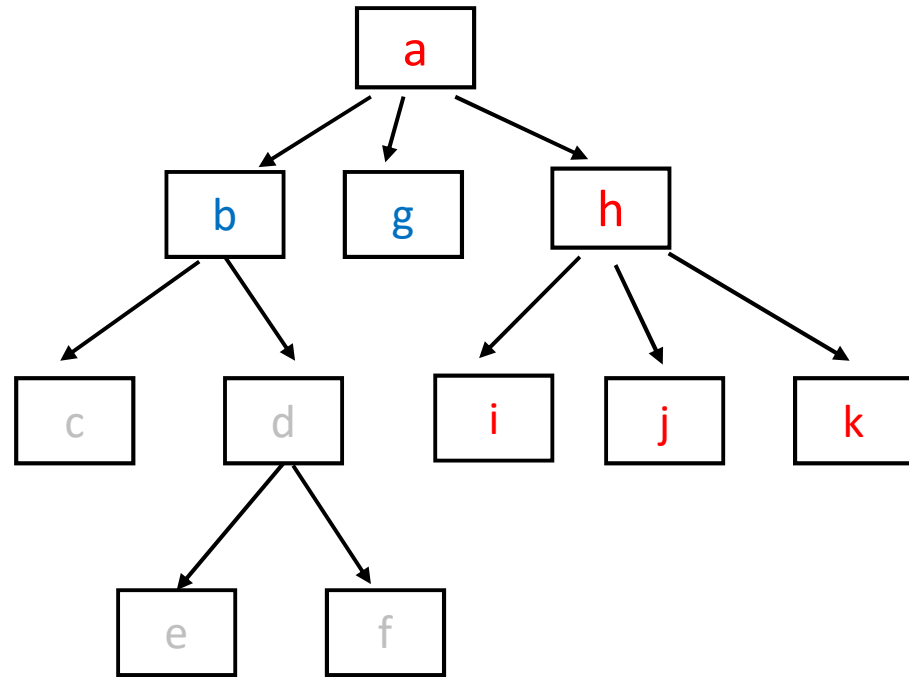
```



```

treeTraversalUsingStack(root){
  initialize empty stack s
  s.push(root)
  ● while s is not empty {
    cur = s.pop()
    visit cur
    for each child of cur
      s.push(child)
  }
}

```



```

          k
        j j j
      h i i i i
    g g g g g g g g
  a _ b b b b b b b b

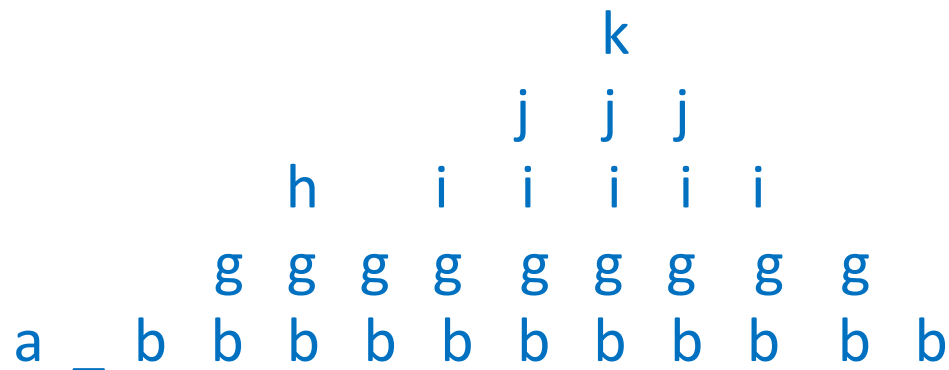
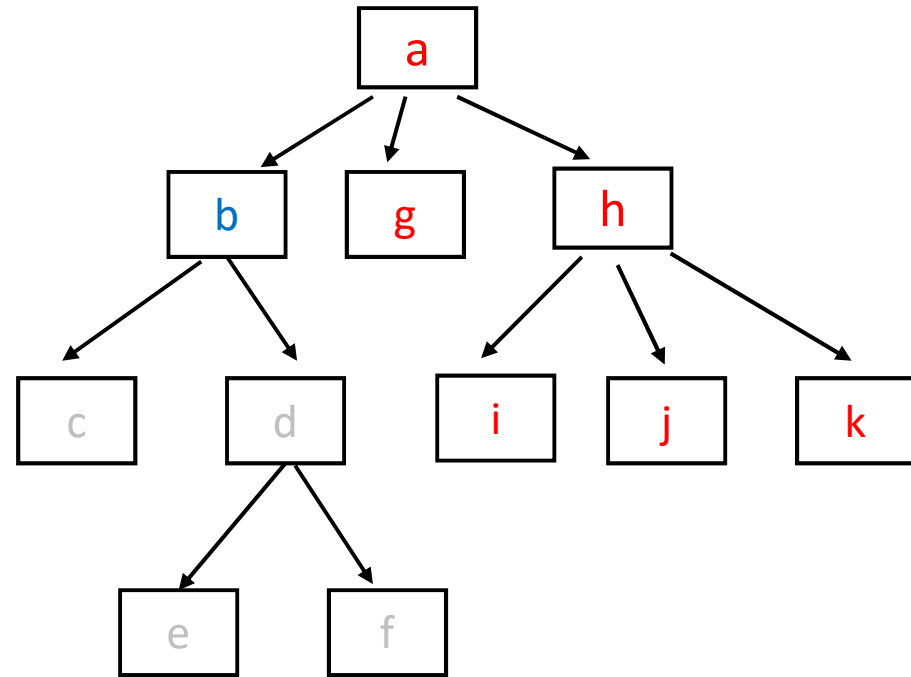
```



```

treeTraversalUsingStack(root){
  initialize empty stack s
  s.push(root)
  ● while s is not empty {
    cur = s.pop()
    visit cur
    for each child of cur
      s.push(child)
  }
}

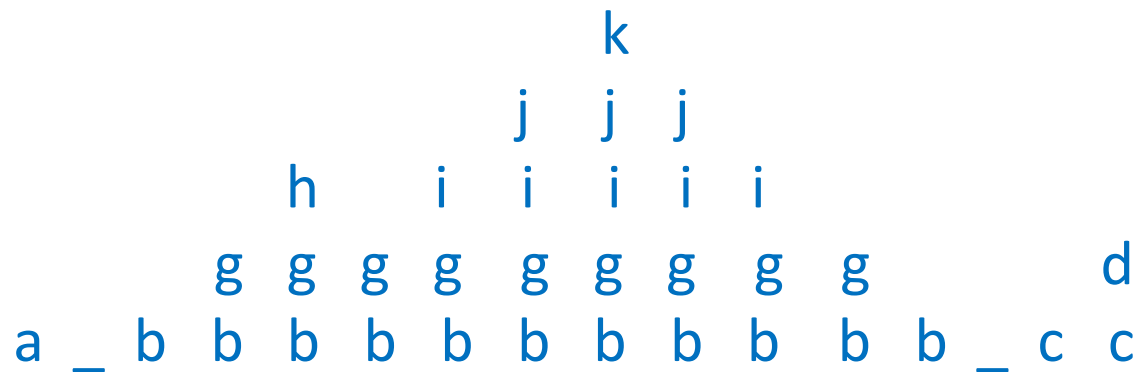
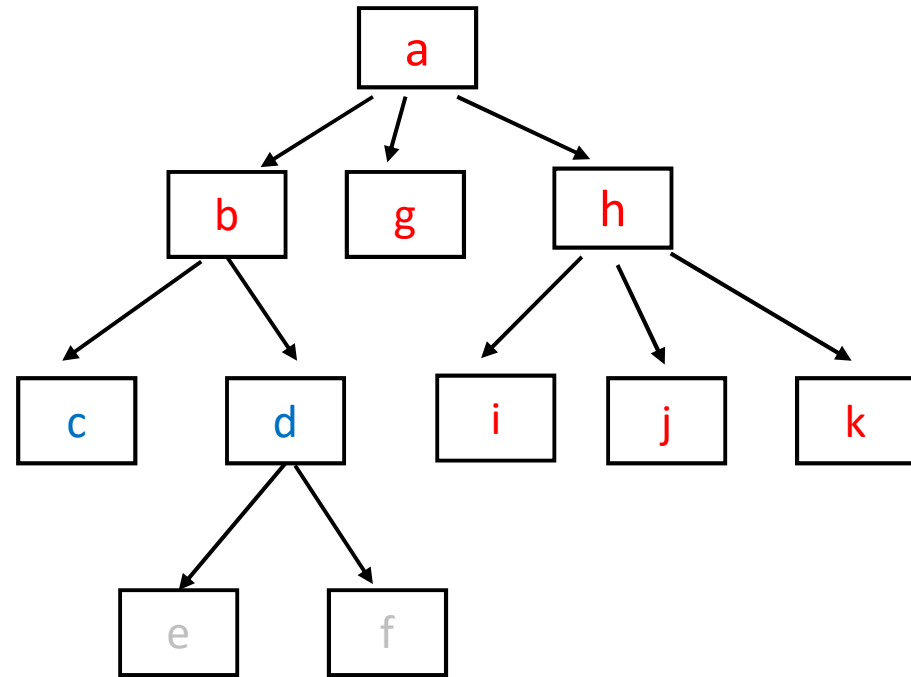
```



```

treeTraversalUsingStack(root){
  initialize empty stack s
  s.push(root)
  while s is not empty {
    cur = s.pop()
    visit cur
    for each child of cur
      s.push(child)
  }
}

```

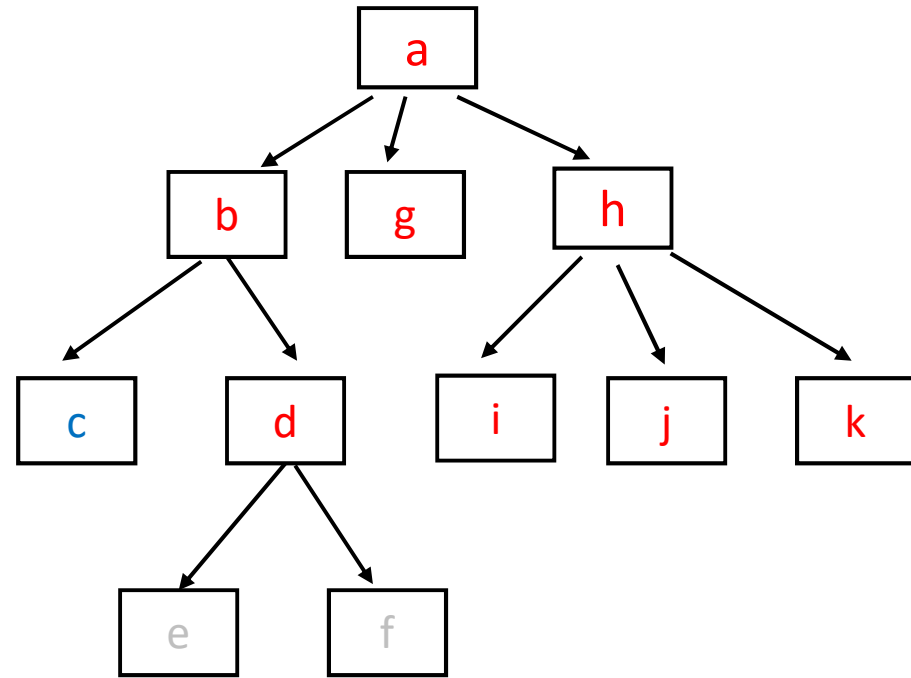




```

treeTraversalUsingStack(root){
  initialize empty stack s
  s.push(root)
  while s is not empty {
    cur = s.pop()
    visit cur
    for each child of cur
      s.push(child)
  }
}

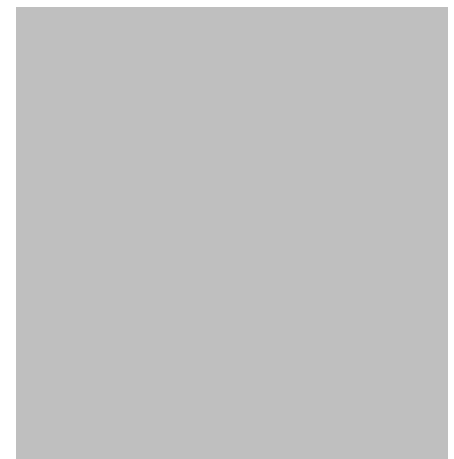
```



```

          k
        j j j
      h i i i i
    g g g g g g g g g d
a _ b b b b b b b b b _ c c c

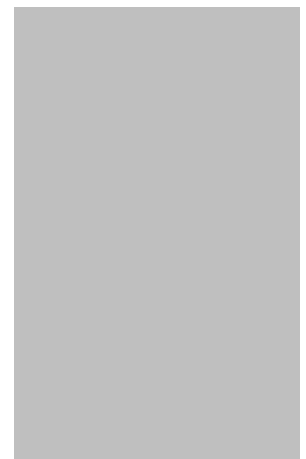
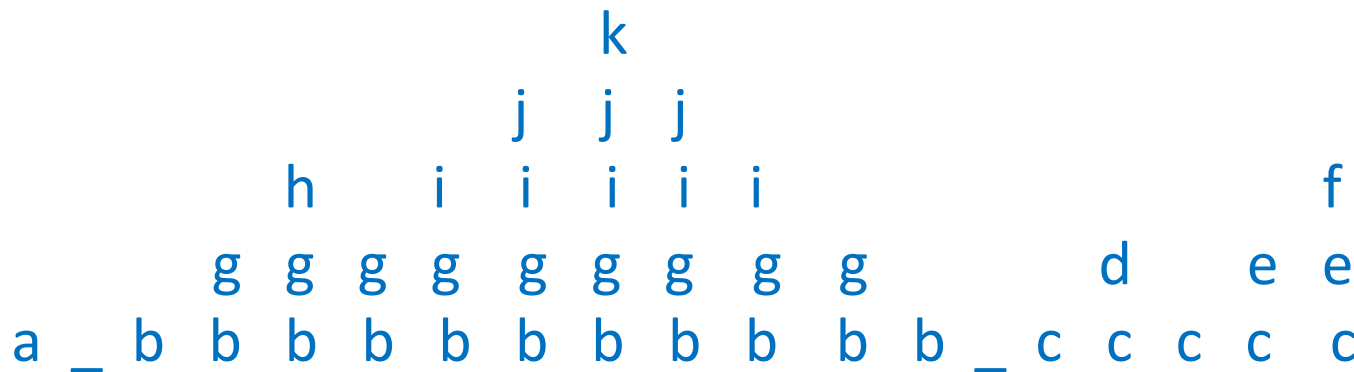
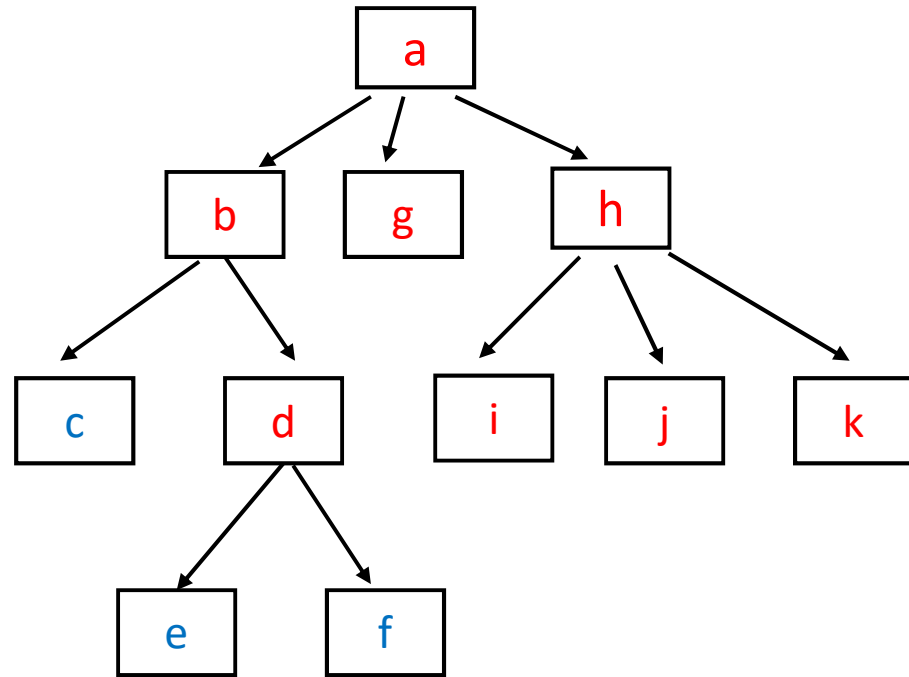
```



```

treeTraversalUsingStack(root){
  initialize empty stack s
  s.push(root)
  while s is not empty {
    cur = s.pop()
    visit cur
    for each child of cur
      s.push(child)
  }
}

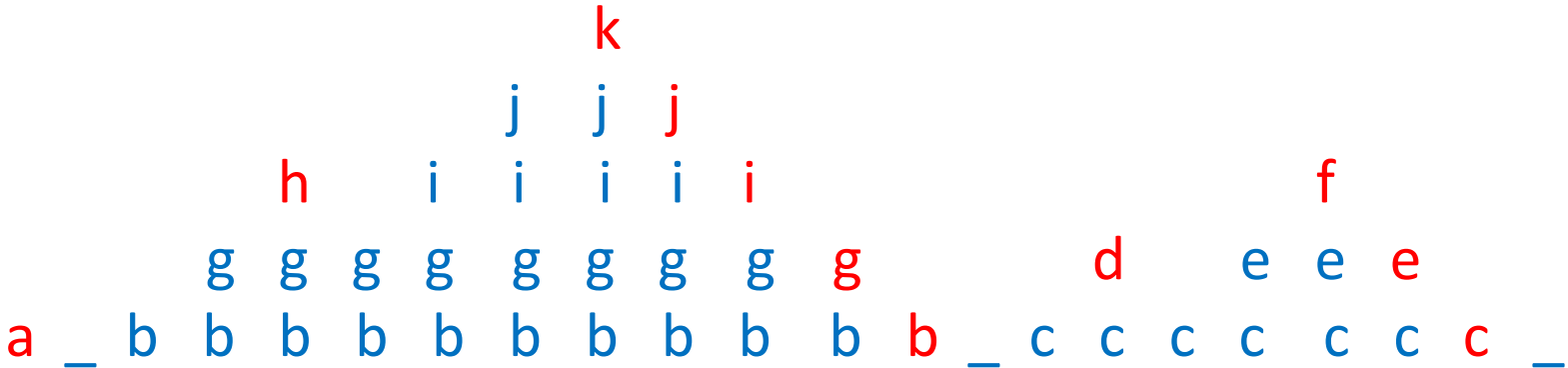
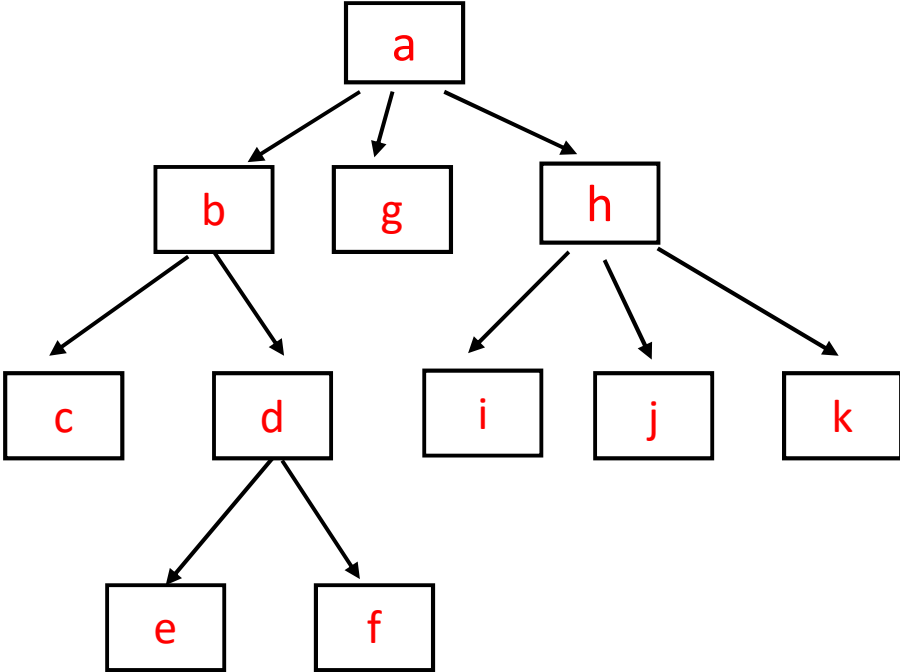
```



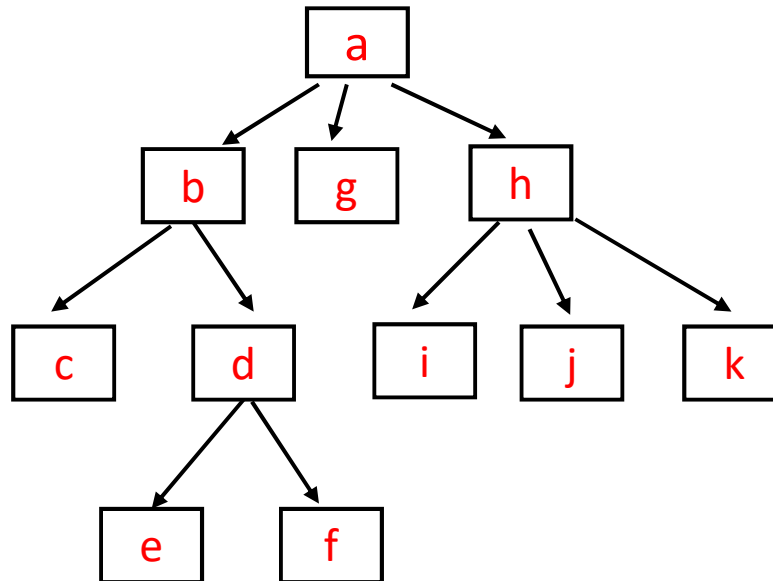
```

treeTraversalUsingStack(root){
  initialize empty stack s
  s.push(root)
  while s is not empty {
    cur = s.pop()
    visit cur
    for each child of cur
      s.push(child)
  }
}

```



The stack based method is also depth first,  
but we are visiting children from right to left



recursive preorder : abcdefghijk

recursive postorder : cefdbgijkha

non-recursive (stack) : **ahkjigbdfec**

# What if we use a queue instead?

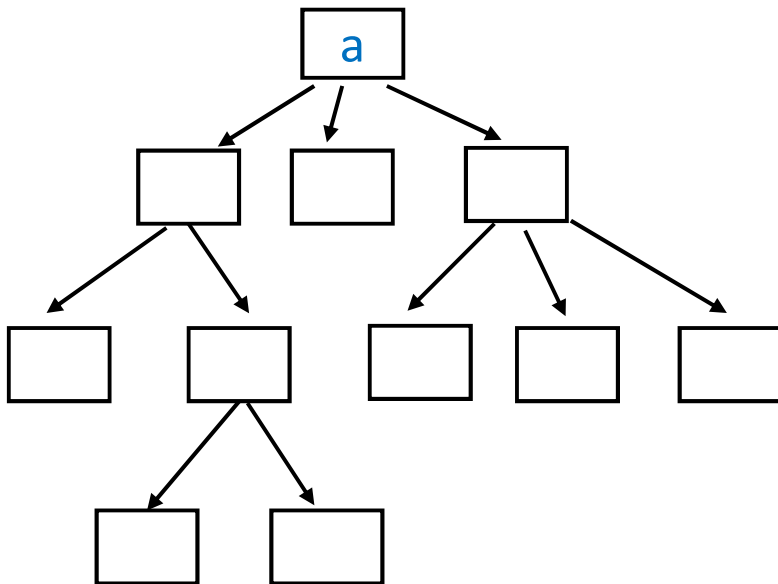
```
treeTraversalUsingStack(root){
  initialize empty stack s
  s.push(root)
  while s is not empty {
    cur = s.pop()
    visit cur
    for each child of cur
      s.push(child)
  }
}
```

```
treeTraversalUsingQueue(root){
  initialize empty queue q
  q.enqueue(root)
  while q is not empty {
    cur = q.dequeue()
    visit cur
    for each child of cur
      q.enqueue(child)
  }
}
```

```

treeTraversalUsingQueue(root){
  initialize empty queue q
  q.enqueue(root)
  ● while q is not empty {
    cur = q.dequeue()
    visit cur
    for each child of cur
      q.enqueue(child)
  }
}

```



Queue state  
at start of the  
while loop

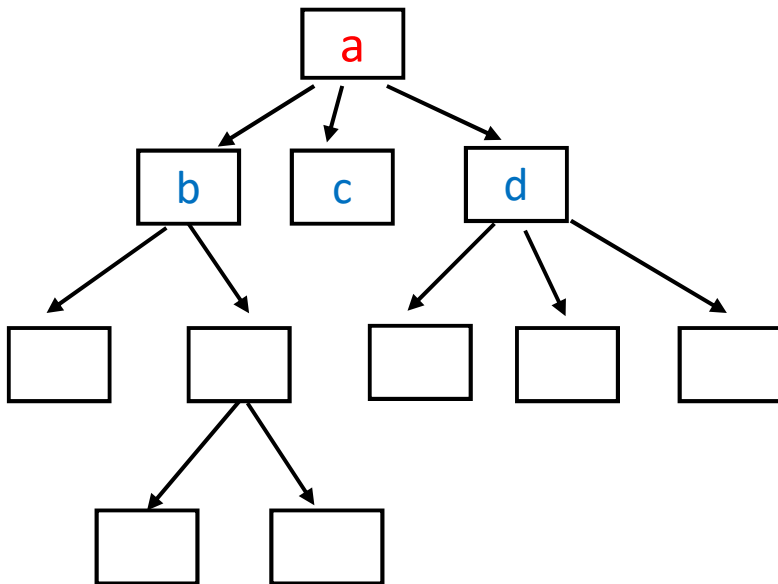
a



```
treeTraversalUsingQueue(root){
  initialize empty queue q
  q.enqueue(root)
  ● while q is not empty {
    cur = q.dequeue()
    visit cur
    for each child of cur
      q.enqueue(child)
  }
}
```

Queue state  
at start of the  
while loop

a  
b c d



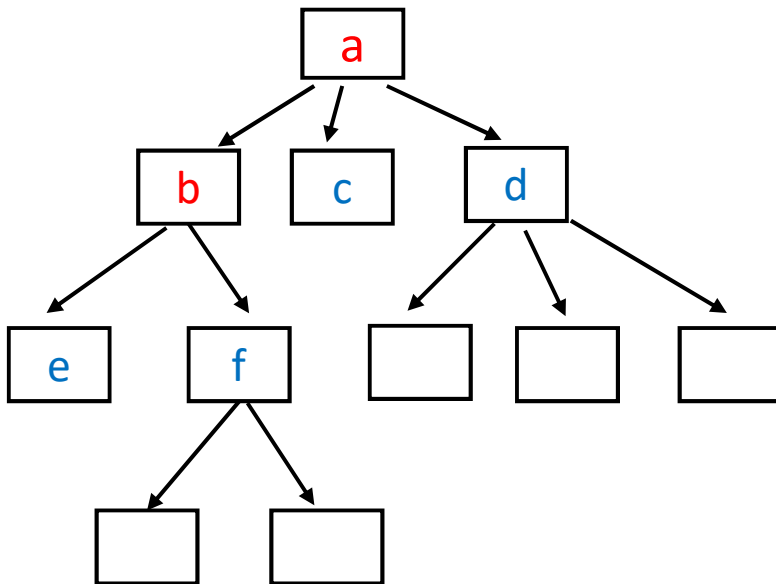
```

treeTraversalUsingQueue(root){
  initialize empty queue q
  q.enqueue(root)
  ● while q is not empty {
    cur = q.dequeue()
    visit cur
    for each child of cur
      q.enqueue(child)
  }
}

```

Queue state  
at start of the  
while loop

a  
b c d  
c d e f

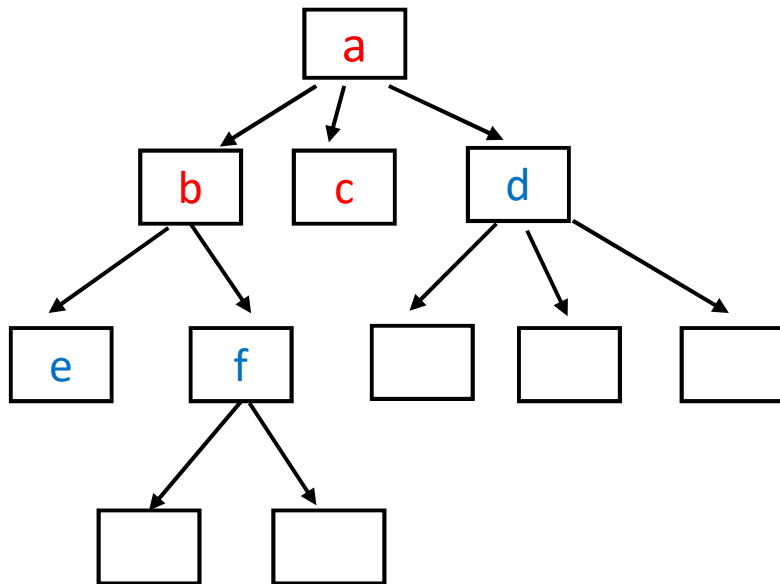




```

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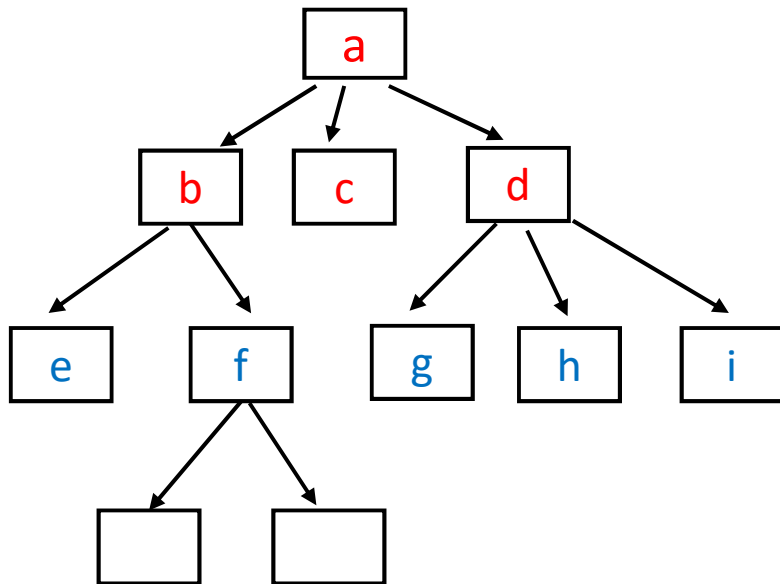


Queue state  
at start of the  
while loop

a  
b c d  
c d e f  
d e f



```
treeTraversalUsingQueue(root){
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Queue state  
at start of the  
while loop

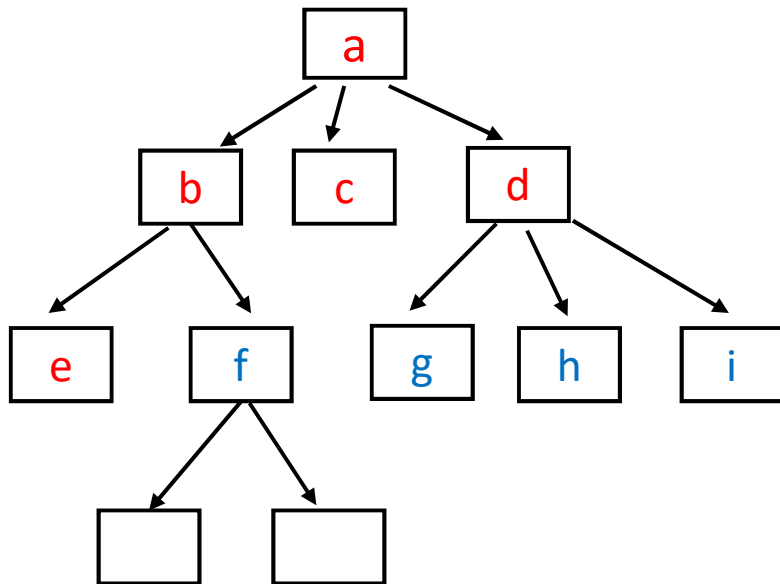
a  
b c d  
c d e f  
d e f  
e f g h i



```

treeTraversalUsingQueue(root){
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  q.enqueue(root)
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}

```



Queue state  
at start of the  
while loop

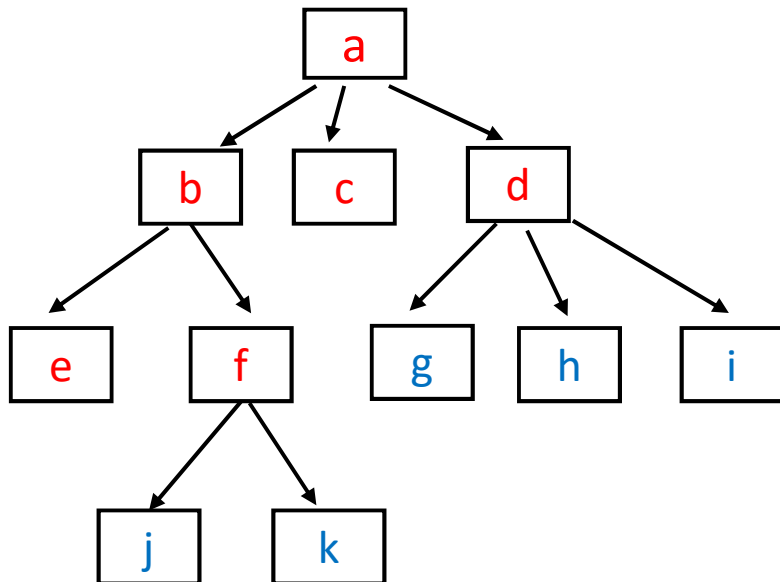
a  
b c d  
c d e f  
d e f  
e f g h i  
f g h i



```

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}

```



Queue state  
at start of the  
while loop

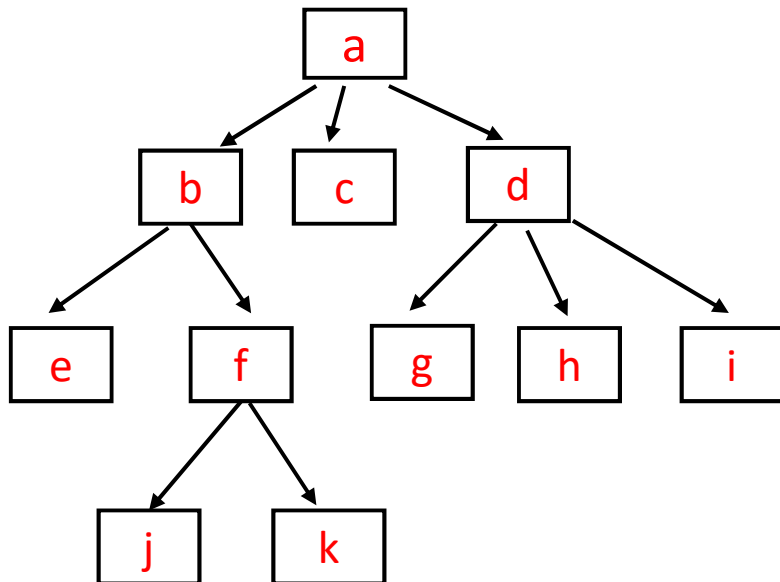
a  
b c d  
c d e f  
d e f  
e f g h i  
f g h i  
g h i j k



```

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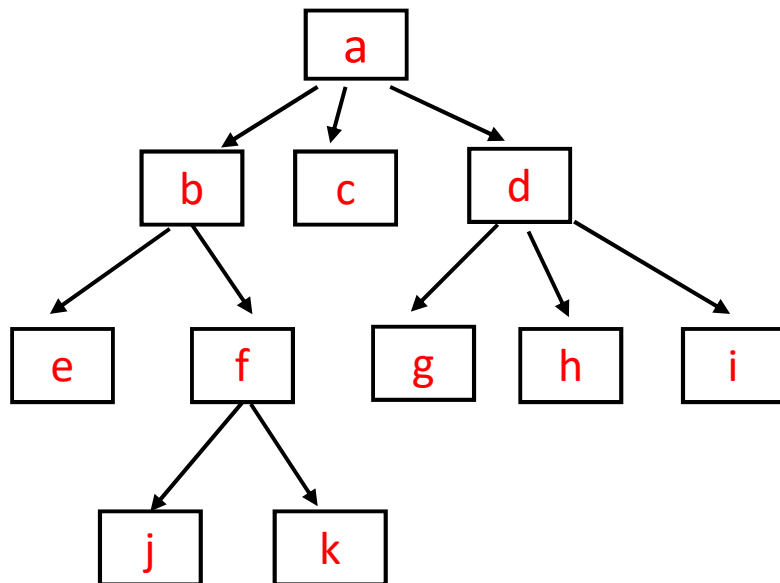


Queue state  
at start of the  
while loop

a  
b c d  
c d e f  
d e f  
e f g h i  
f g h i  
g h i j k  
h i j k  
i j k  
j k  
k

# breadth first traversal

for each level  $i$   
visit all nodes at level  $i$



order visited: **abcdefghijkl**

# Coming up...

## Lectures

Fri. March 11 (lecture 25)  
Expression Trees

Mon. March 14  
Binary Search Trees

Wed & Fri. March 16 & 18  
Heaps

## Tutorial + Assessments

**Tutorial for Assignment 3**  
**today at 6 pm**

Assignment 3  
due Wed. March 16

Quiz 4 (lectures 20-25)  
Fri. March 18