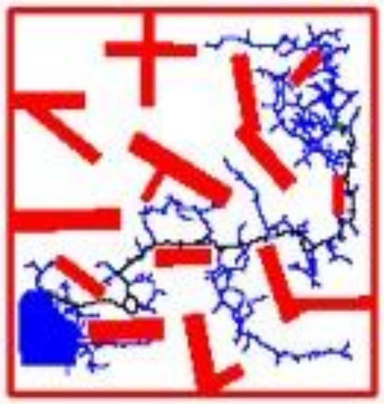
Geometric Path Planning

McGill COMP 765

Oct 12th, 2017



The Motion Planning Problem

- Intuition: Find a safe path/trajectory from start to goal
- More precisely:
  - A path is a series of robot configurations (e.g., joint angles, points in x-y-theta, 6-DOF poses, etc)
  - A trajectory is a path indexed by time
  - We express safety as a set of constraints (e.g., walls, impassable terrain, unsafe flight configurations, self-penetration of arm links, etc.)

Planning Algorithm Properties

- We can measure the performance of our planning algorithms in a number of ways:
  - Complete: returns a path if one exists. Variants include:
    - Resolution complete
    - Probabilistically complete
  - Optimal: the path returned is the "best" one under some condition (usually path length, but we can consider others)
  - Computational complexity:
    - Measured w.r.t. number of dimensions of configuration space, size of the world, number of steps in the returned path, etc.
  - Nature of constraints handled:
    - Holonomic
    - Non-holonomic
  - Uncertainty awareness

Challenges for motion planning

- Robots are often high dimensional (e.g., humanoid, robot arm, quadruped) useful paths can require very many motions, and there can be tight constraints between each location (e.g., piano moving)
- The constraints formed by the robot and its environment do not take a clean form that we can easily optimize over
  - Example: if we have a square obstacle on the 2D plane, what is the shape of the constraint in the 6 degree-of-freedom joint space of a manipulator?
- Motion planning research:
  - One of the oldest topics in robotics
  - For many years we did not have efficient algorithms to handle the problems posed by the hardware – approximations and heuristics were abundant
  - Today: recent breakthroughs mean that good libraries exists for many of the problems of interest. Research on pure geometric planning is slightly less and we are ready to tackle uncertainty, differential control, learned models, etc.

Starting Simple: Breadth-first search planning

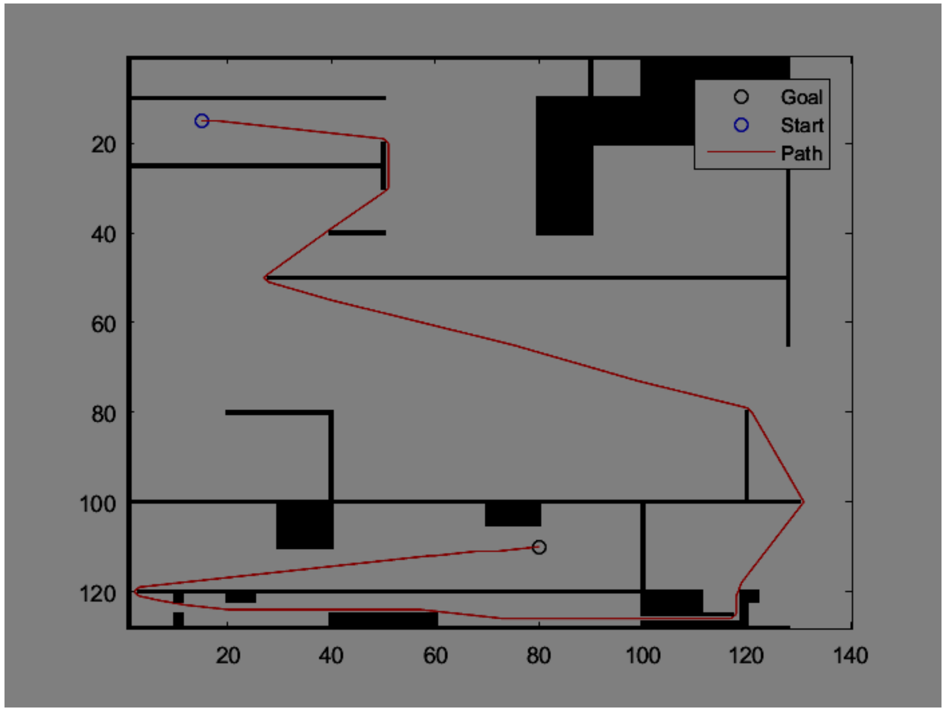
- Breadth-first search applied to a discrete planning problem:
  - From the start, try every possible action and record the result, along with book-keeping, in an "open list" (queue)
  - Until we reach the goal, pop from the open list, try every possible action, and place the results back at the end of the open list
- BFS is a complete planner, it explores all possible paths
- It scales very badly with dimension and path length
- Can only handle discrete or discretized environments

A* Planning

- Same structure as BFS, but add a heuristic function that estimates the cost-to-goal from each location
- The open-list becomes a priority queue on the function:
  - $f(n) = g(n) + h(n)$
  - With g the cost so far and h the heuristic
- The heuristic guides our computation to open nodes in the direction of the goal first
- As long as h is admissible (it never over-estimates), we are guaranteed an optimal solution when we first open the goal
- Still a complete planner as we eventually open all nodes regardless of h

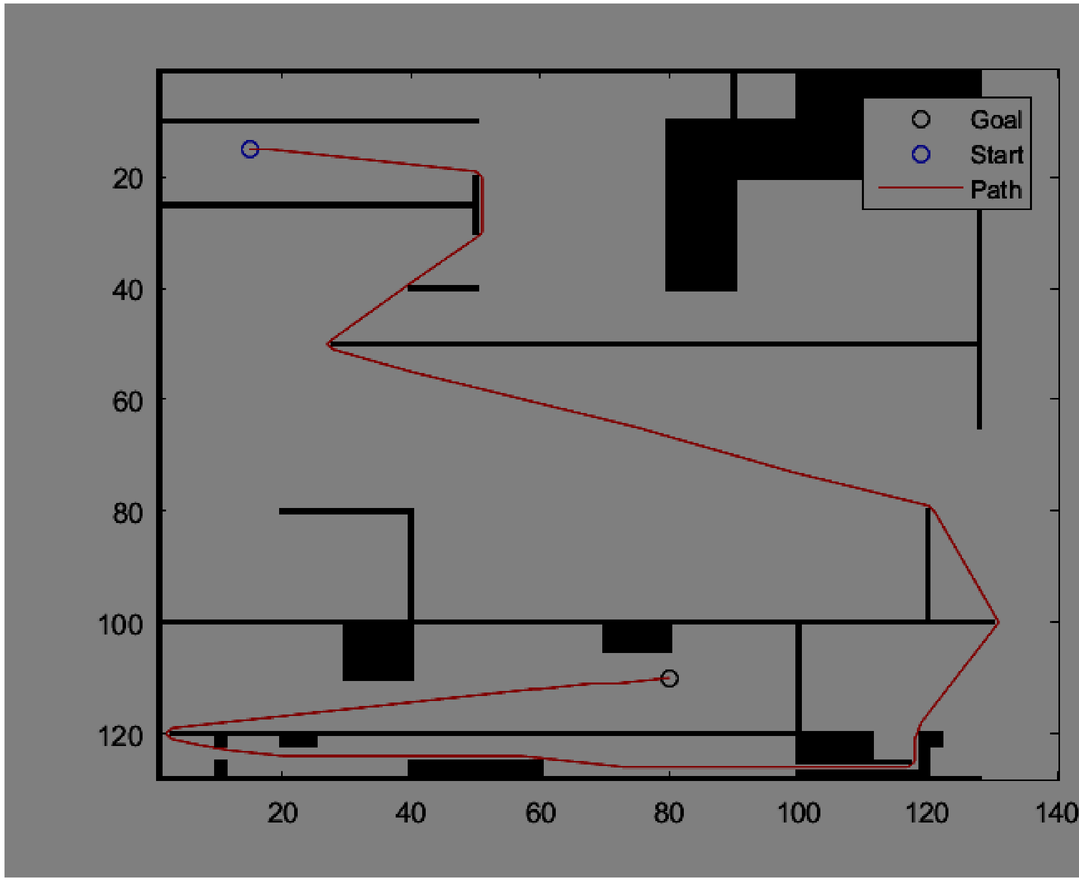
A-star

- A* is used very often as a long-term planner in robotics, when the world can be considered discrete and controls are ignored
- Examples:
  - Self-driving car routes through city maps
  - Navigation through indoor hallways



Issues with A-star

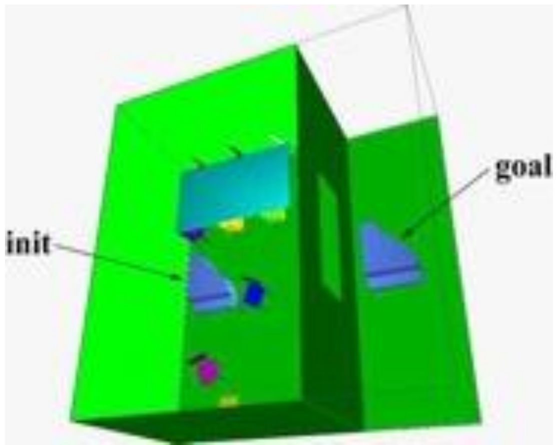
- Re-using computation:
  - In robotics, map is often evolving (D*, Stentz *et al.*)
  - Same path segments will be shared by many paths (roadmaps and decompositions, soon)



Poor scaling to higher dimensions

Robot-Specific Methods

(1) Cell decomposition	←	**Goal** account for all of the free space
(2) Potential Fields	←	**Goal** Create flexible local control strategies
(3) Bug algorithms	←	Limited knowledge path planning with a behavioral approach
(4) Roadmap approaches	←	**Goal** reduce the N-dimensional configuration space to a set of one-D paths to search.

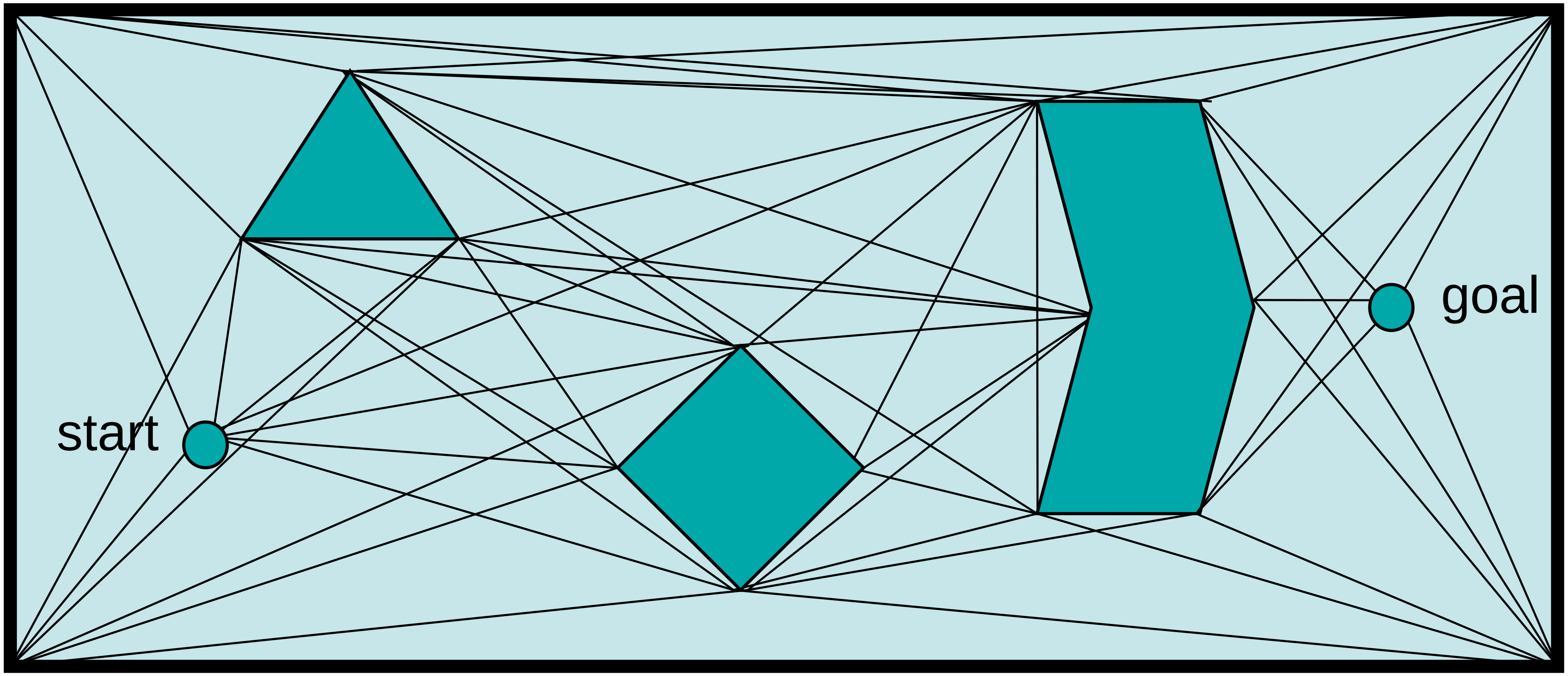


Cell Decomposition Planners

- Basic idea, decompose the overall planning process into local regions
- Simple computations can tell us the path through the region (may or may not be optimal)
- A global computation can be performed to re-assemble the entire path, and often we can re-use our local computation for multiple queries
- Examples:
  - Voronoi diagrams
  - Visibility graphs

The Visibility Graph

- Opened vertices = {start}. Repeat until you're done:
  - Add edges from opened vertices to visible obstacles
  - If goal visible from latest vertex added, you're done.



Since the map was in *C*-space, each line potentially represents part of a path from the start to the goal.

Potential Field Planners

- A more local and reactive method: utilize the constraints and information nearby to form a simple local motion decision
- Questions:
  - How to formulate these local decisions in a coherent way?
  - Can we prove any properties or guarantees about the overall plans?
- Potential fields are an intuitive approach based on the motions of physical bodies propelled by smooth forces

Potential Field Method

Potential Field (Working Principle)

– The goal location generates an attractive potential – pulling the robot towards the goal

– The obstacles generate a repulsive potential – pushing the robot away from the obstacles

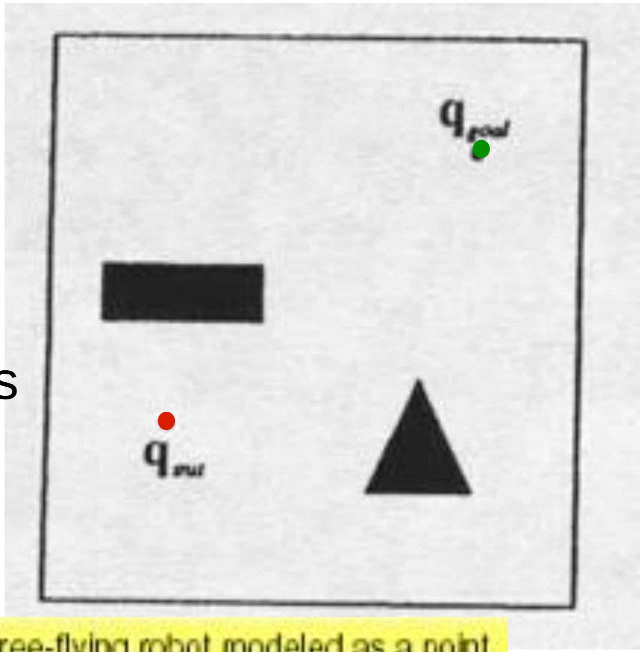
– The **negative gradient of the total potential** is treated as a force applied to the robot

-- Let the sum of the forces control the robot

Artificial **Potential**

$$U(q) = \underbrace{U_{goal}(q)}_{\text{attractive potential}} + \underbrace{\sum U_{obstacles}(q)}_{\text{repulsive potential}}$$

C-obstacles



Artificial **Force Field**

$$F(q) = -\nabla U(q)$$

Negative gradient

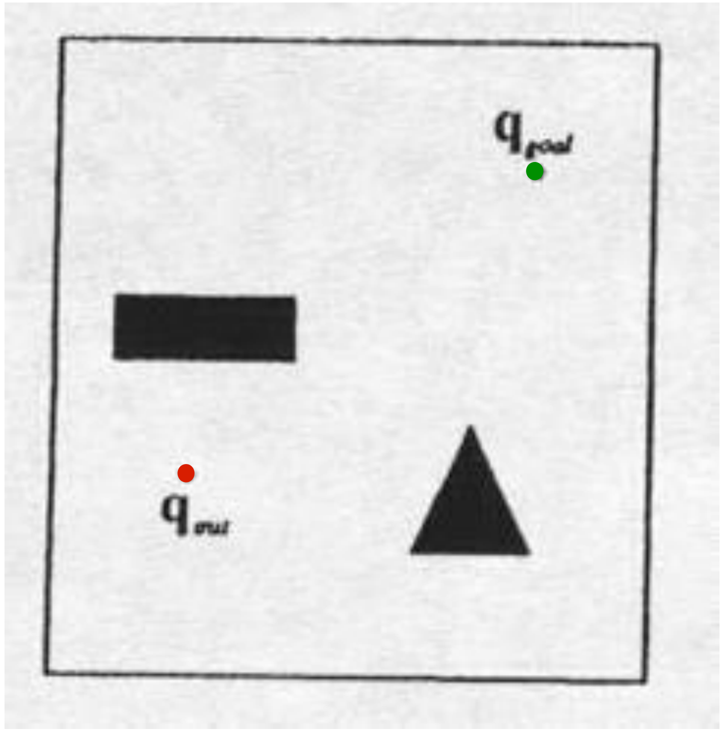
Example: free-flying robot modeled as a point

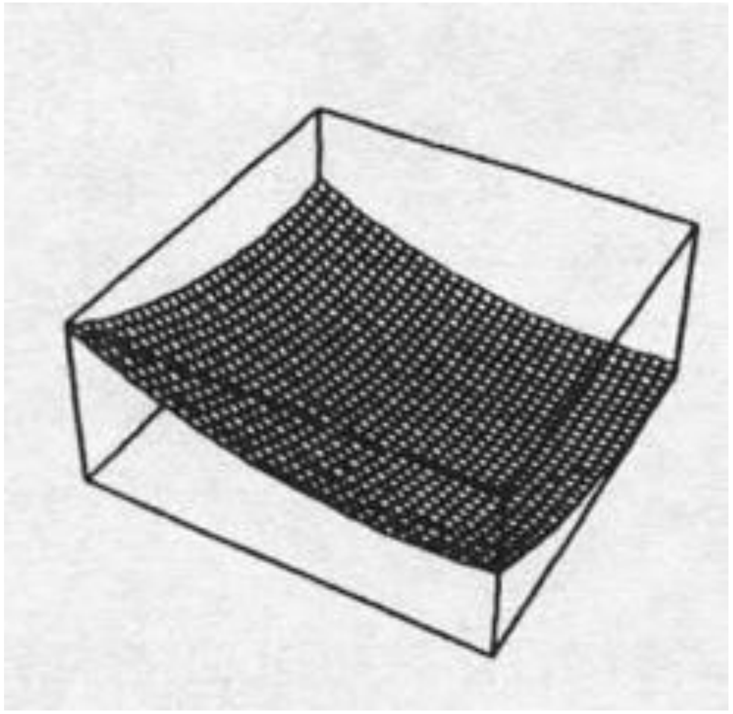
$$F(q) = -\nabla U(q) = -\begin{bmatrix} \partial U / \partial x \\ \partial U / \partial y \end{bmatrix}$$

Local techniques

Potential Field methods

- compute a repulsive force away from obstacles
- compute an attractive force toward the goal





Potential Field Method

- Compute an attractive force toward the goal

  - Attractive Potential

$$U_{goal}(q) = \frac{1}{2}\xi \| q - q_{goal} \|^2$$

$$F_{att}(q) = -\xi (q - q_{goal})$$

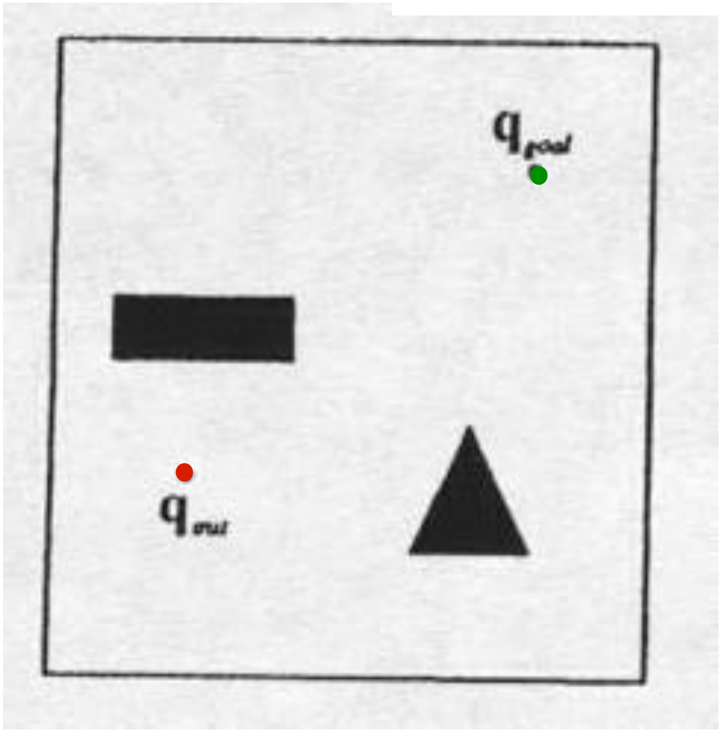
Parabolic

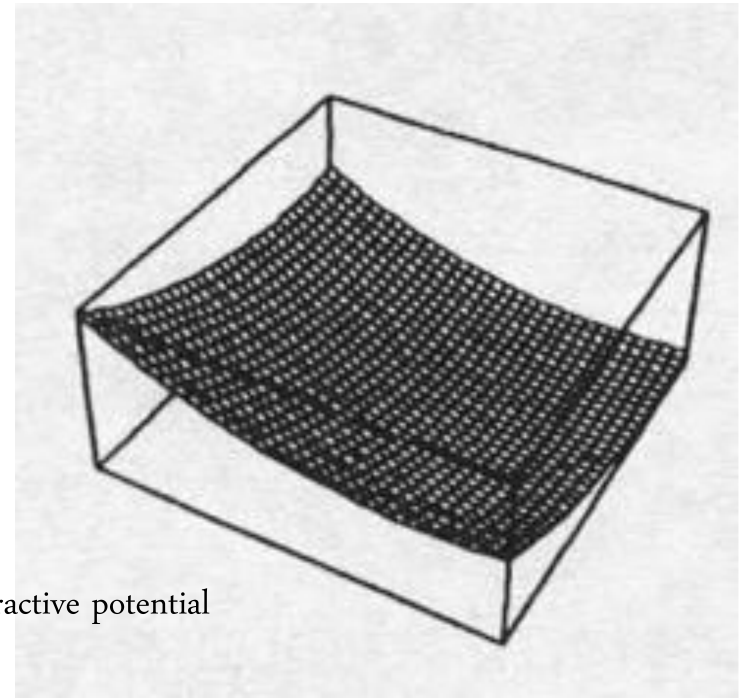
Positive or null

Minimum at q_{goal}

Tends to zero when the robot gets closer to the goal configuration

C- obstacles

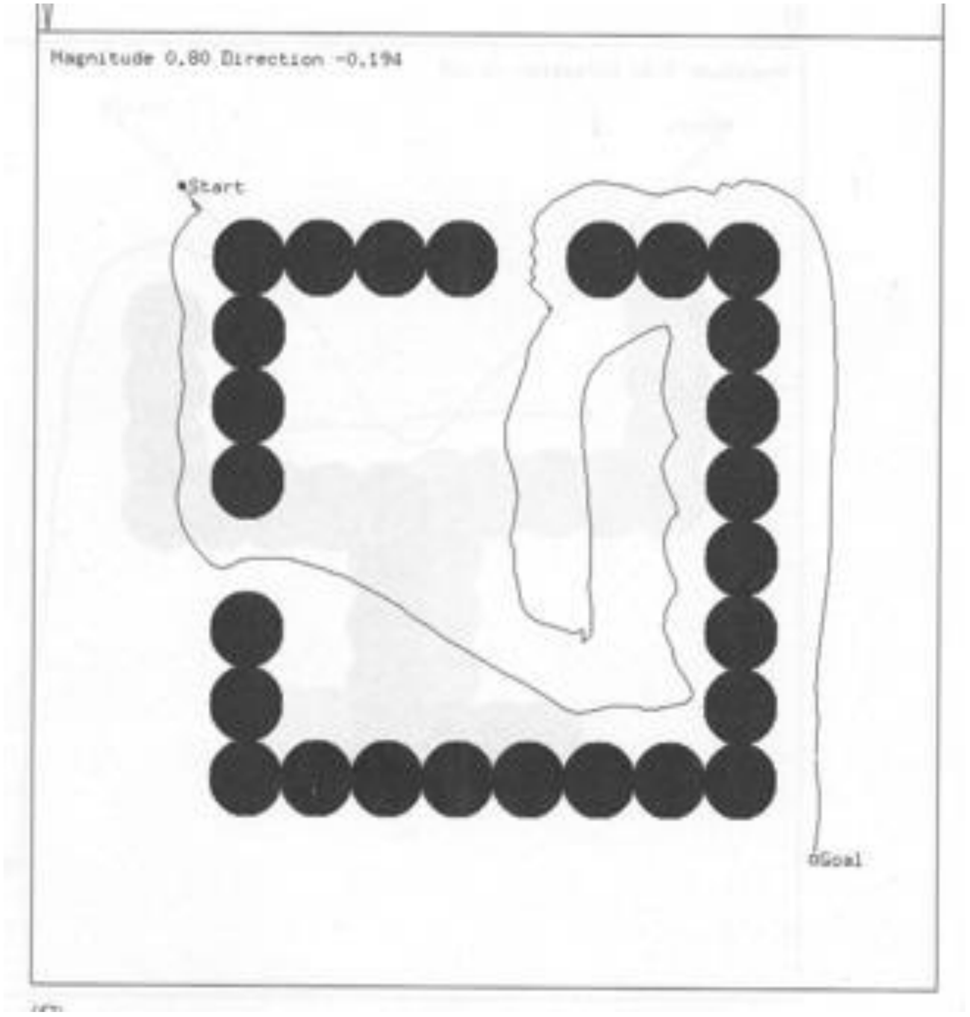




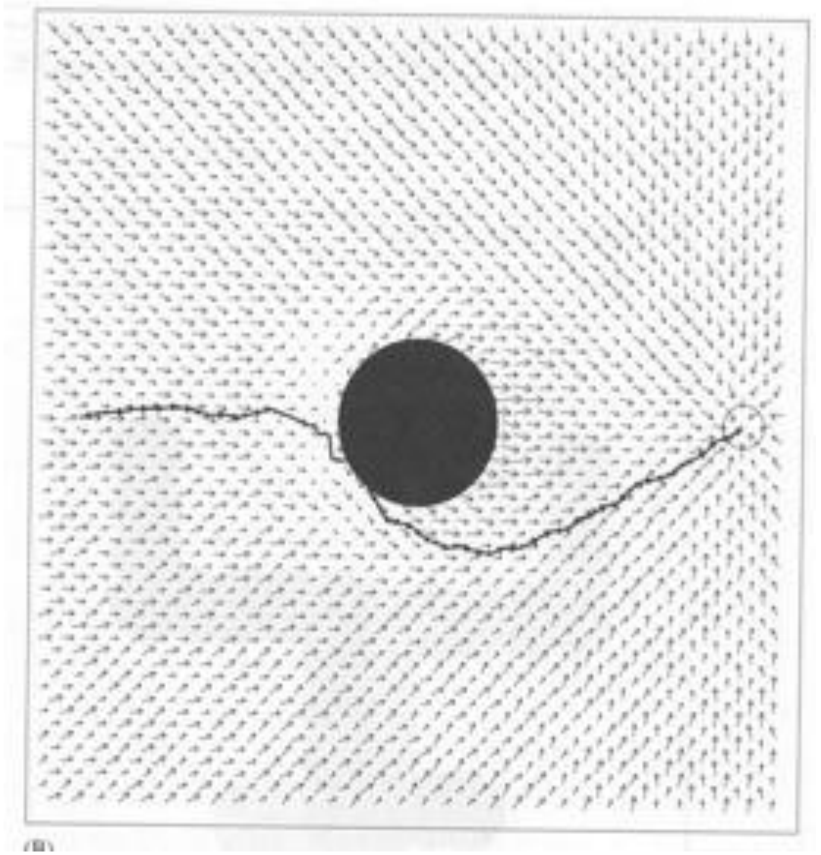
Attractive potential

Random search

Often combined with potential field methods to escape minima



"Filling in" local minima

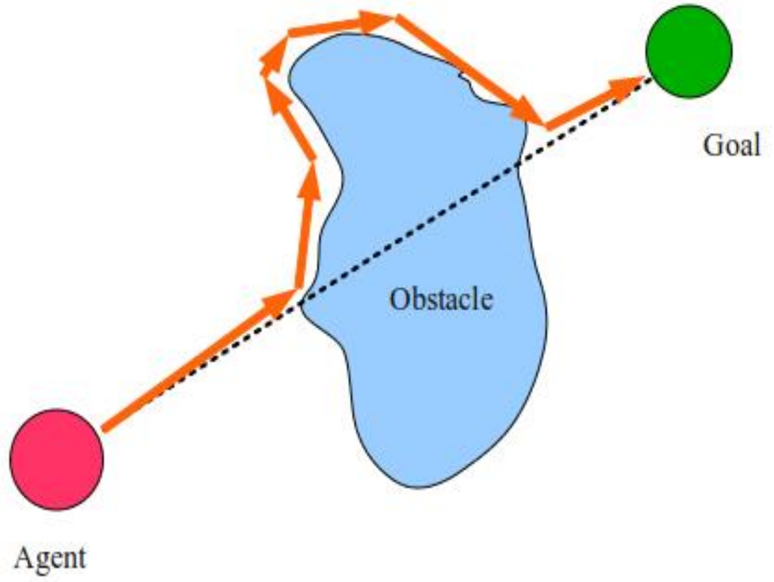


Random walks

random walks are not perfect...

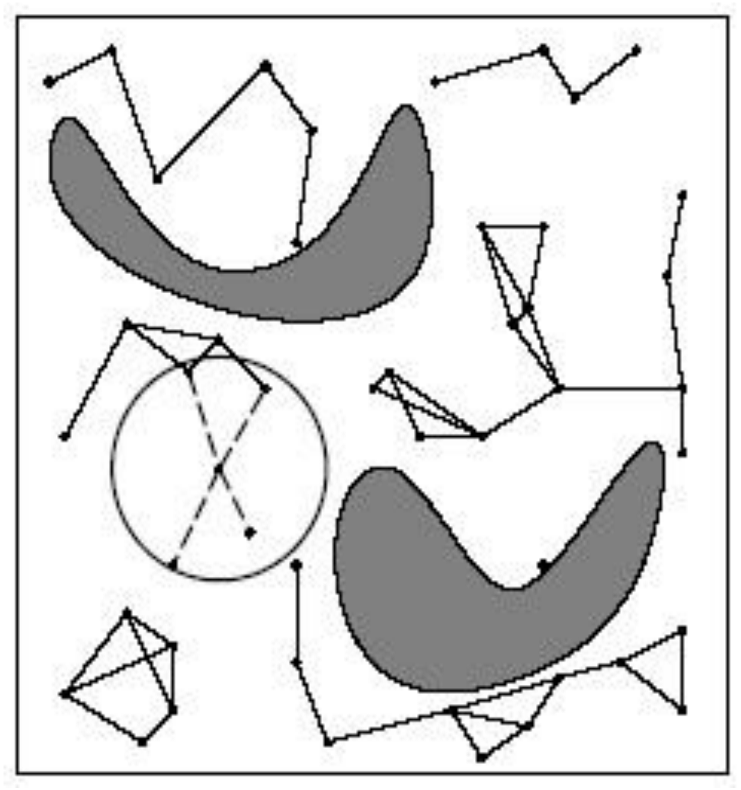
Bug Algorithms

- Idea: can we come up with a simple set of rules that allow us to perform behavior-based travel to goal (no explicit global plan?)
- Yes: follow the intuition of human labyrinth solving: “Always turn left”
- For the most part, this works well, but we can devise adversarial complex obstacles where smarts are needed:
    - BUG B approach…



Probabilistic Roadmaps (PRMs)

Kavraki, Latombe, Overmars, Svestka, 1994



- Developed for high-dimensional spaces
- Avoid pitfalls of classical grid search
- Random sampling of C_{free}
- Find neighbors of each sample (radius parameter)
- Local planner attempts connections
- “Probabilistic completeness" achieved

Other PRM variants: Obstacle-Based PRM (Amato, Wu, 1996); Sensor-based PRM (Yu, Gupta, 1998); Gaussian PRM (Boor, Overmars, van der Stappen, 1999); Medial axis PRMs (Wilmarth, Amato, Stiller, 1999; Pisula, Ho, Lin, Manocha, 2000; Kavraki, Guibas, 2000); Contact space PRM (Ji, Xiao, 2000); Closed-chain PRMs (LaValle, Yakey, Kavraki, 1999; Han, Amato 2000); Lazy PRM (Bohlin, Kavraki, 2000); PRM for changing environments (Leven, Hutchinson, 2000); Visibility PRM (Simeon, Laumond, Nissoux, 2000).

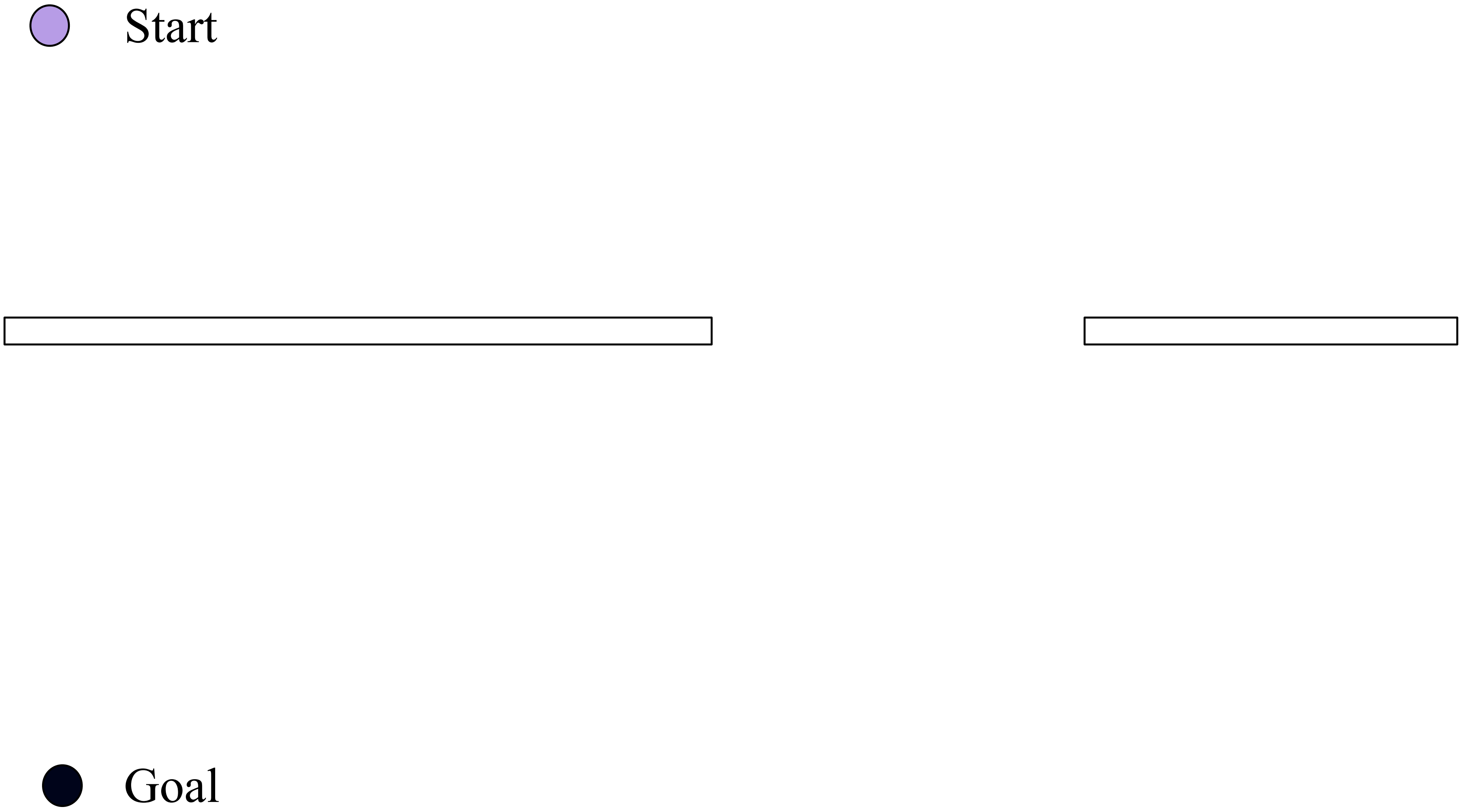
Rapidly-exploring Random Trees

- A point P in C is randomly chosen.
- The nearest vertex in the RRT is selected.
- A new edge is added from this vertex in the direction of P, at distance
- The further the algorithm goes, the more space is covered.

RRT-Connect

- We grow two trees, one from the beginning vertex and another from the end vertex
- Each time we create a new vertex, we try to greedily connect the two trees

RRT-Connect: example



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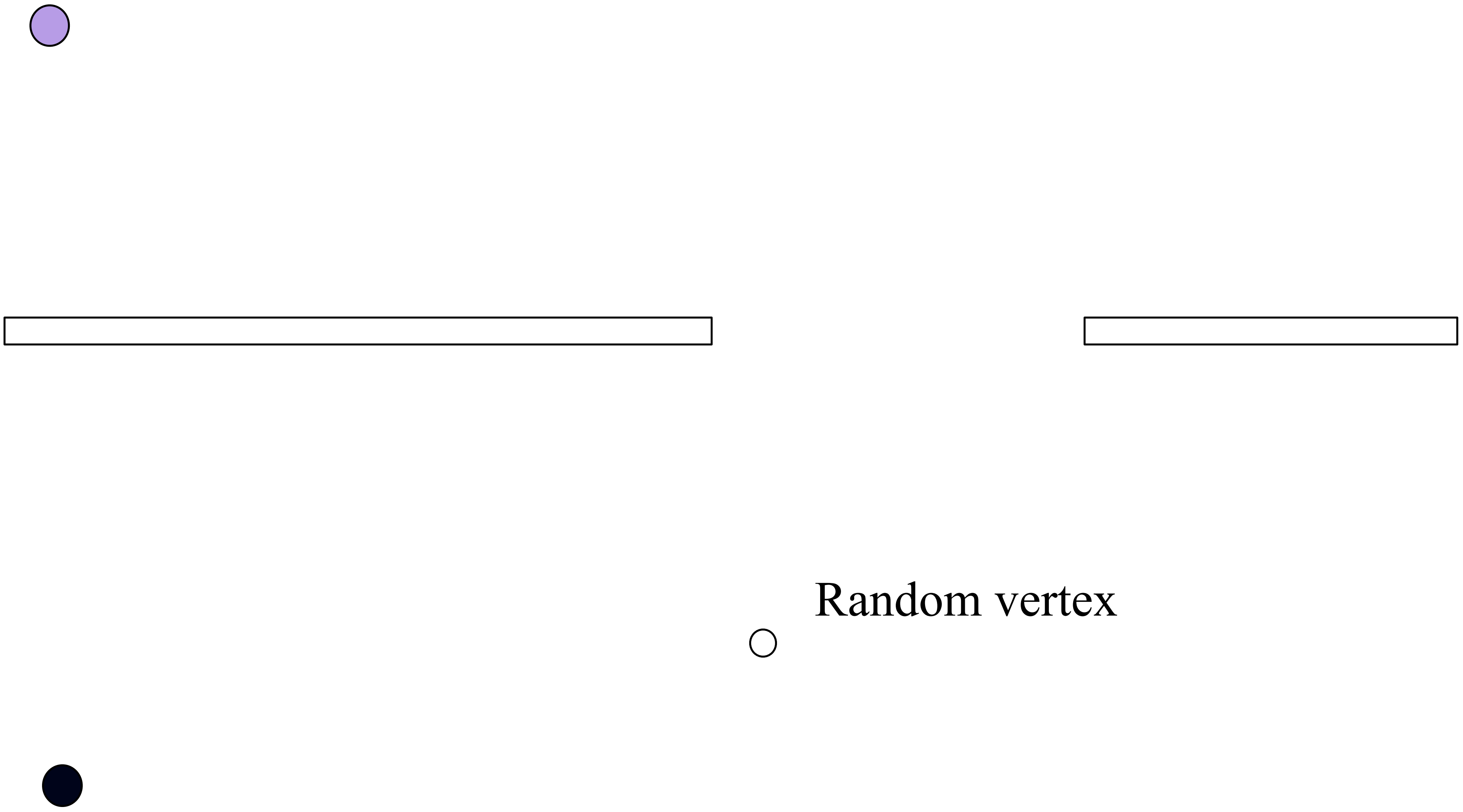

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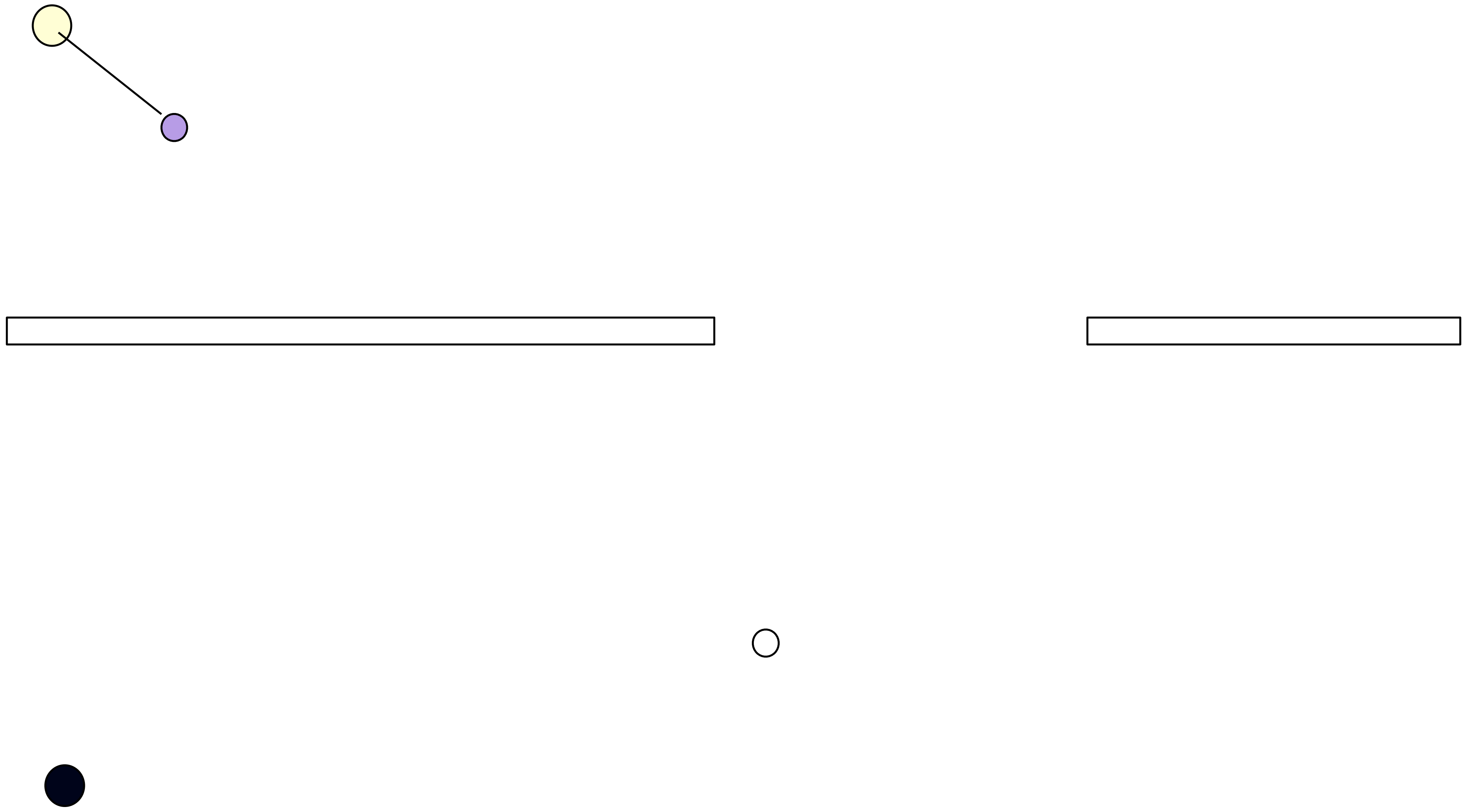

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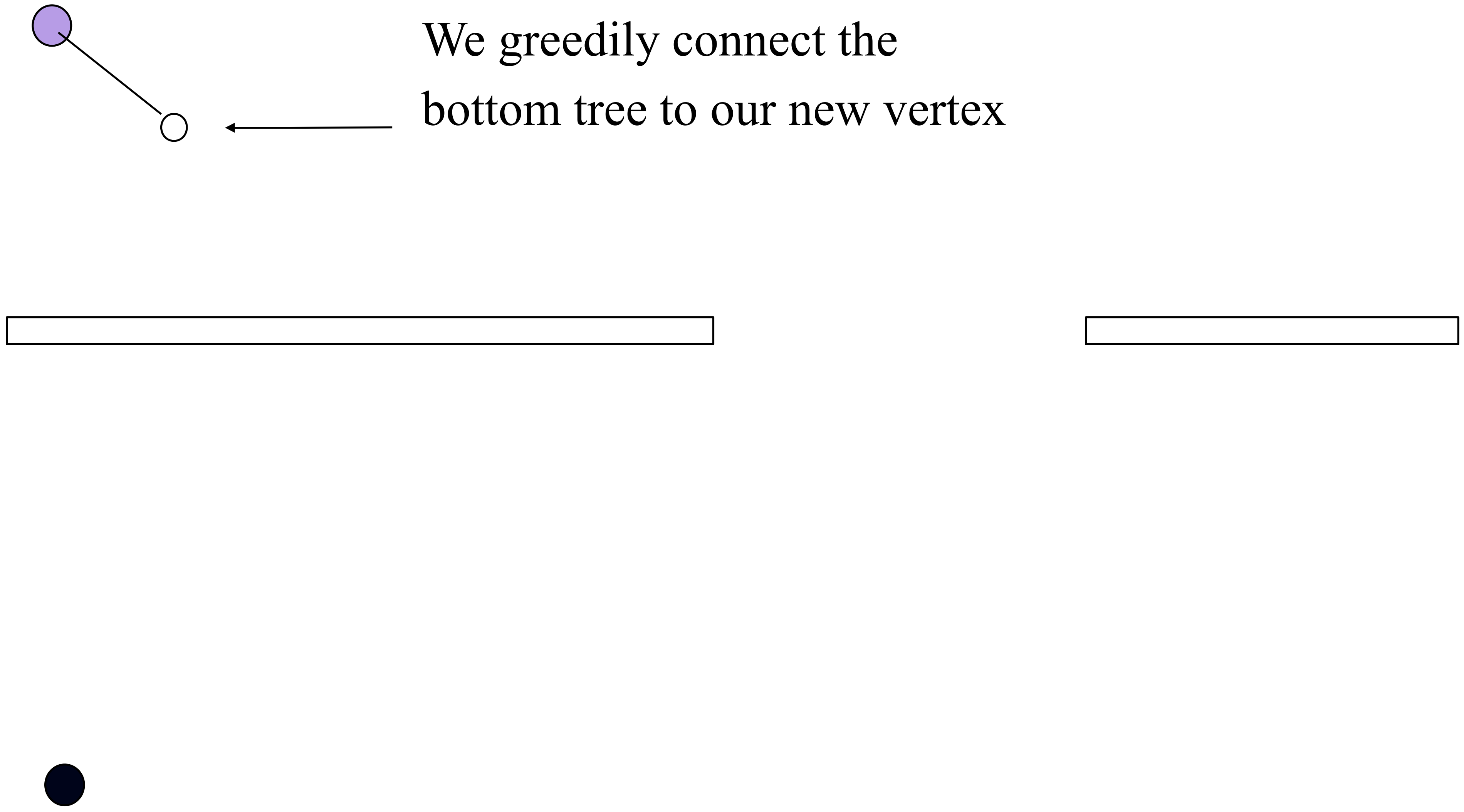
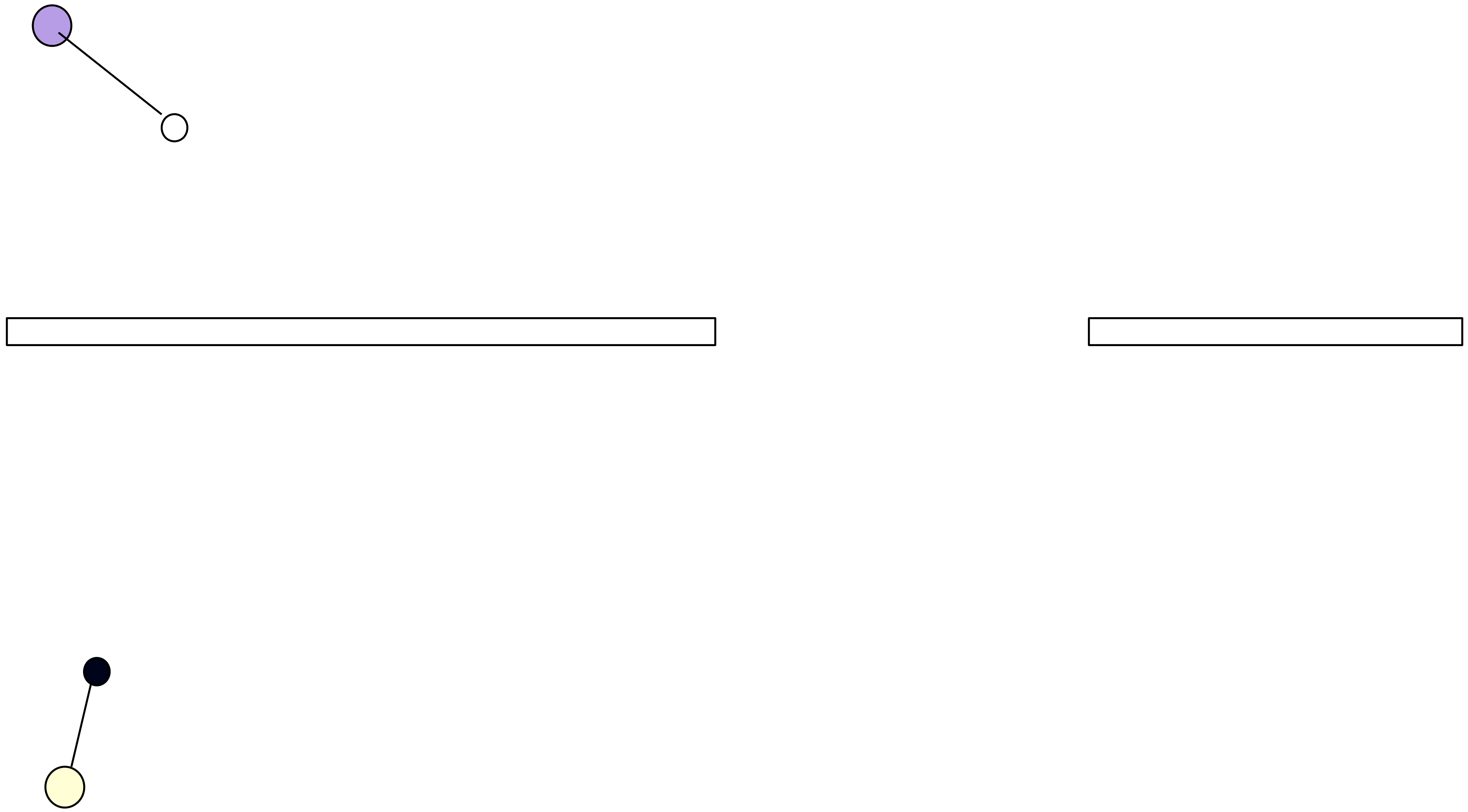


We greedily connect the bottom tree to our new vertex

RRT-Connect: example



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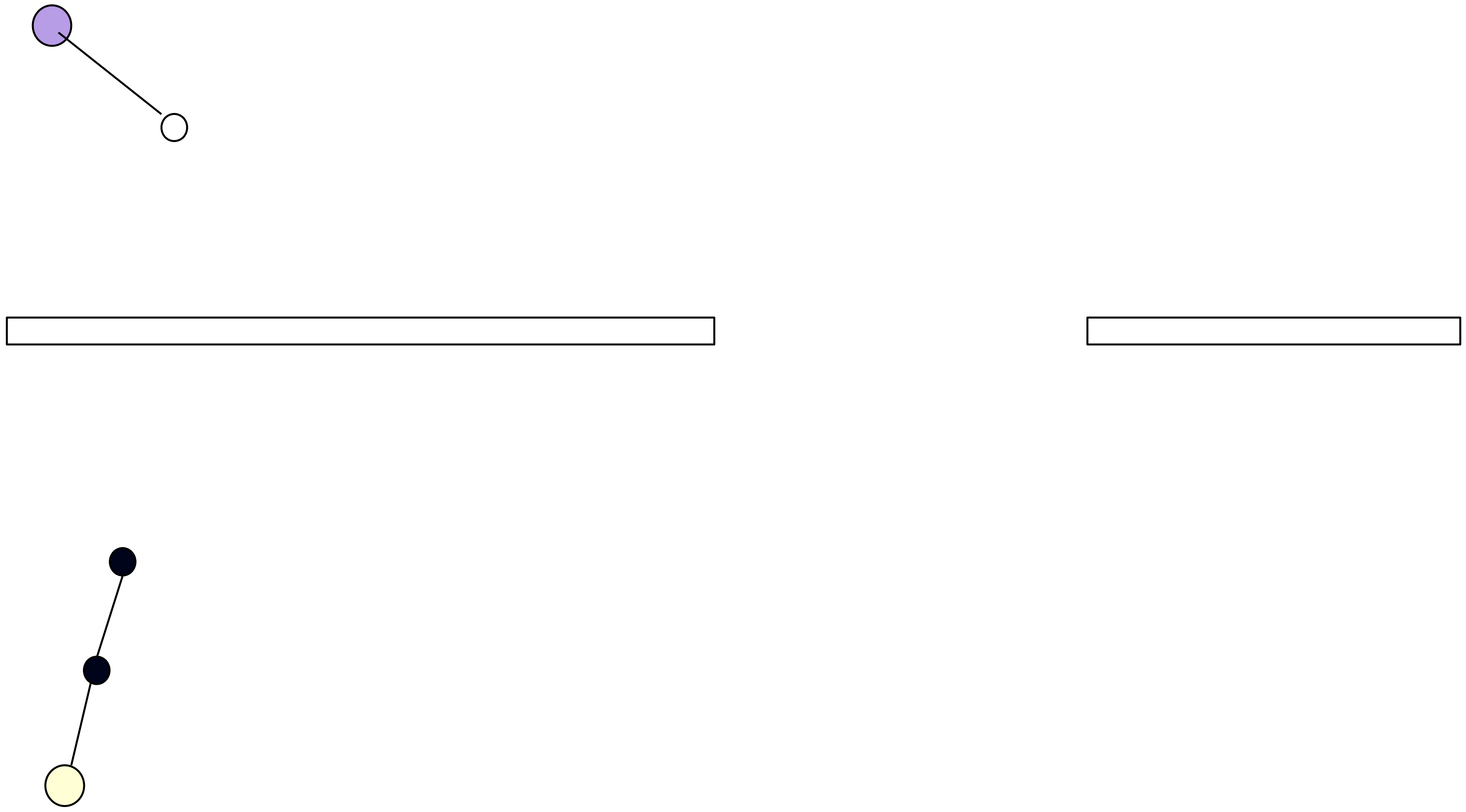

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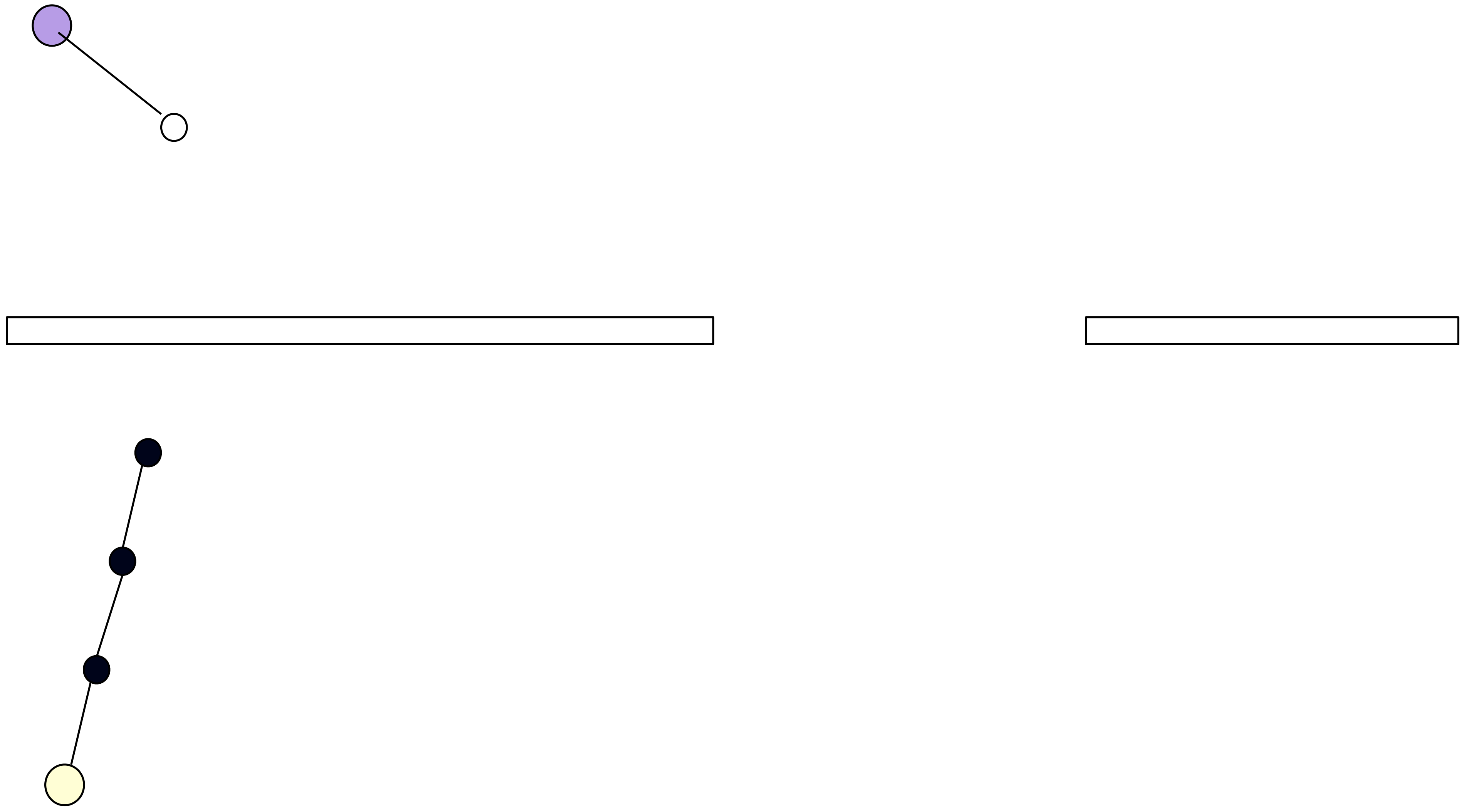

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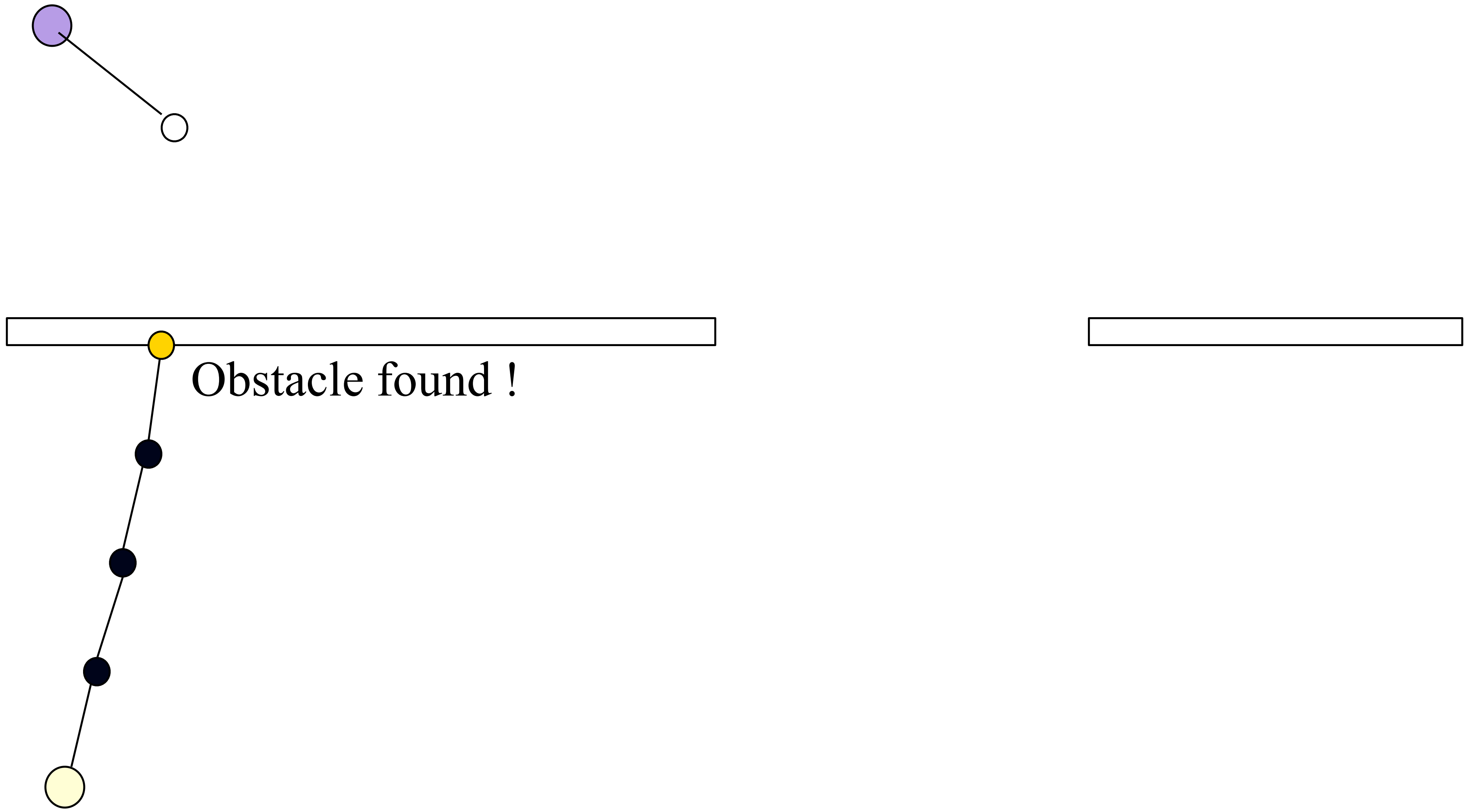

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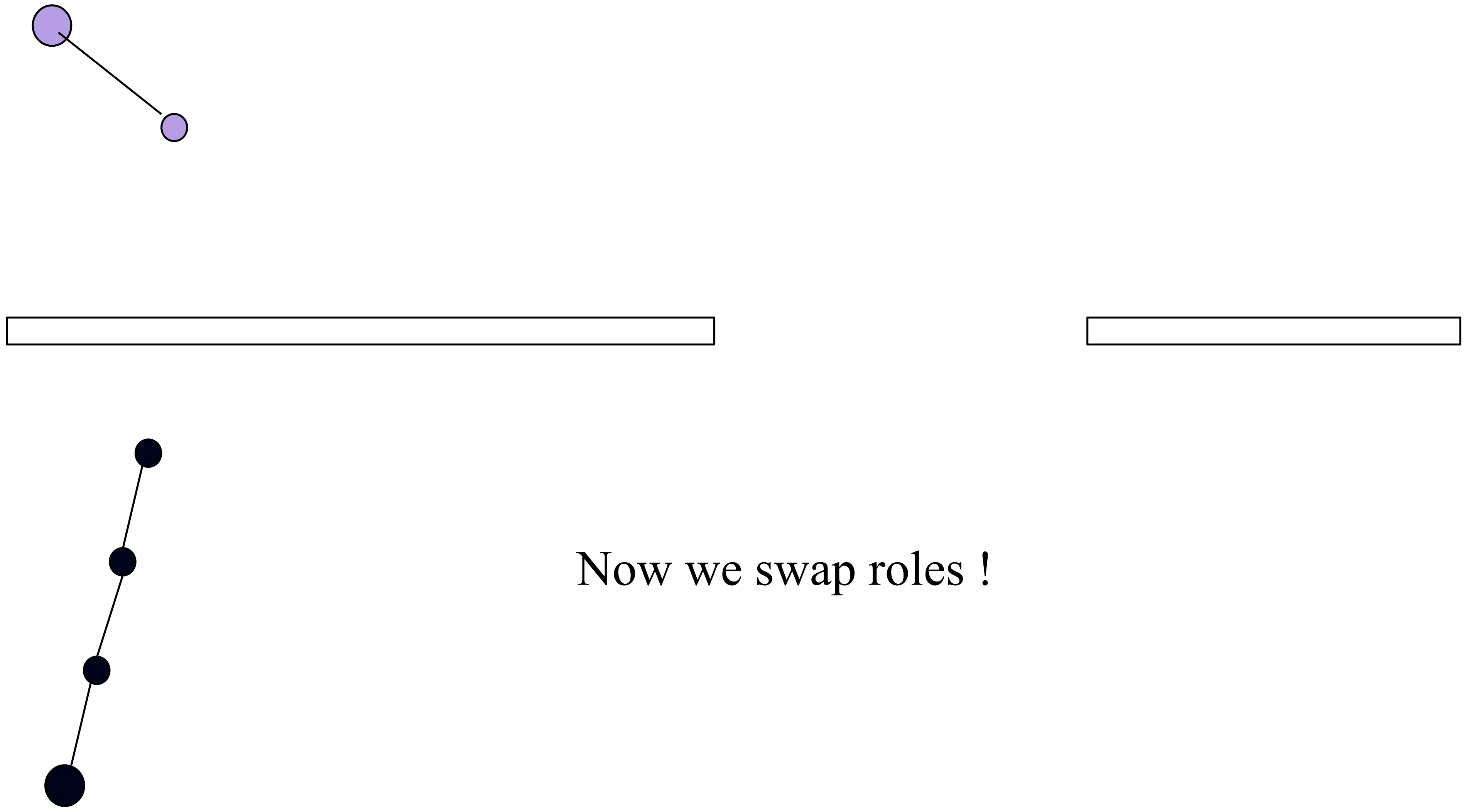
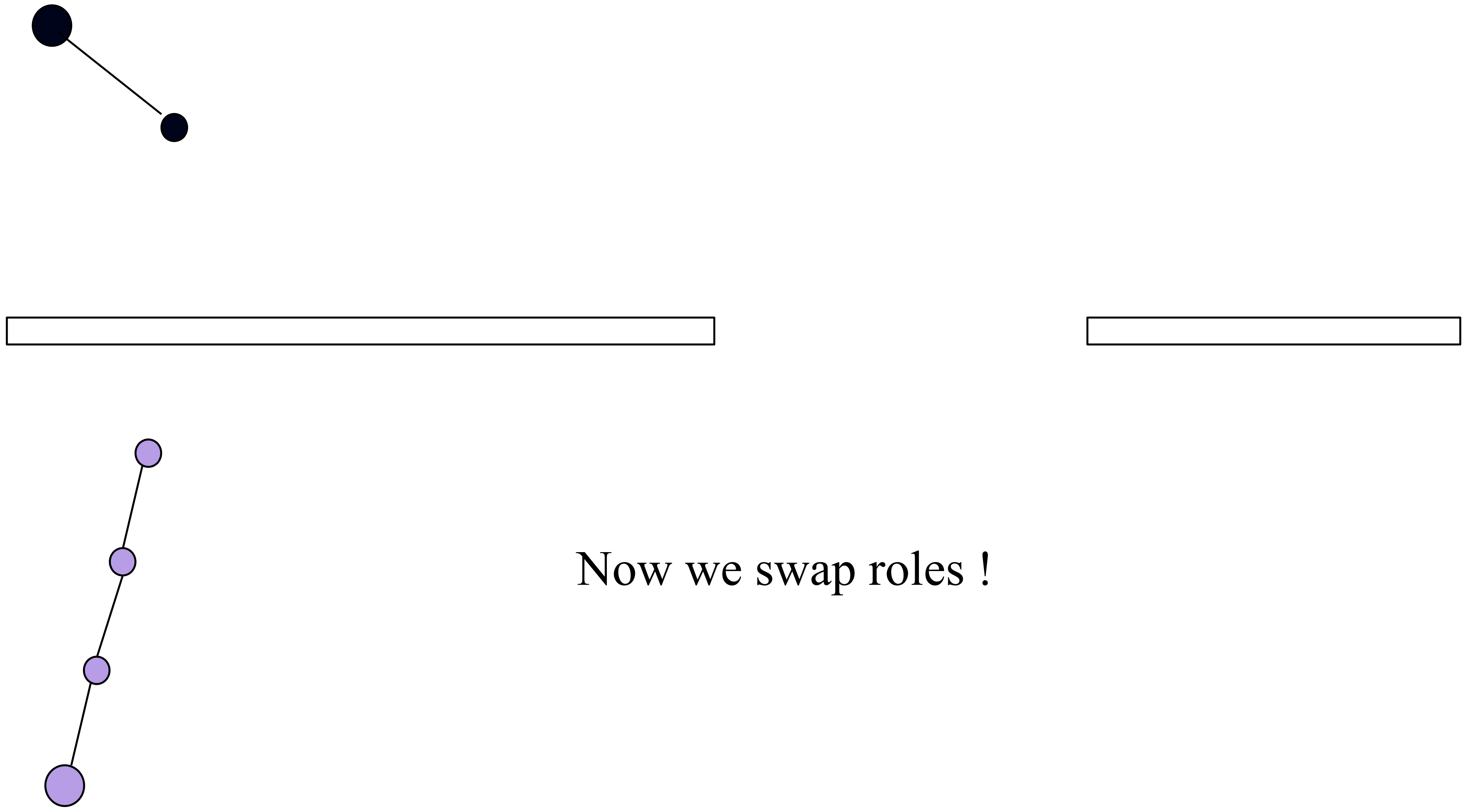


Now we swap roles !

RRT-Connect: example



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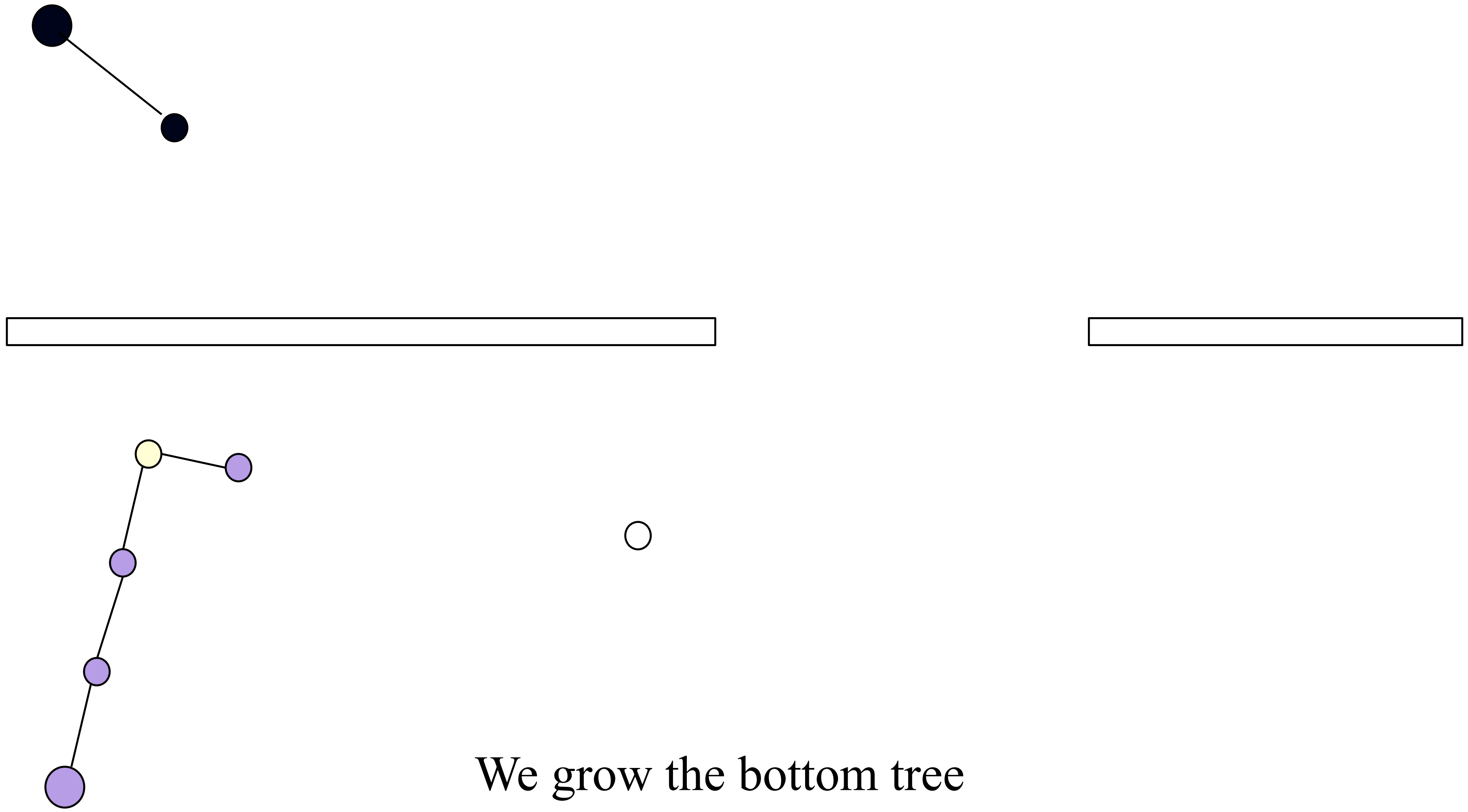
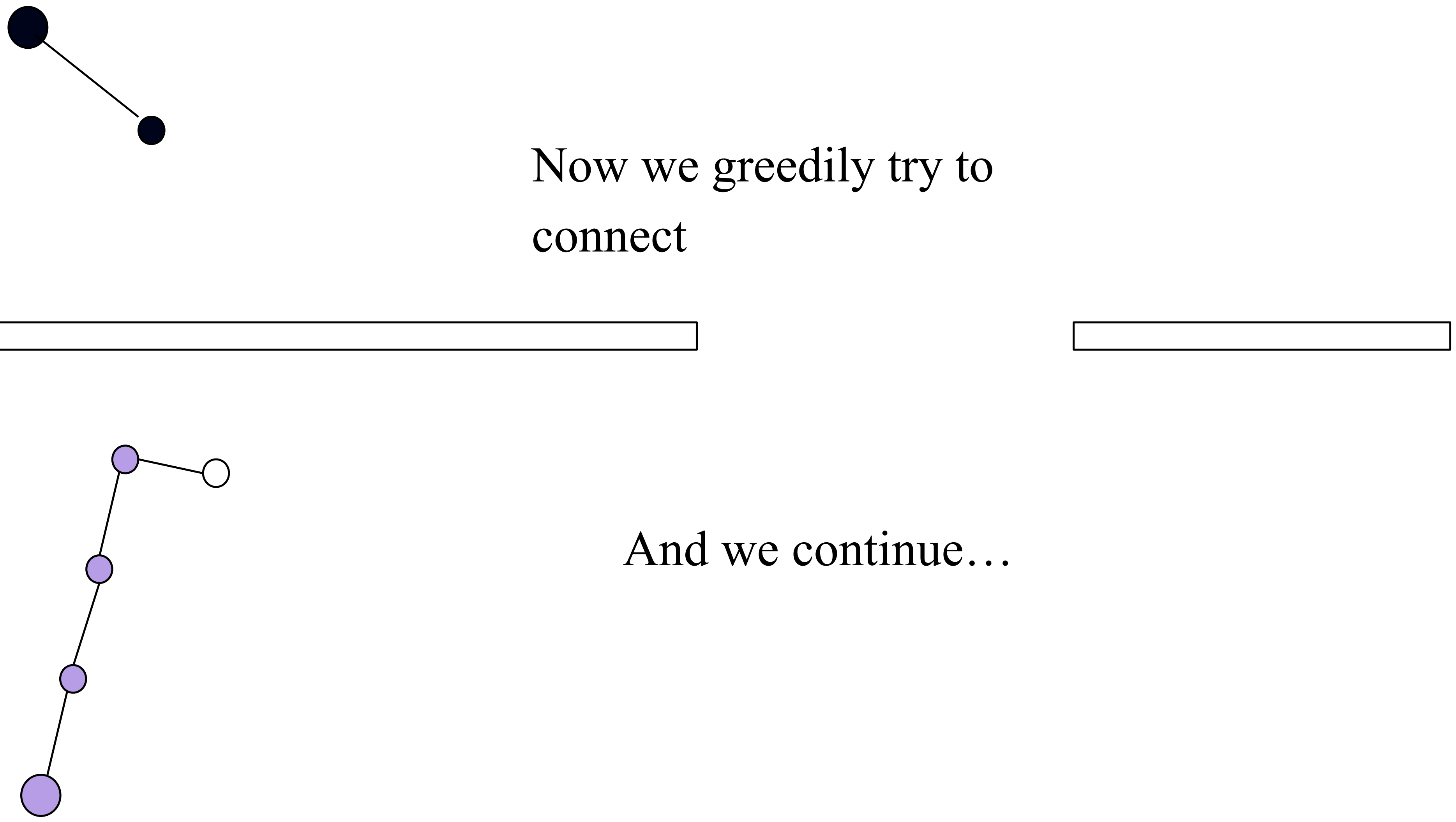


We grow the bottom tree

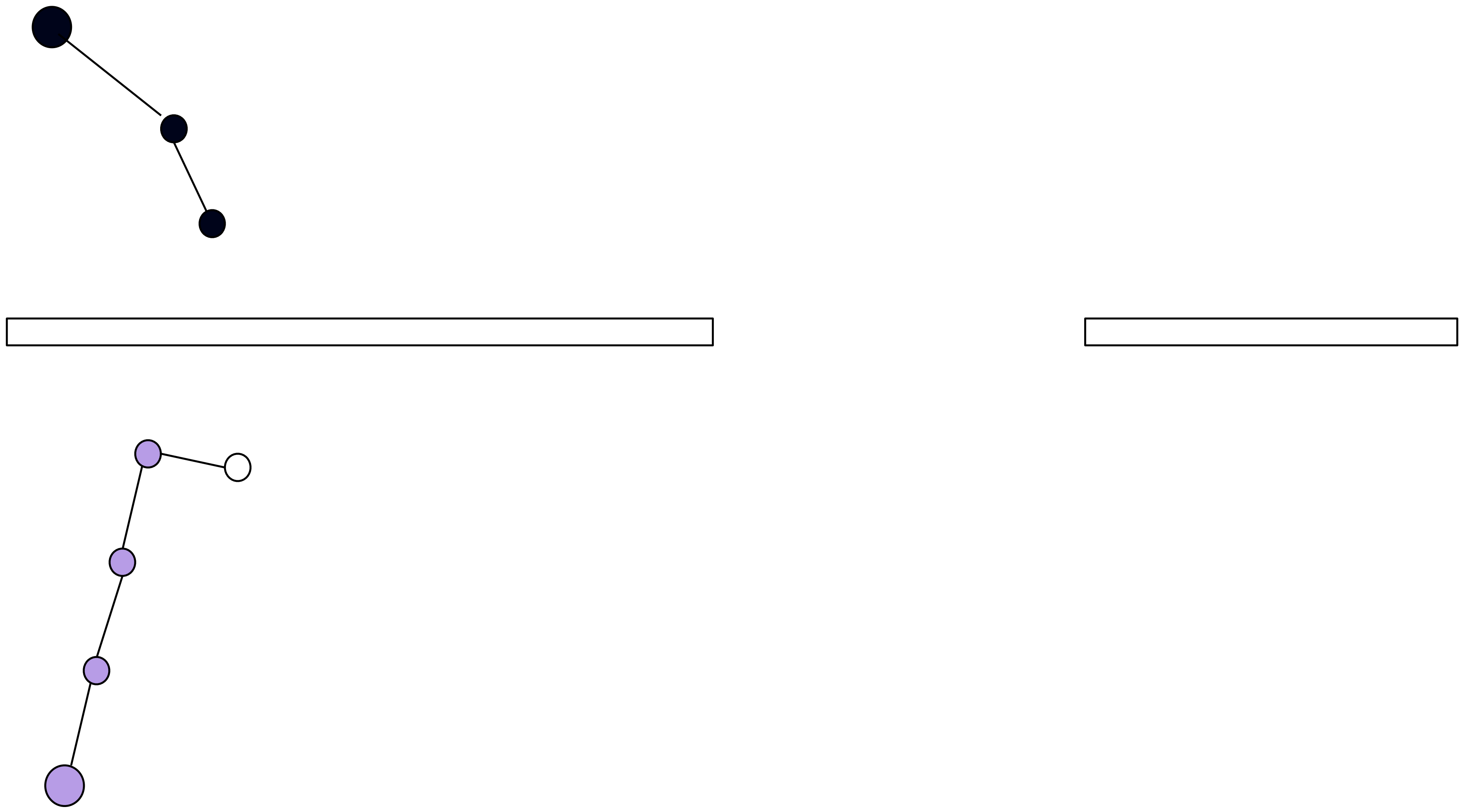
RRT-Connect: example



Now we greedily try to connect

And we continue…

RRT-Connect: example



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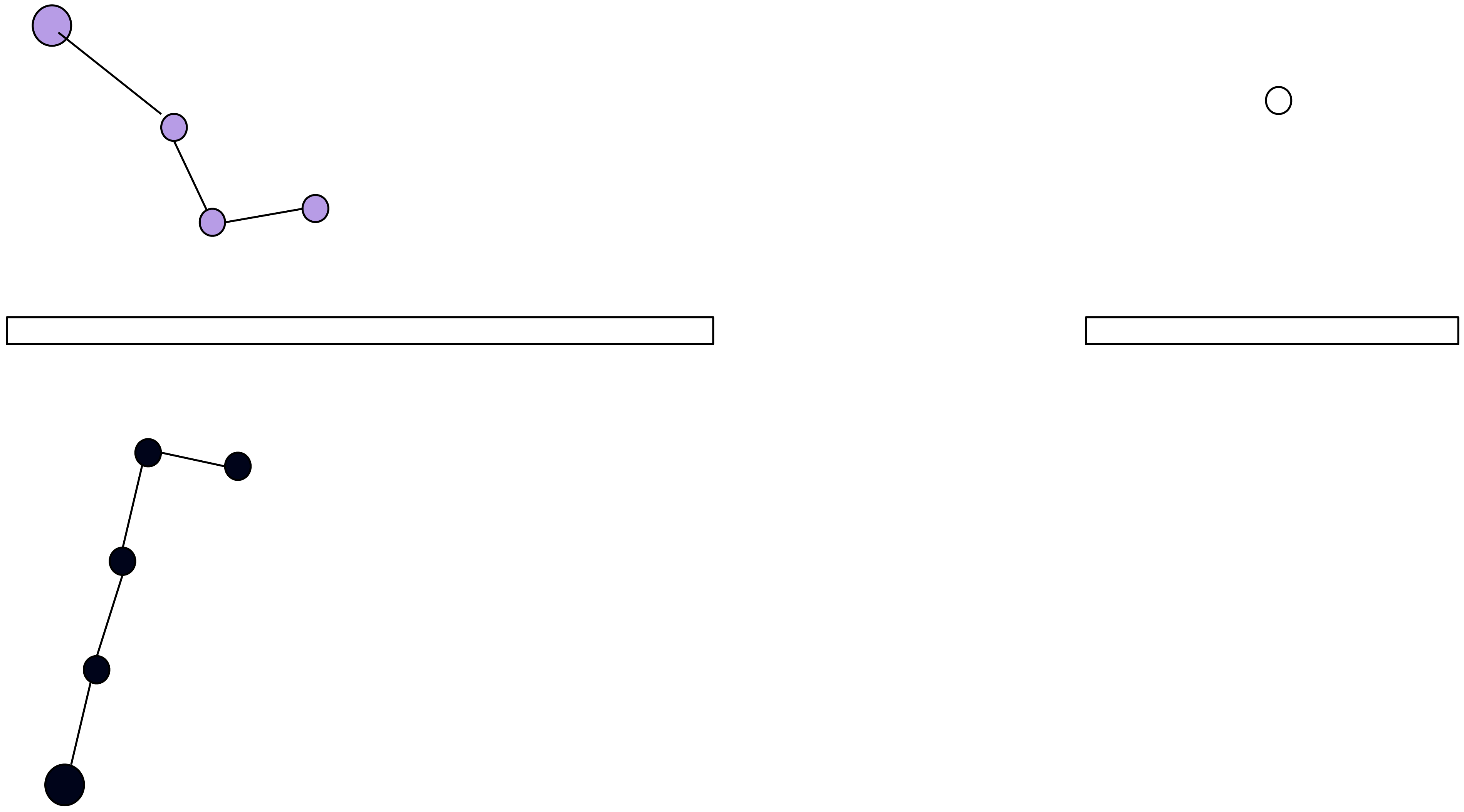

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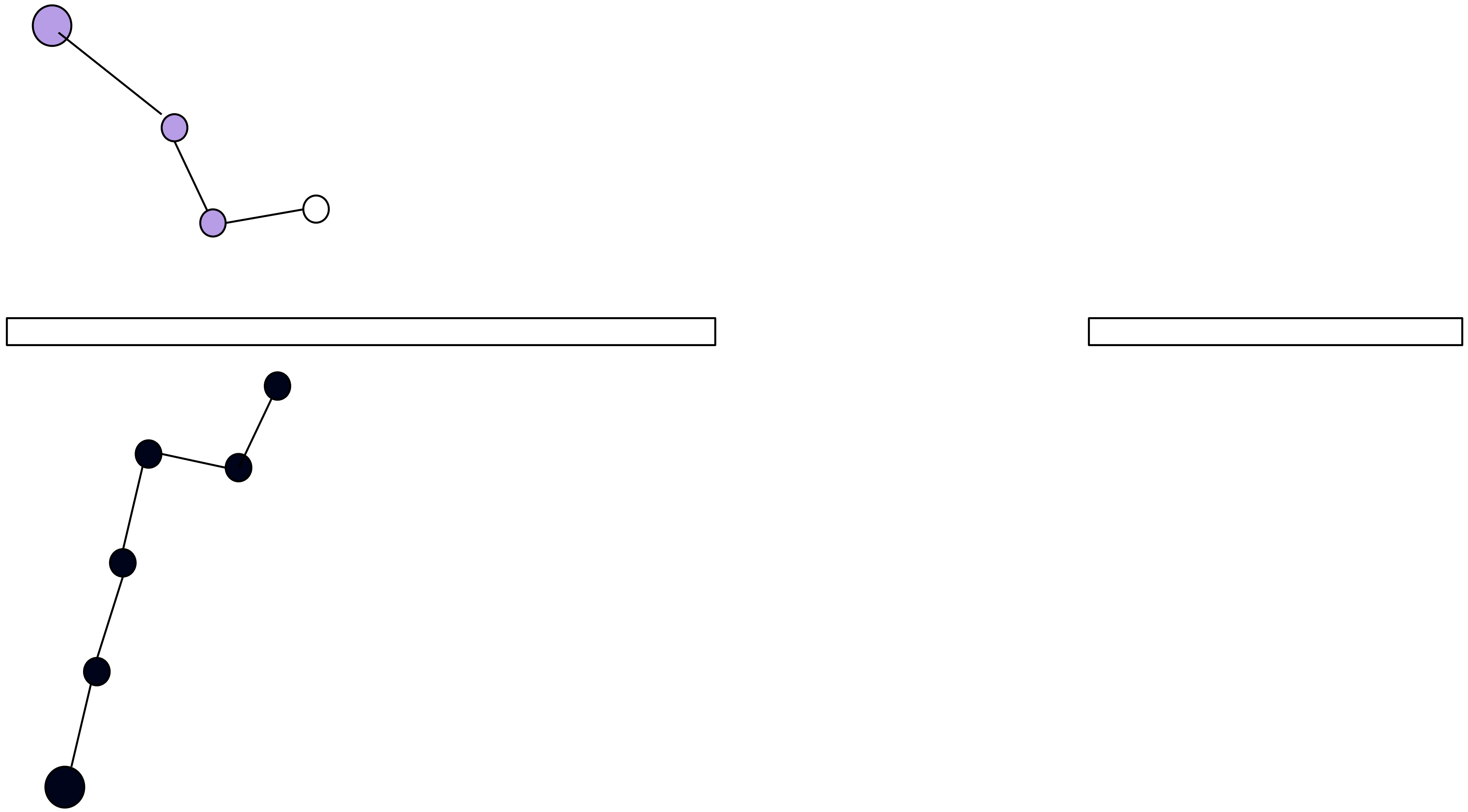

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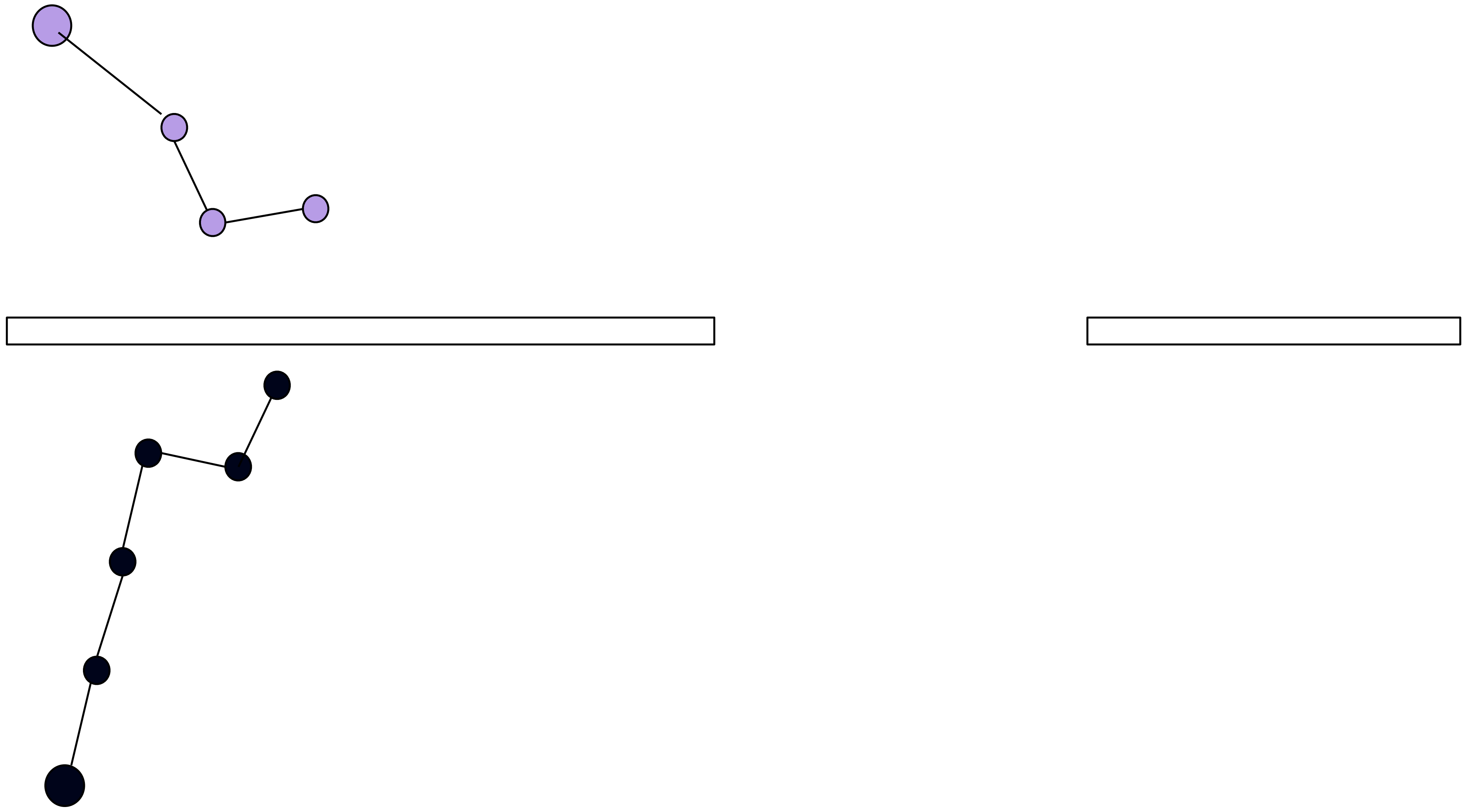

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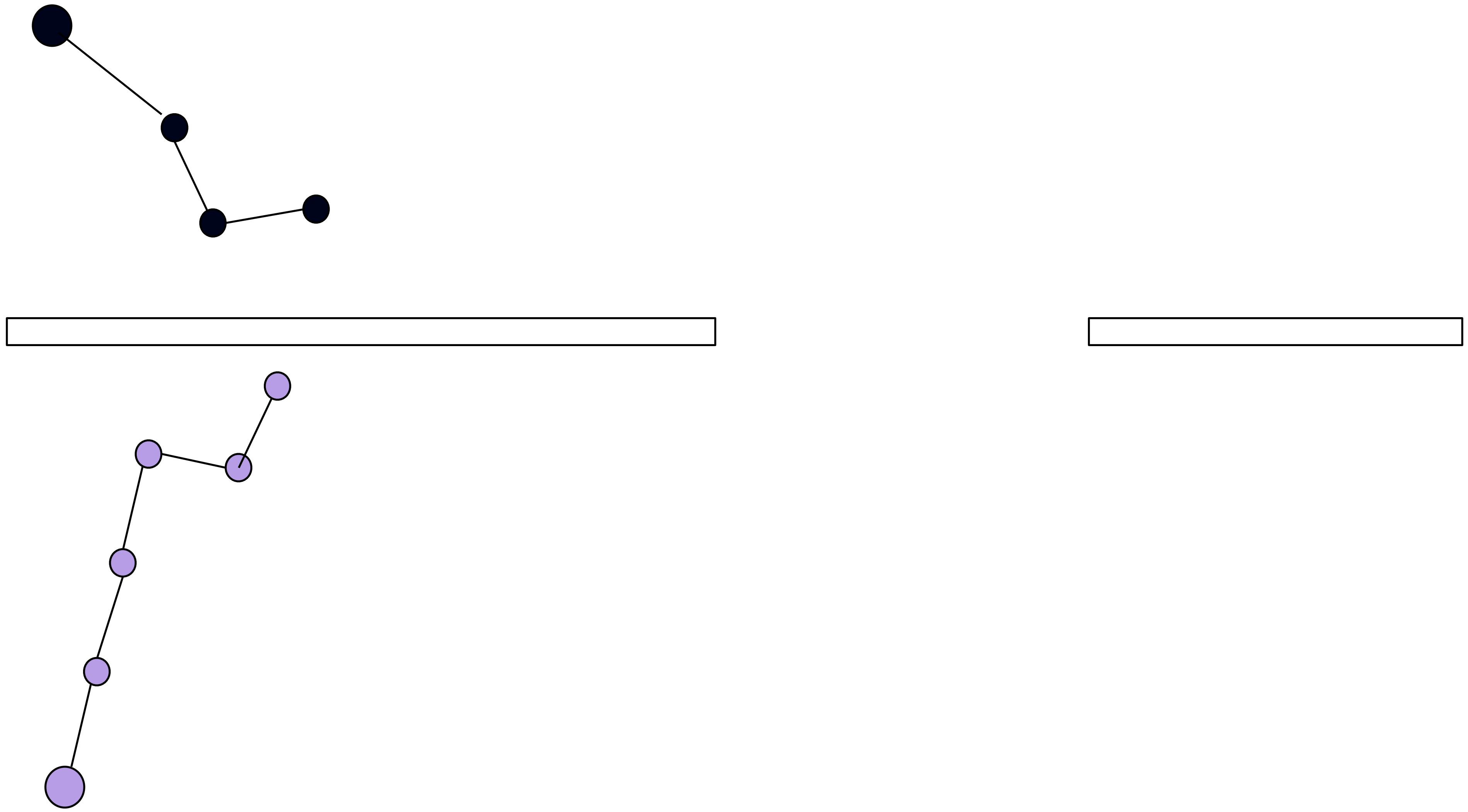

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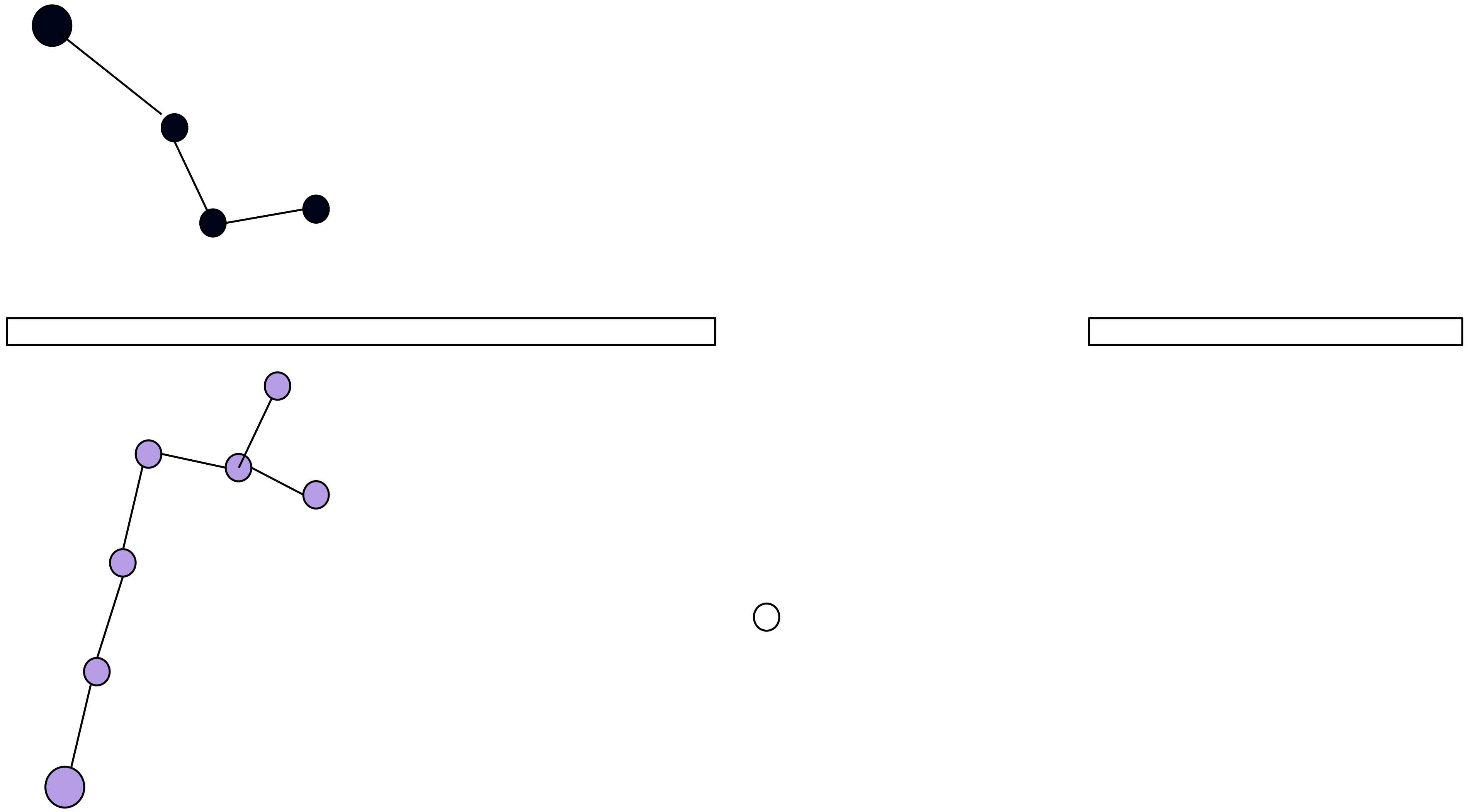

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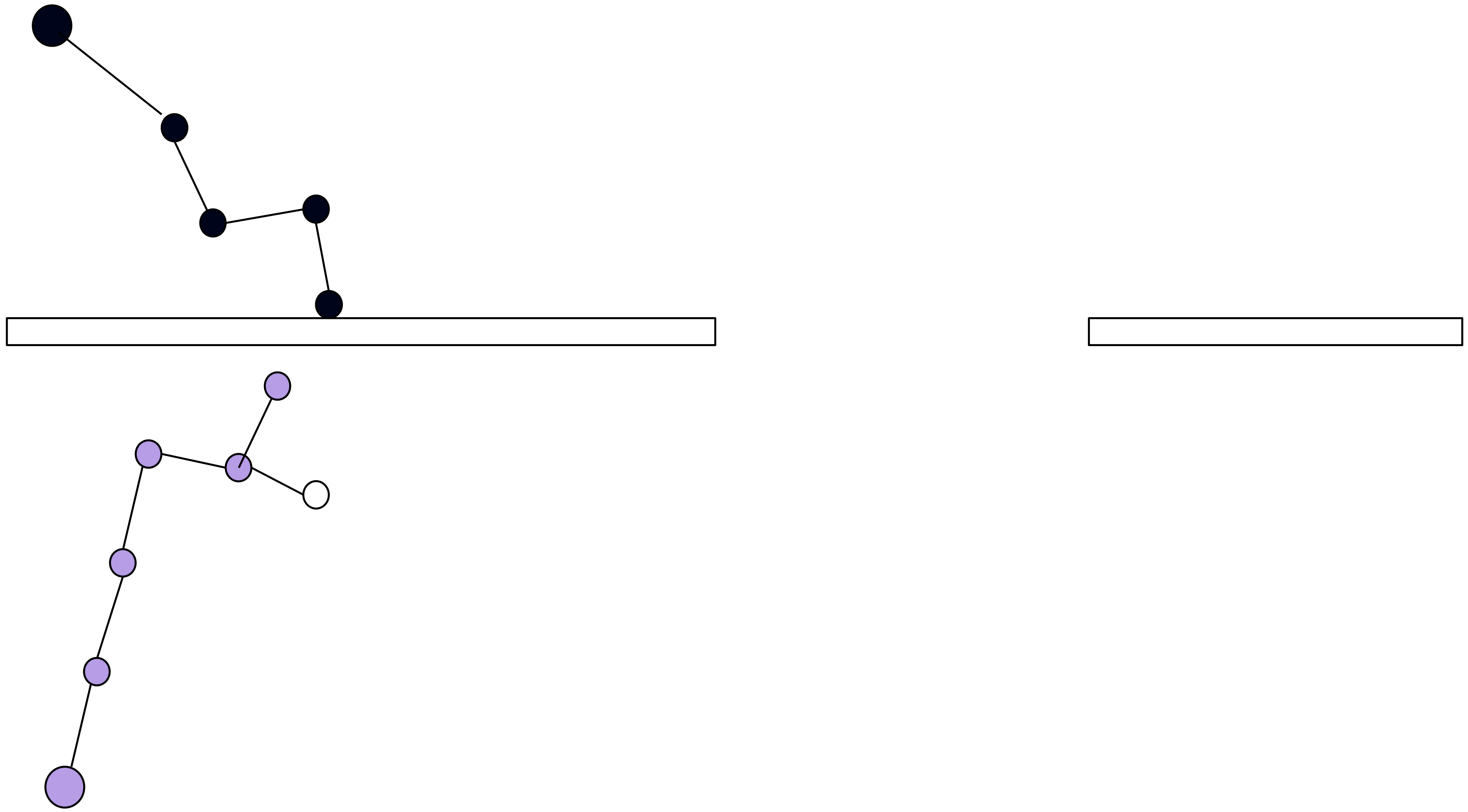

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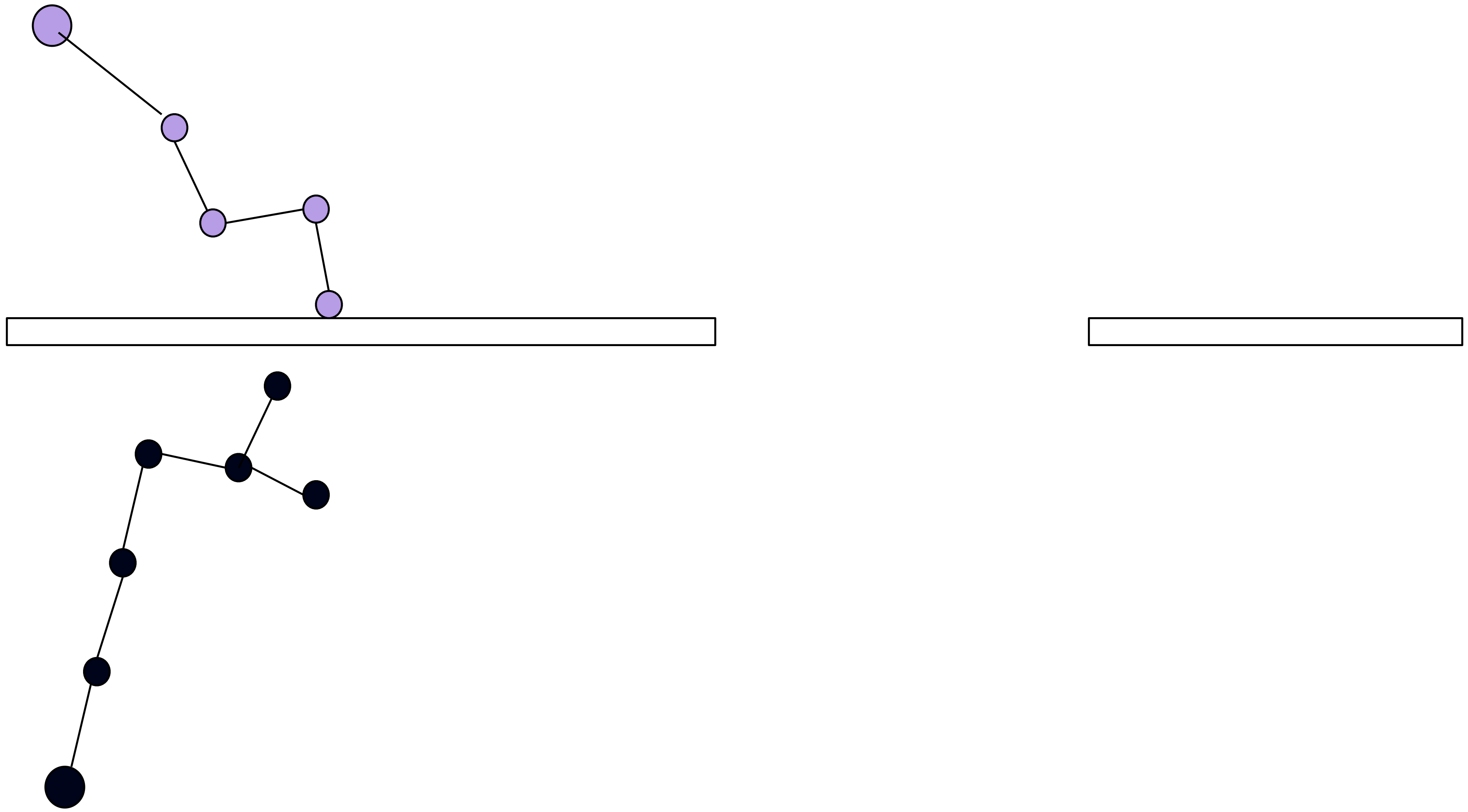

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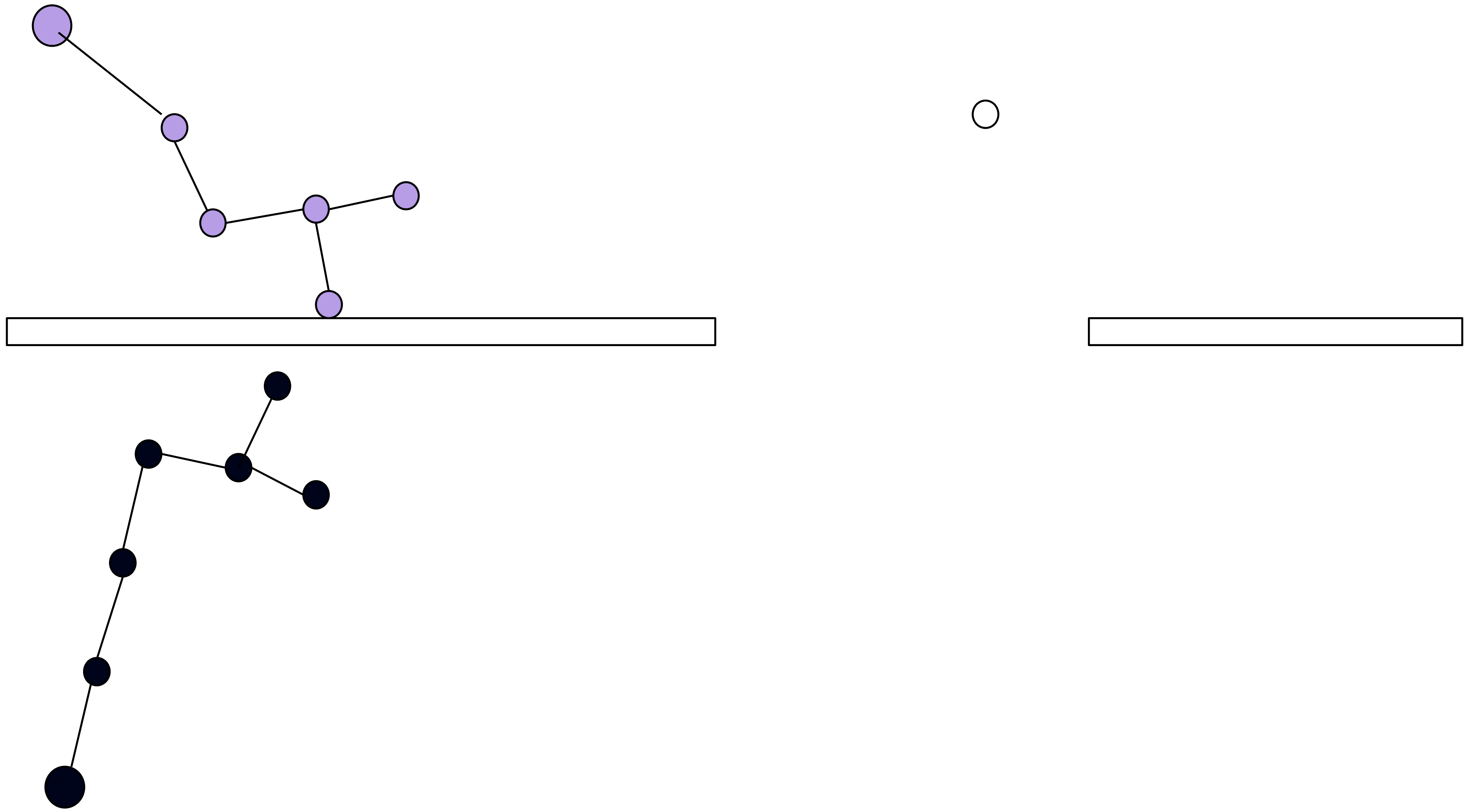

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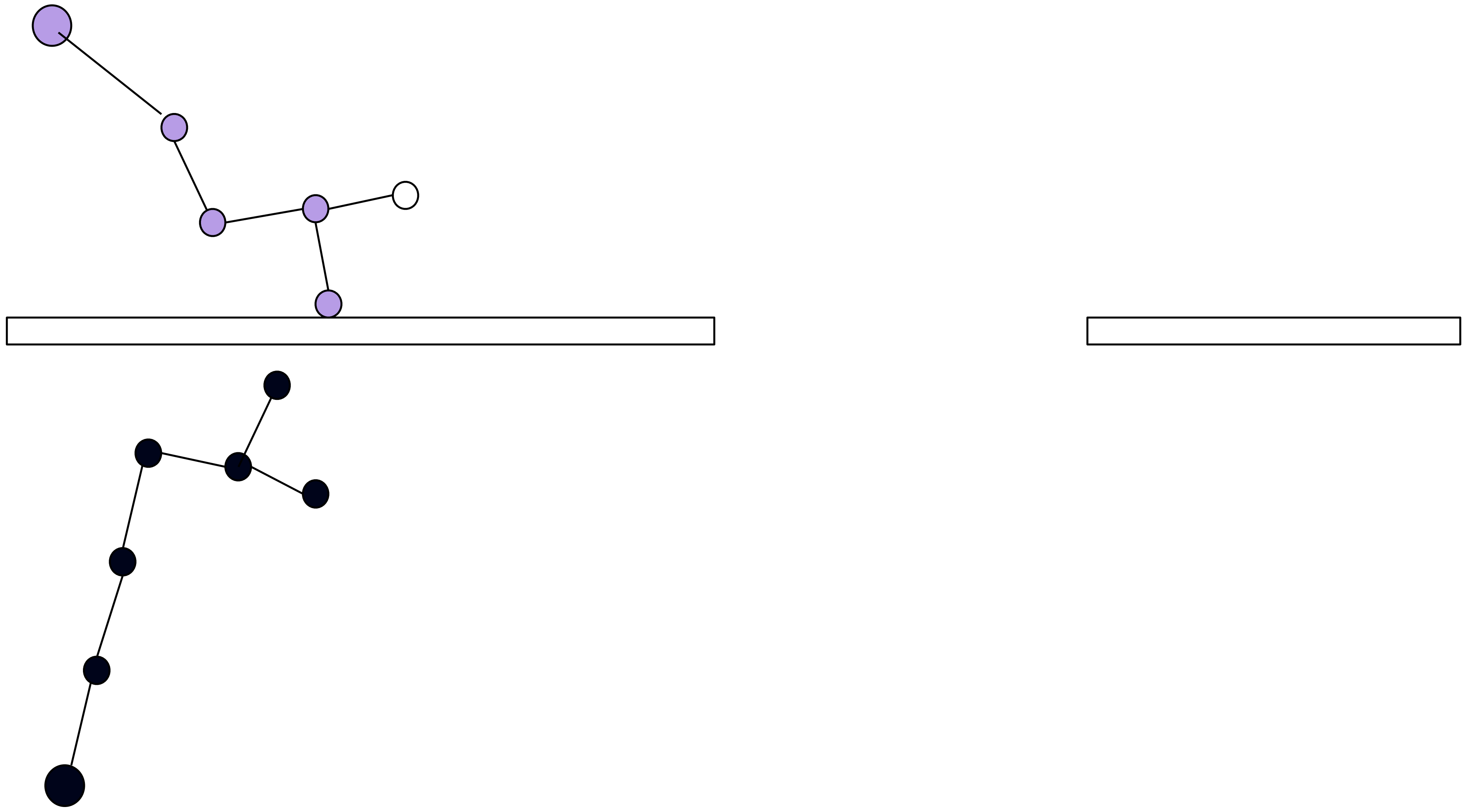

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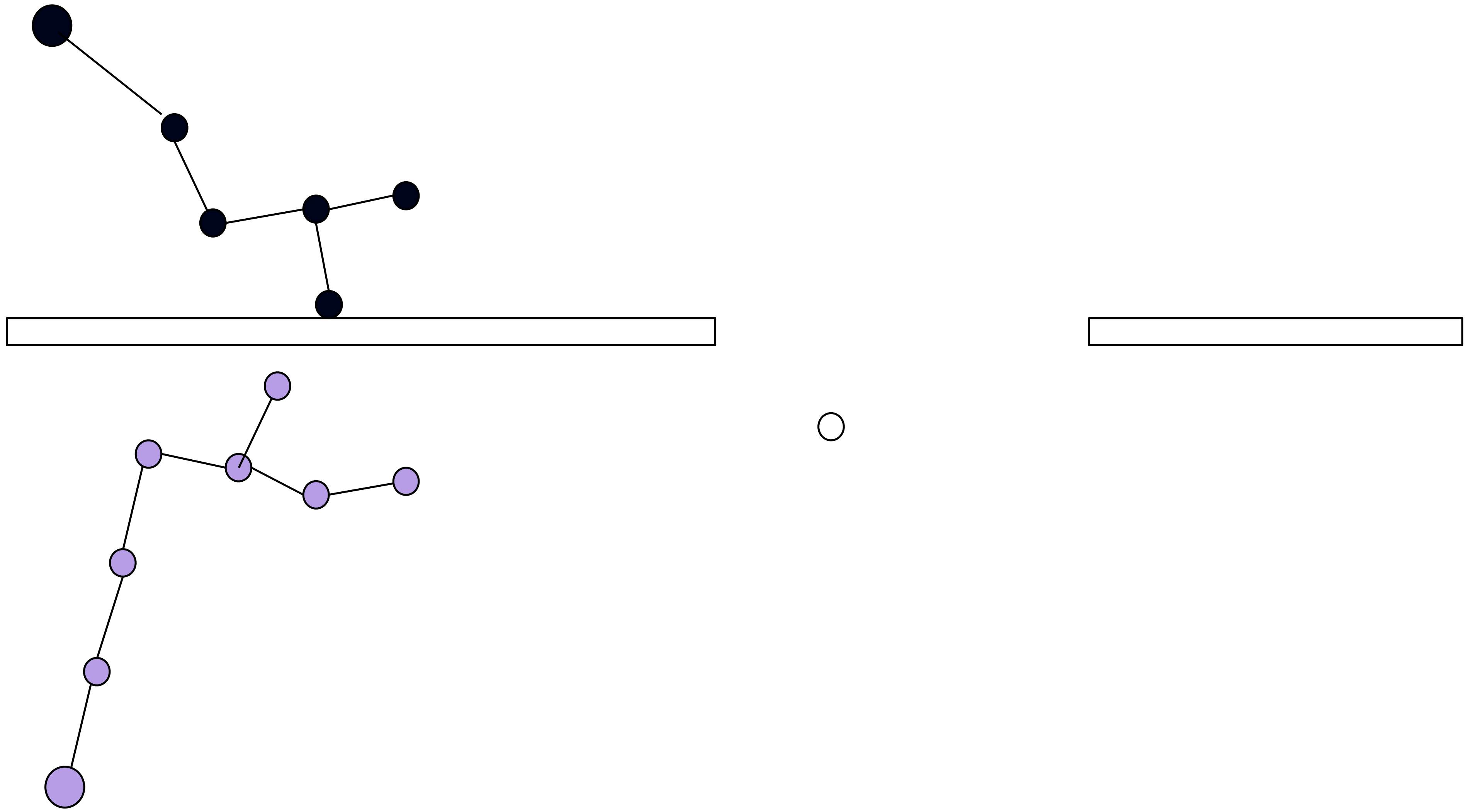

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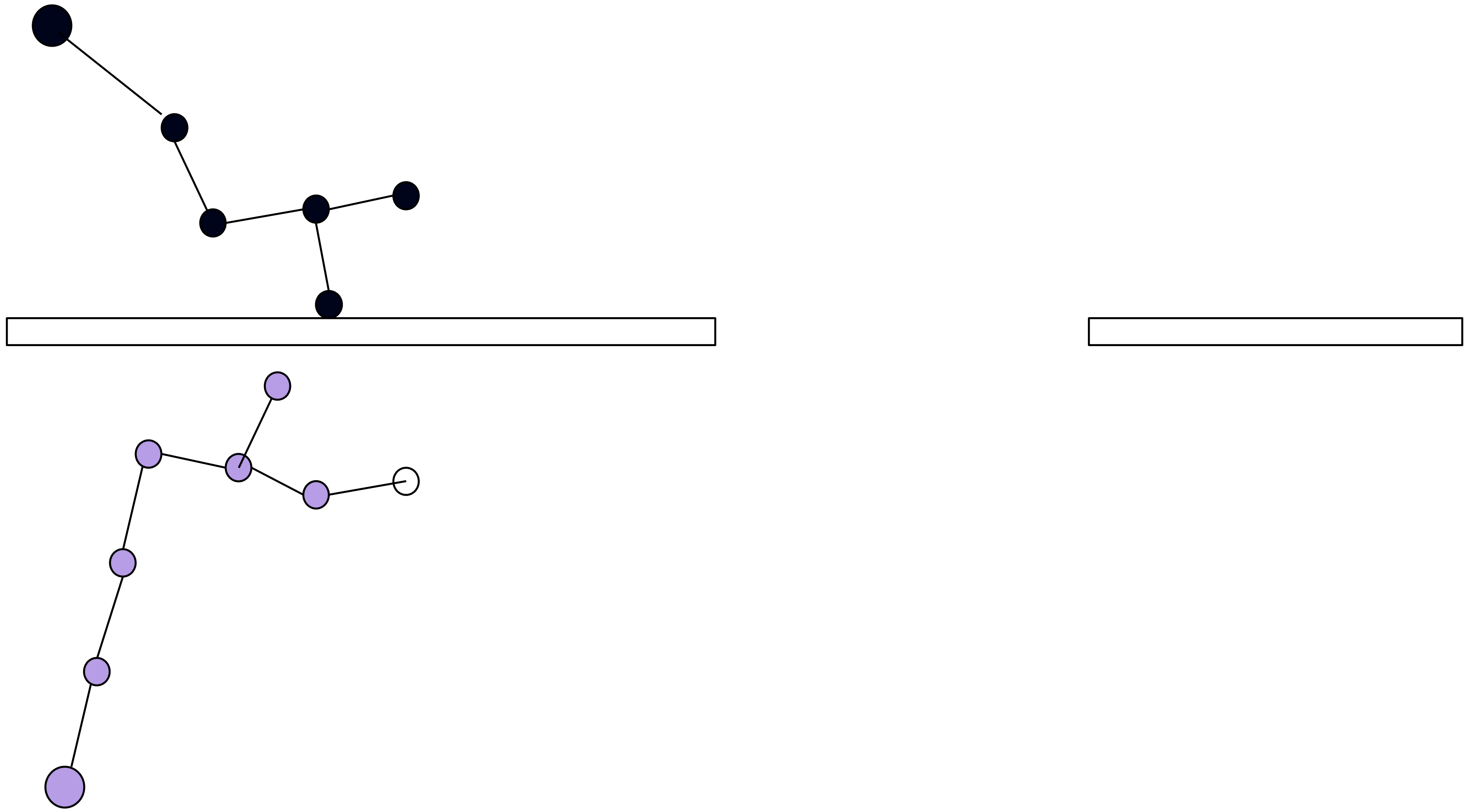

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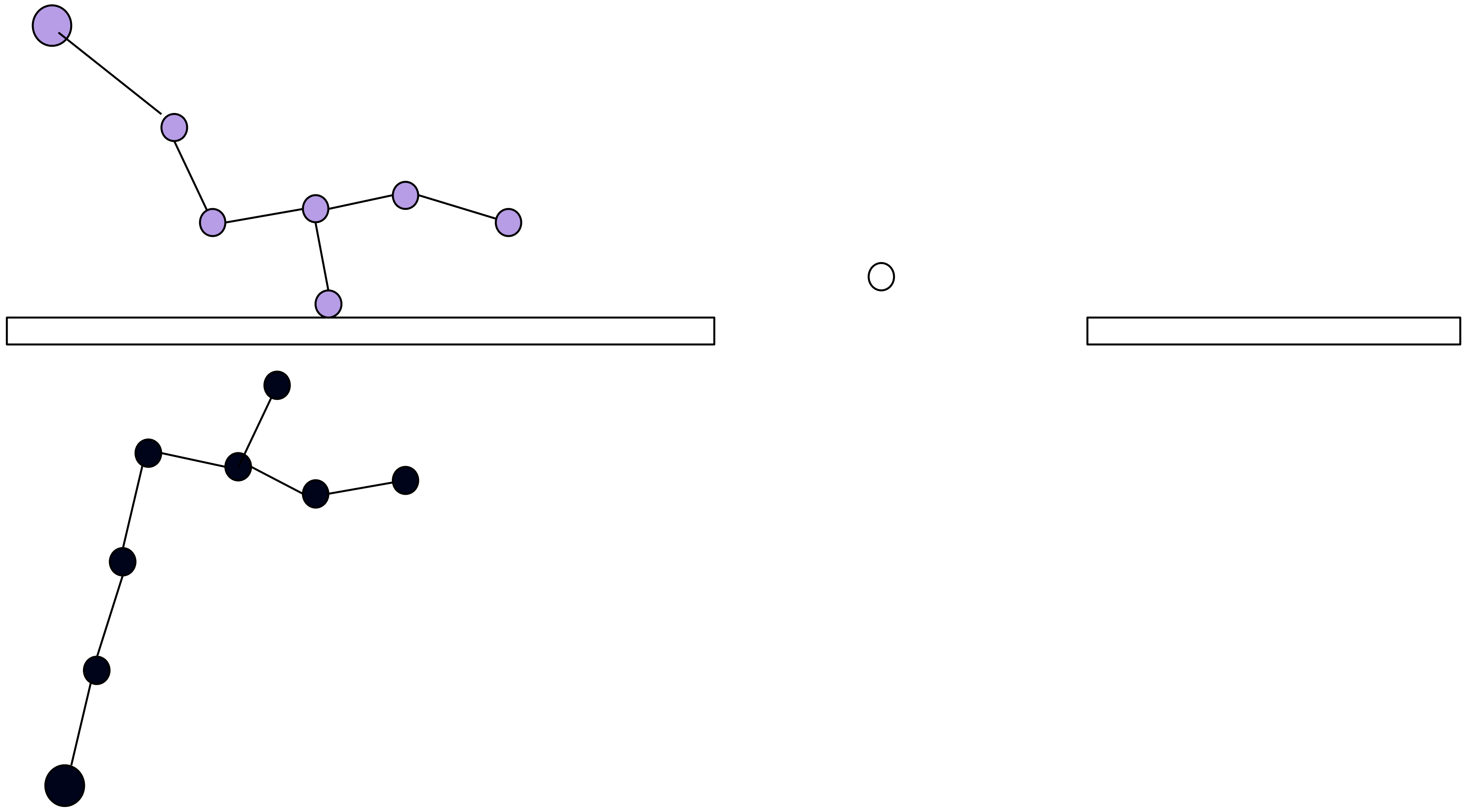

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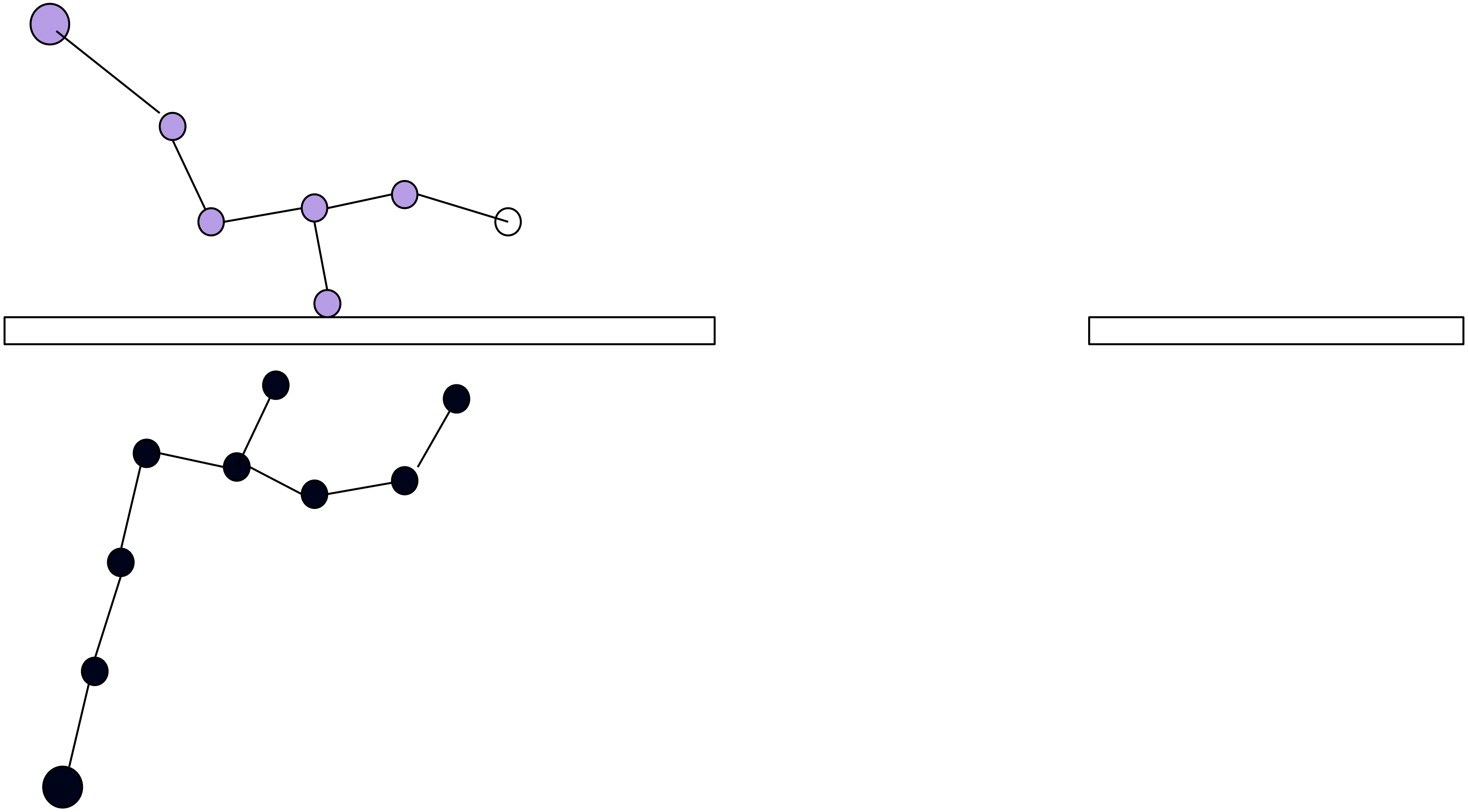

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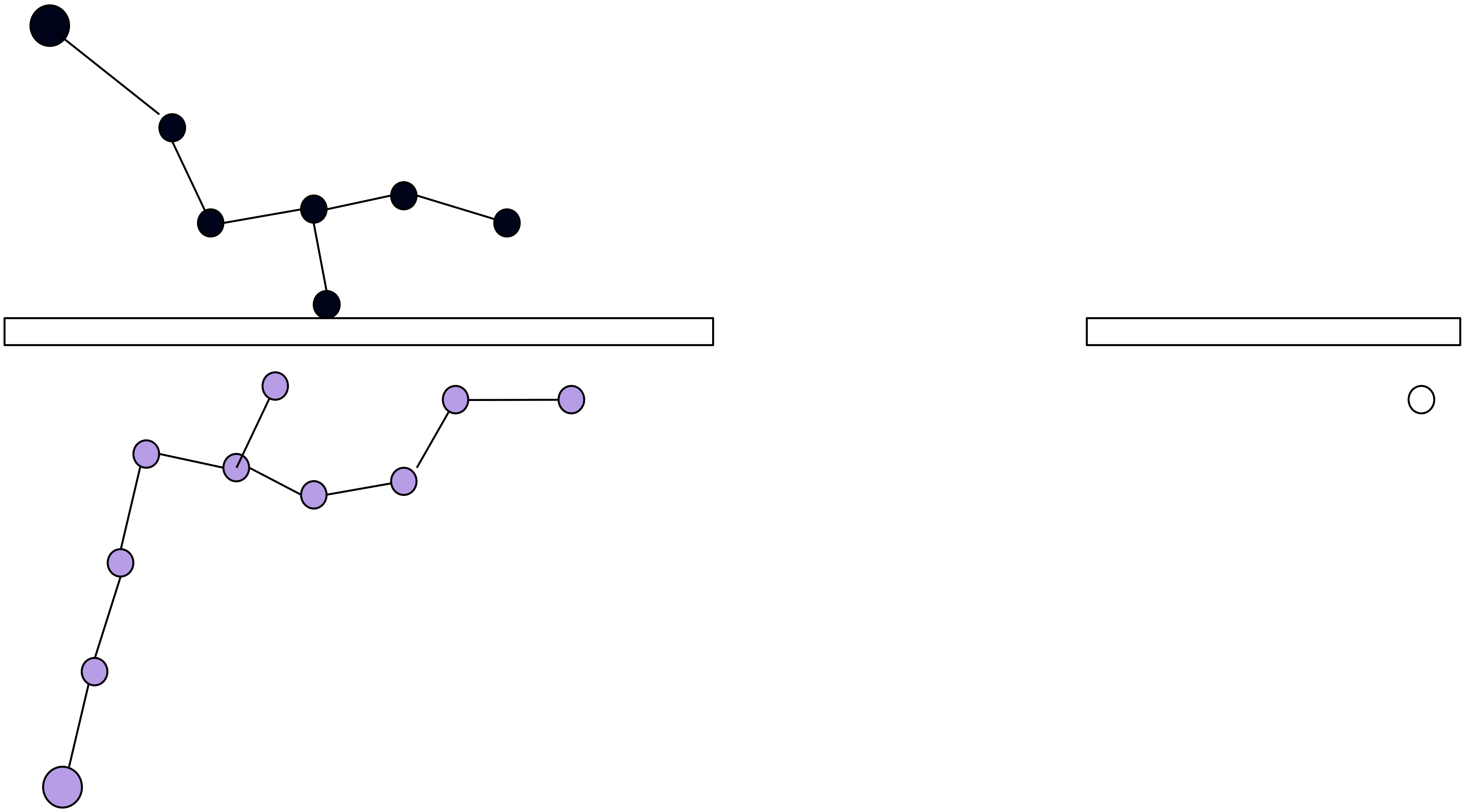

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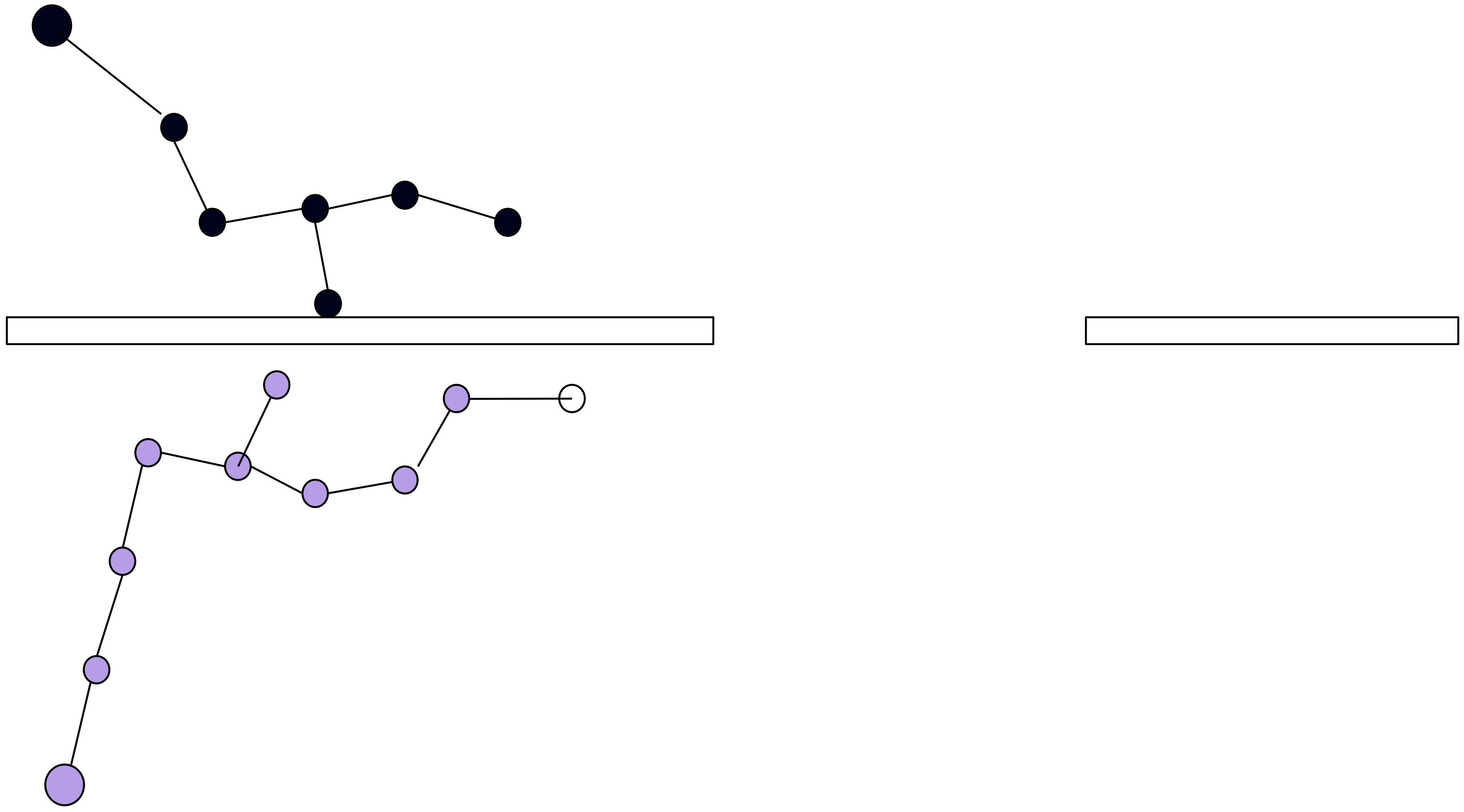

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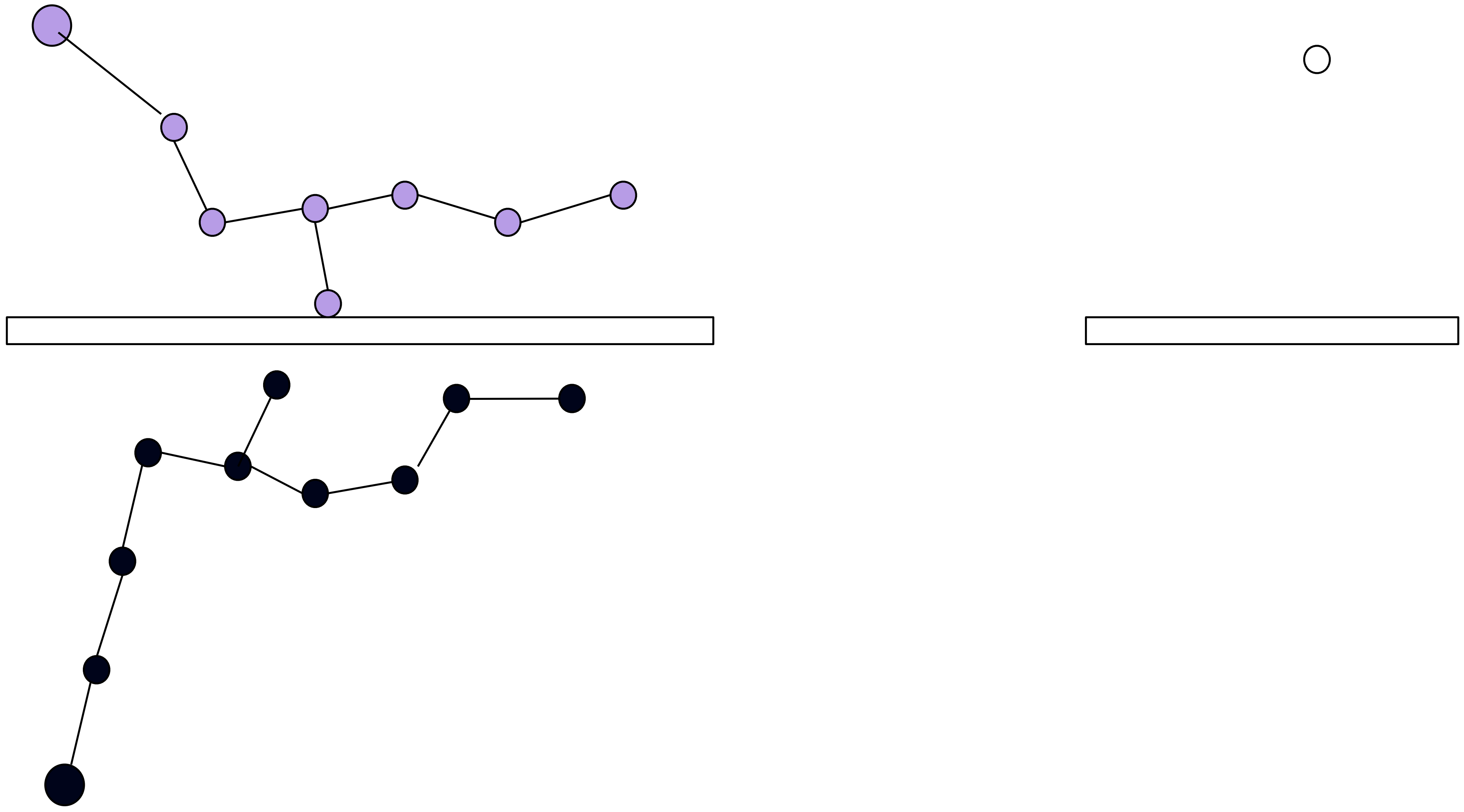

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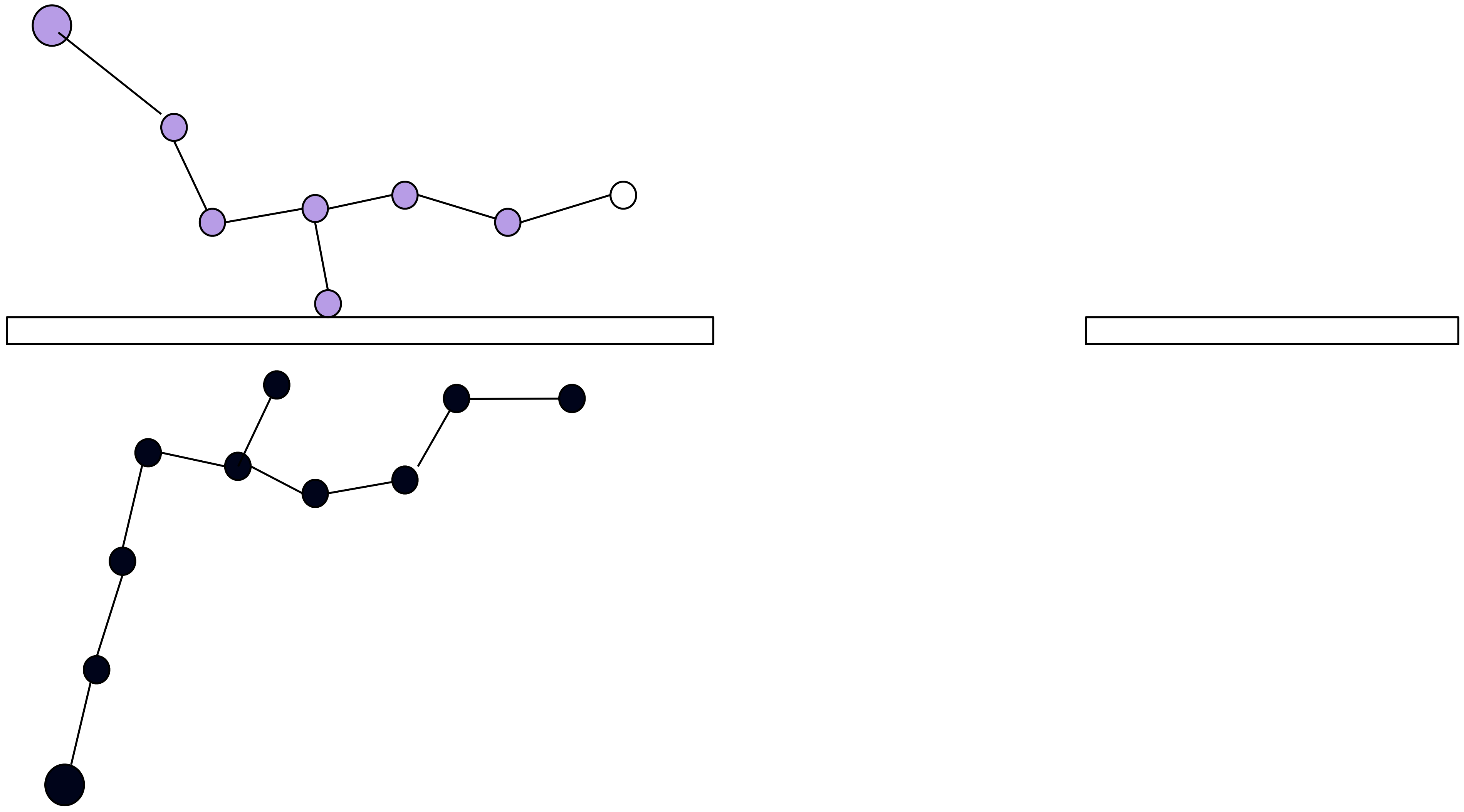

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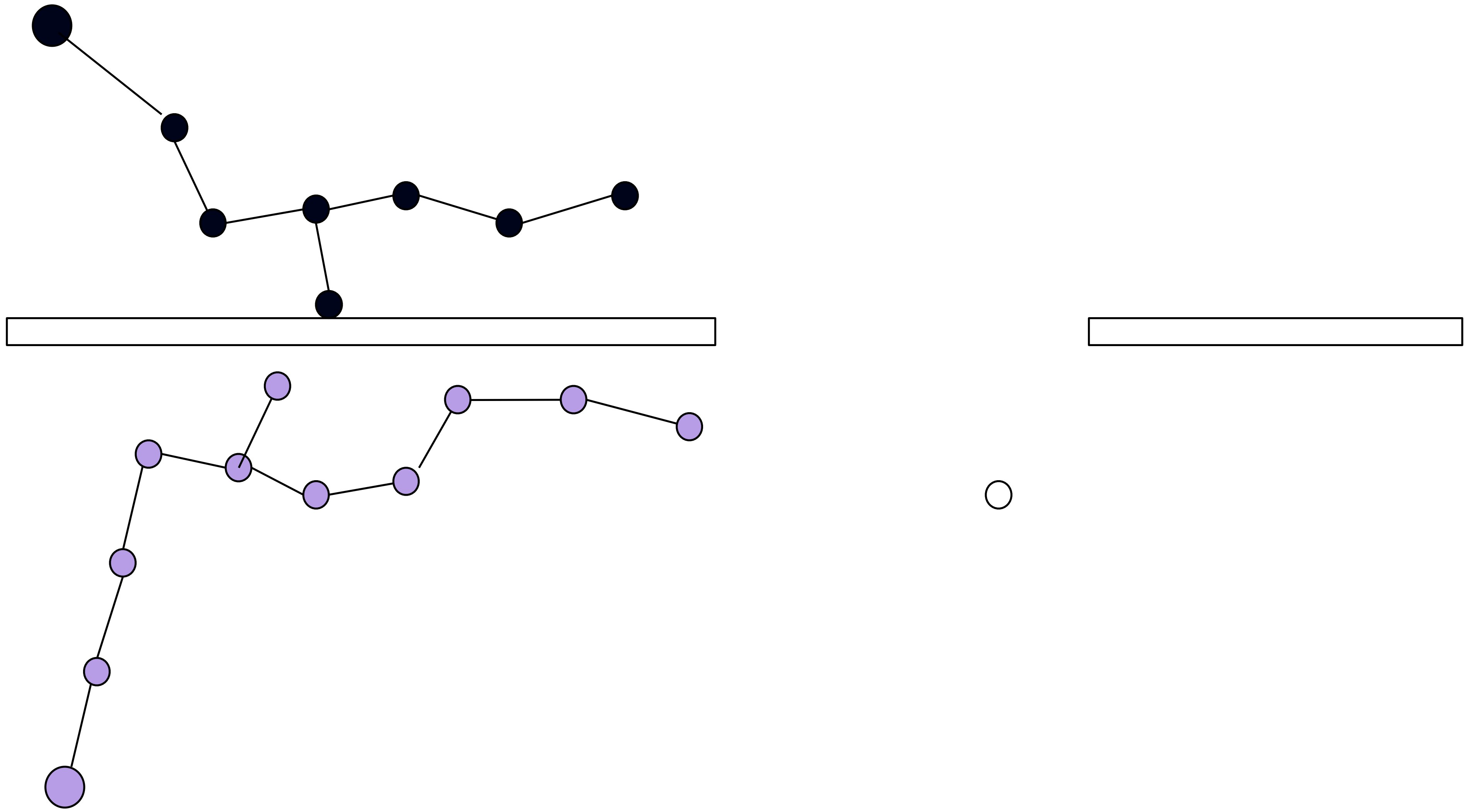

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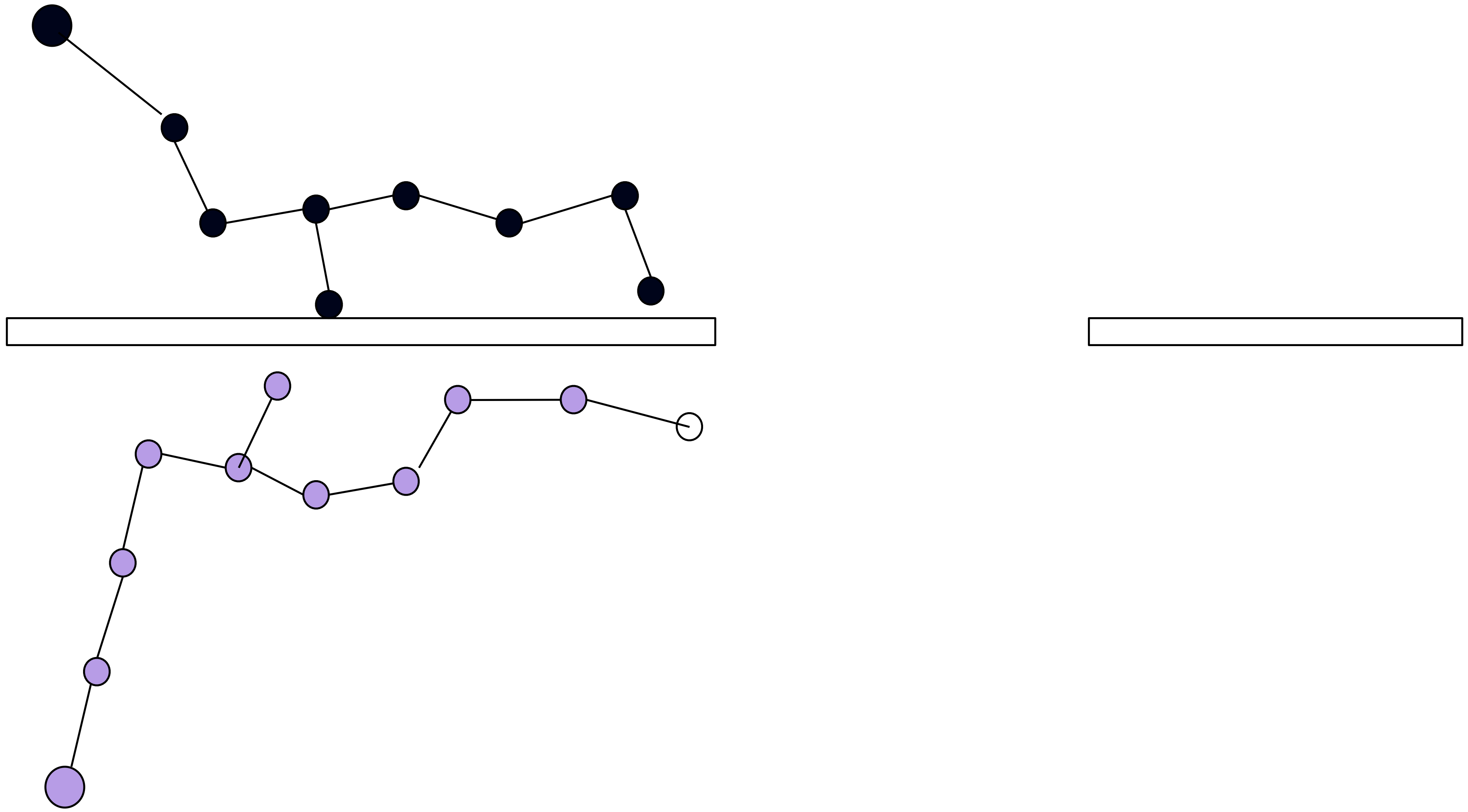

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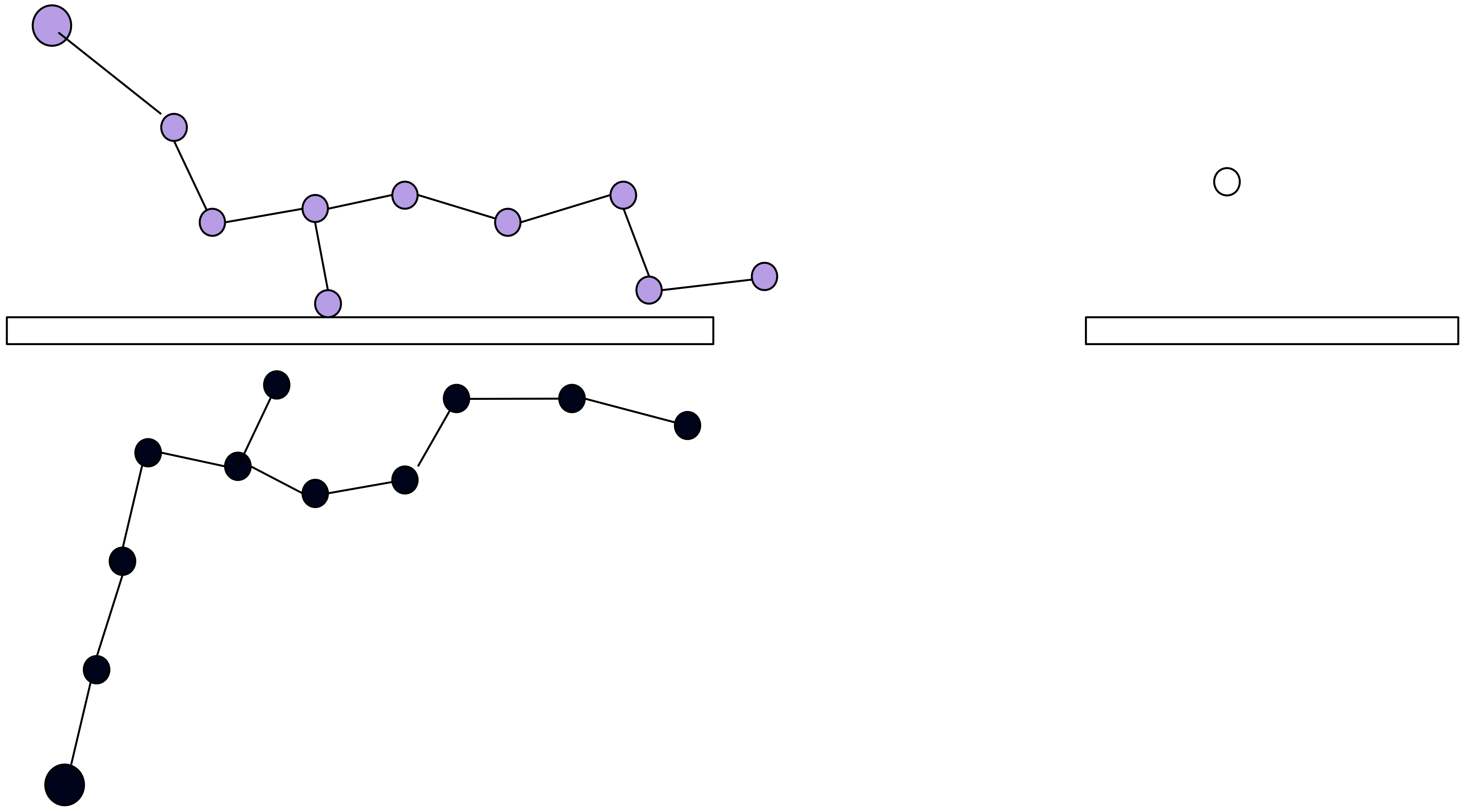

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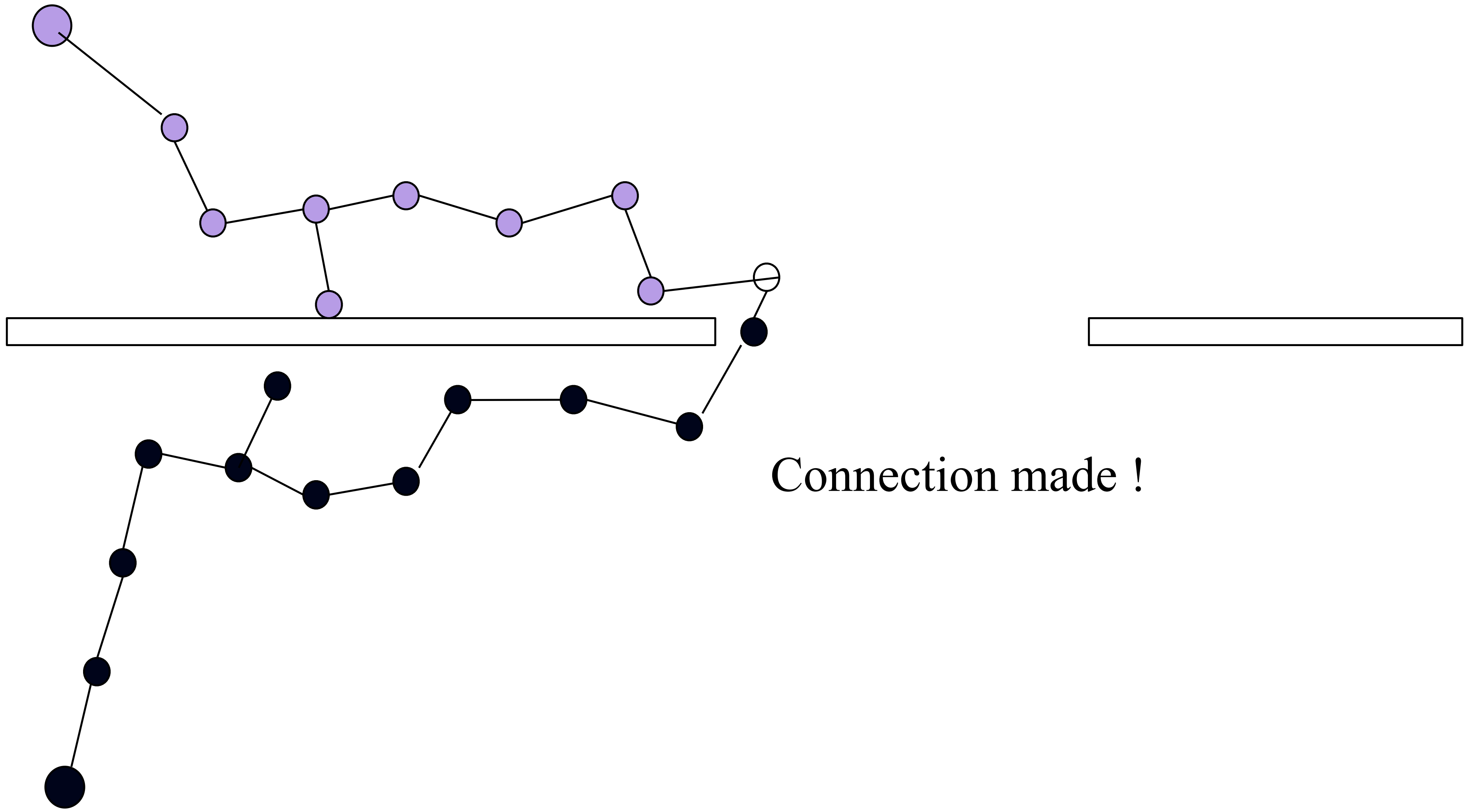

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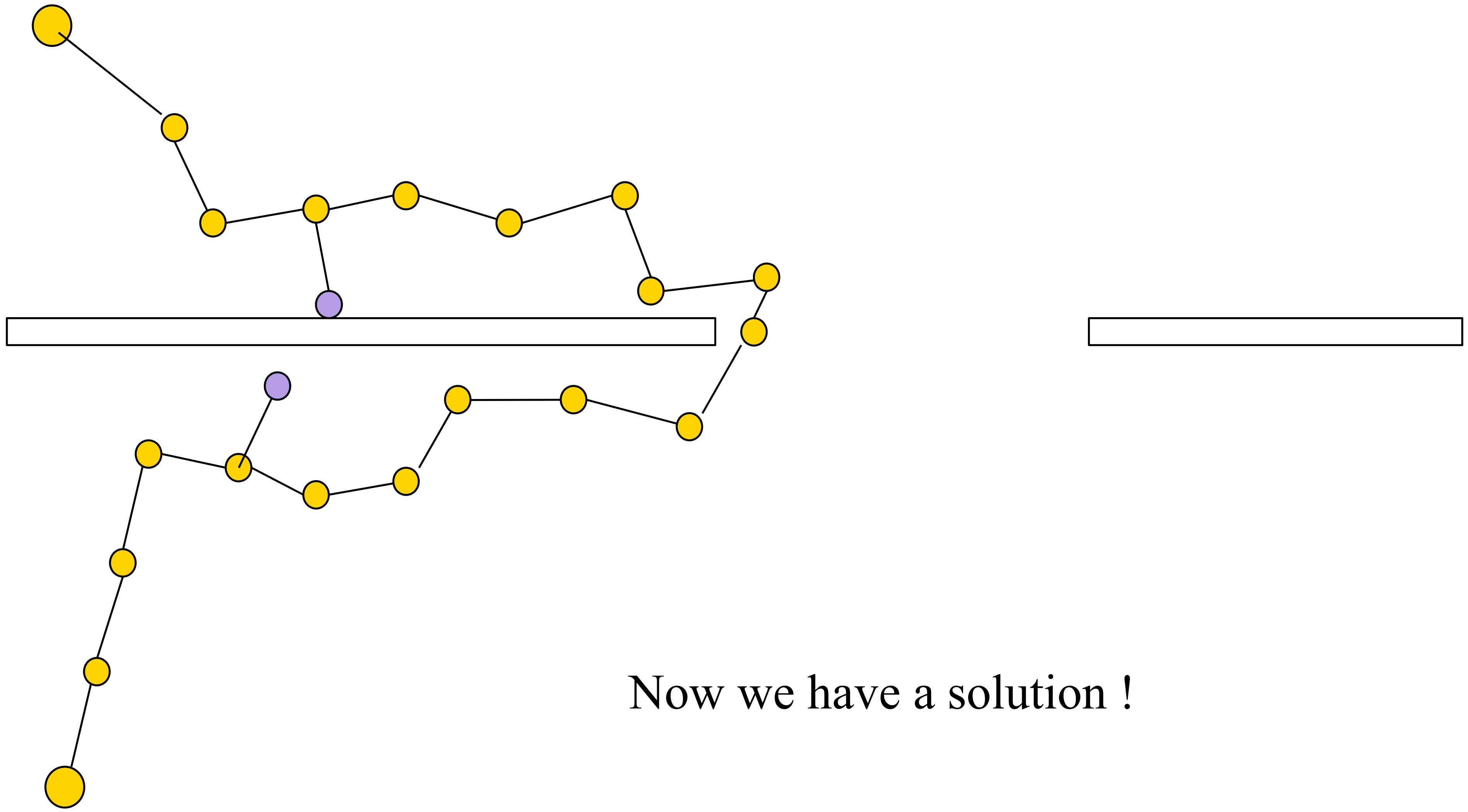
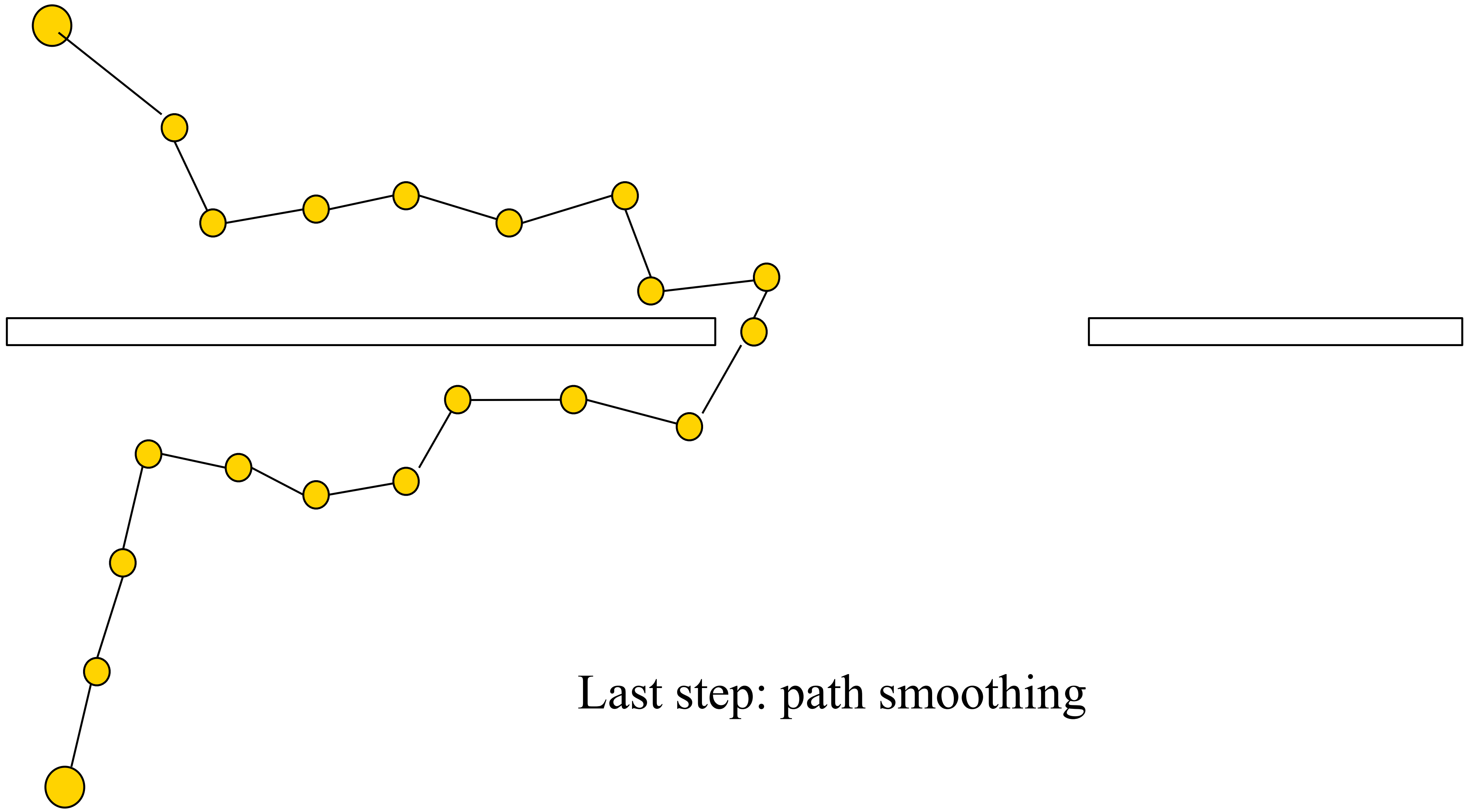


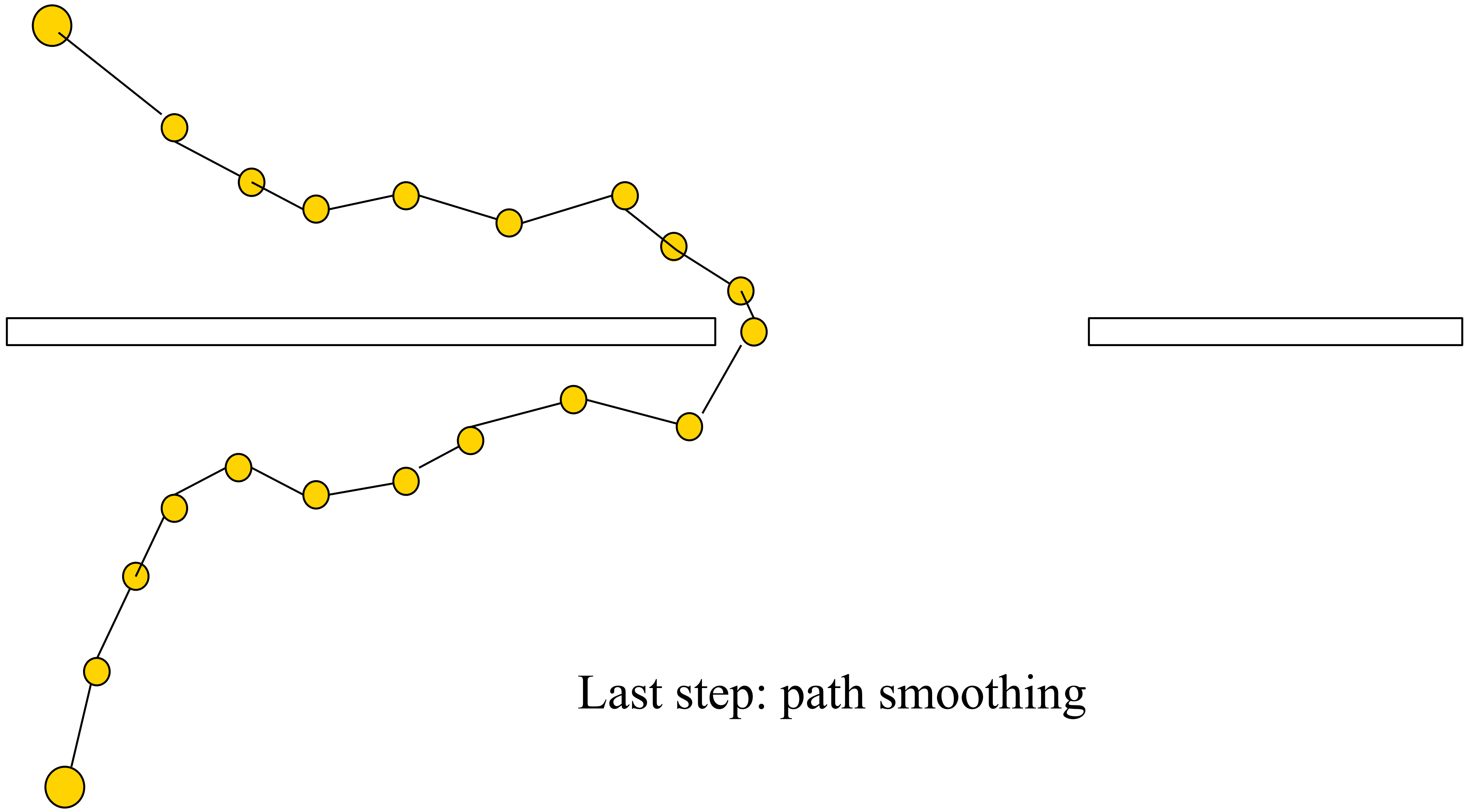
Now we have a solution !

RRT-Connect: example



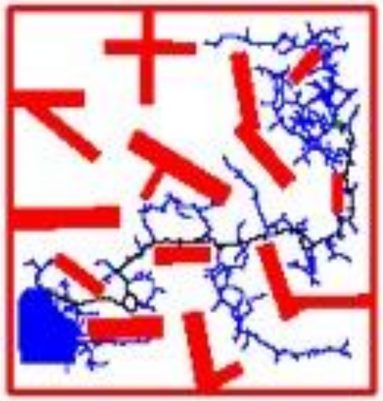
Last step: path smoothing

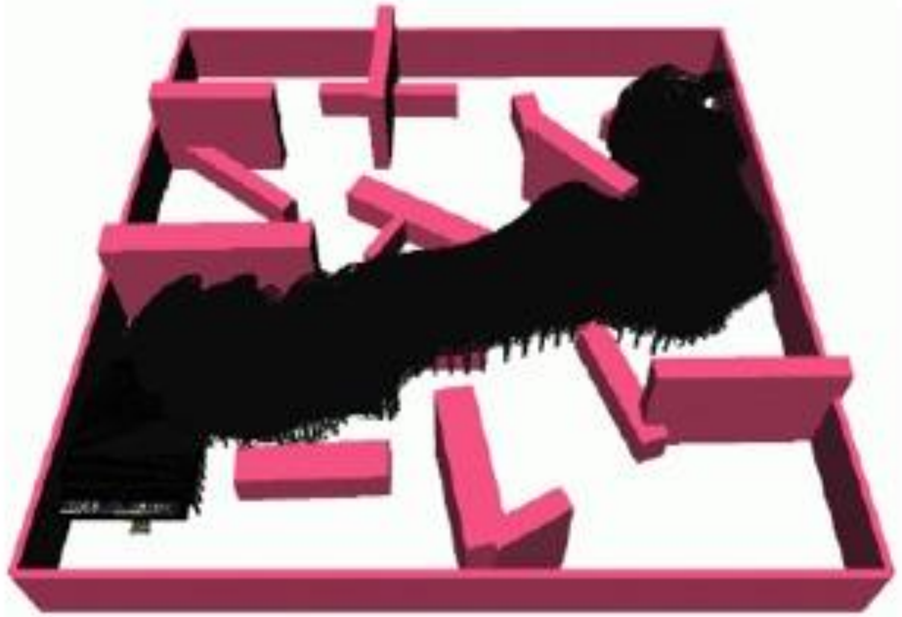
RRT-Connect: example



Last step: path smoothing

Examples: 6D





Using RRTs

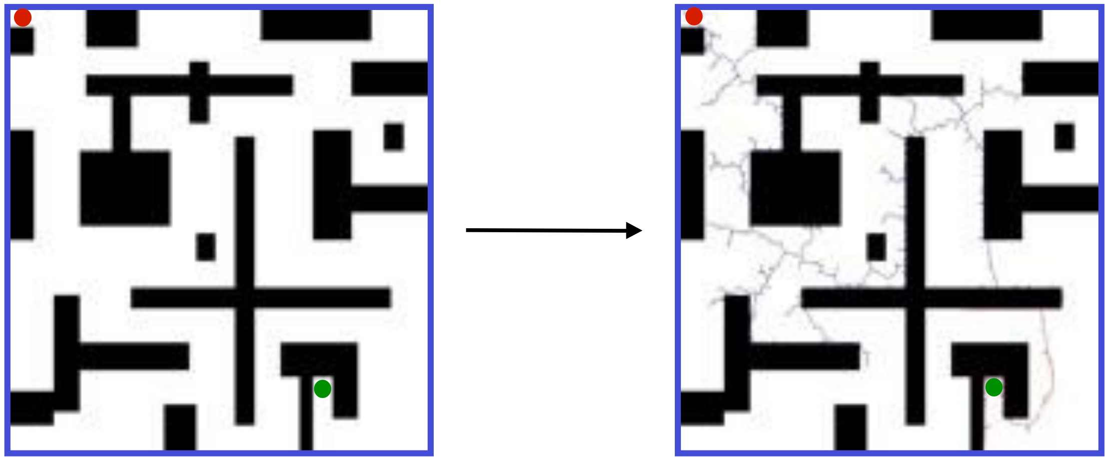
How can RRTs be used to help find paths?

- Can be used on their own (*eventually* will get close to the goal)

- Can be used with potential field methods

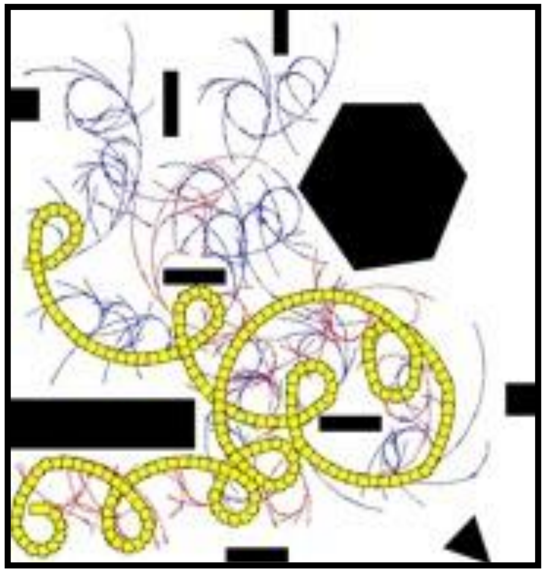
- Every so often (1 in 20 times), choose the goal to be the next free-space point chosen

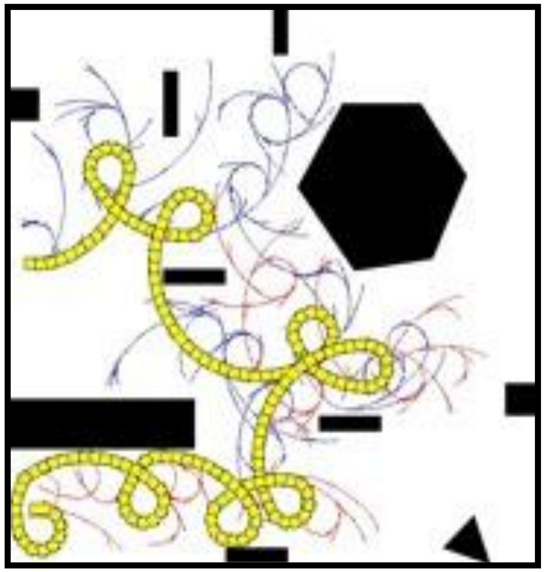
- Bias the random configuration chosen *toward the goal*.

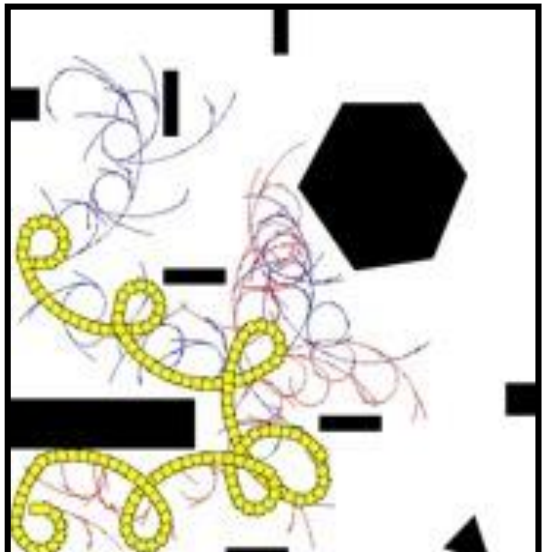


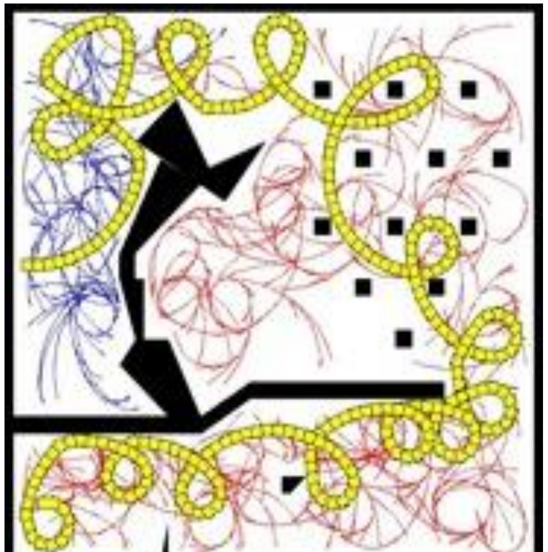
Additional complexity

Nonholonomic vs. Locally Uncontrollable $\longrightarrow$ forward-only left-only car









Also: moving obstacles

dynamic constraints

Conclusion and readings

- RRTs are one of the most attractive planners developed to date
- Much research has been generated from applying them to a variety of problems in robotics and AI, beyond simple motion planning
- Next Tuesday, we will read two recent research papers that follow-on from concepts in optimal control and planning:
  - LQR-Trees: Feedback Motion Planning on Sparse Randomized Trees by Russ Tedrake.
  - Sampling-based Algorithms for Optimal Motion Planning by Sertac Karaman and Emilio Frazzoli.
- Unless presenters appear over the weekend, I will lead the discussions, but please be prepared with questions!