Topological Mapping with Weak Sensory Data

Gregory Dudek, Dimitri Marinakis
Centre for Intelligent Machines, McGill University
3480 University St, Montreal, Quebec, Canada H3A 2A7
{dudek, dmarinak}@cim.mcgill.ca

Abstract

In this paper, we consider the exploration of topological environments by a robot with weak sensory capabilities. We assume only that the robot can recognize when it has reached a vertex, and can assign a cyclic ordering to the edges leaving a vertex with reference to the edge it arrived from. Given this limited sensing capability, and without the use of any markers or additional information, we will show that the construction of a topological map is still feasible. This is accomplished through both the exploration strategy which is designed to reveal model inconsistencies and by a search process that maintains a bounded set of believable world models throughout the exploration process. Plausible models are selected through the use of a ranking heuristic function based on the principle of Occam's Razor. We conclude with numerical simulations demonstrating the performance of the algorithm.

Introduction

In this paper we address a fundamental problem in mobile robotics: the mapping of an unknown environment. In particular, we are interested in constructing a topological map in the absence of metric (positional) information and using limited sensory data. We represent the world as an undirected graph in which vertices represent discrete places and edges navigable paths between them. We assume that the robot can consistently assign a cyclic ordering to the edges leaving a vertex with reference to the edge it arrived from, however, it is unable to associate a unique label with any place or edge. Given this limited sensing capability, and without the use of any markers or additional information, we will show that the construction of a topological map is still feasible.

As the wealth of literature addressing the SLAM problem in mobile robotics suggests, this problem of mapping a previously unknown environment in the face of imperfect sensory data has proved to be a challenging task. One of the key problems is that of *closing the loop* or determining whether a currently observed landmark or region corresponds to a previously visited location or a new portion of the world being explored; (*e.g.* (Newman & Ho 2005) or (Martinelli, Tomatis, & Siegwart 2005)). In this work we examine an aspect of this problem by considering an extreme case in which the robot has almost no ability to characterize its surroundings or obtain meaningful odometry measurements.

The study of a robot equipped only with the sensing ability to assign a consistent cyclic ordering to edges in a graph-like world has been examined previously in (Dudek *et al.* 1993) and (Dudek, Freedman, & Hadjres 1996). In this work, a mapping strategy is presented is which the robot constructs an *exploration tree* that enumerates consistent world hypotheses at each step of an exploration process. The authors classified the potential correspondence errors that could be made during the construction of this tree into three classes. One, errors of type OLD-LOOKS-NEW, in which the current location is assumed to be newly explored, but was actually visited earlier; two, errors of type MIS-CORRESPONDENCE in which the current location is thought to be a certain previously visited area, but is actually a different previously visited area; and three, errors of type NEW-LOOKS-OLD, in which a location is assumed to have been previously visited, but is actually new.

The authors discussed the fact that in a complete hypothesis tree, there will always exist a model which assumes that each place visited is a new location; *i.e* multiple errors of the type OLD-LOOKS-NEW. Among the three types, this class of errors is unique since the models they generate can not be shown inconsistent given the sensing capabilities considered. The work concludes by suggesting a heuristic to be used during the exploration process that prunes all models of size greater than $(\gamma s + C)$ where s is the current largest incomplete model. Later work (Dudek *et al.* 1997) (Rekleitis, Dujmović, & Dudek 1999) considered a version of the problem in which the robot with the same limited perceptual abilities was capable of placing and recognizing one or more markers. It was shown, that unlike the marker-less version, one can resolve potentially incorrect correspondences and therefore unambiguously map a finite world (given adequate exploration).

In this paper, we re-visit the marker-less problem. The main contributions of the paper are:

1. new exploration strategies that attempt to reduce correspondence errors where possible; and
2. a beam-style search through consistent models in the exploration tree that maintains a bounded set of likely hy-

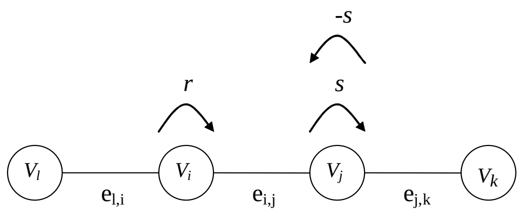


Figure 1: Diagram showing relationship of visited vertices in the context of the transition fuction δ.

potheses based on the principle of Occam's Razor.[1]

We show though simulations that the new exploration strategy combined with the particle filter style of inference performs much better than the earlier approaches.

The topological mapping problem has been well explored in mobile robotics. Early work in this area such as (Kuipers & Byun 1991) constructed a topological network description of the environment by identifying and then linking distinctive places and paths based on the sensory input and control strategies of the robot. Later, work such as (Shatkay & Kaelbling 1997) addressed the topological mapping problem with statistical formulations and techniques. The outcome of these approaches is generally a graph where vertices represent distinct locations or landmarks in the region and edges indicate navigability.

Practical applications of topological mapping must provide a method for the robot to reliably identify a topological node, (or landmark) in the world being explored. In (Choset & Nagatani 2001), sonar data is used to identify and position the robot on the Voronoi graph, the vertices of which correspond to topological nodes. In (Kuipers & Beeson 2002) place recognition is achieved through a multi-process bootstrapping technique that includes sensory clustering and probabilistic inference. Other approaches consider the extraction of features from vision or other sensory data (Se, Lowe, & Little 2001) (Sala *et al.* 2005) (Giguere *et al.* 2006). In this work, we leave for the moment this problem of identifying when the robot has reached a vertex, and focus solely on the correspondence problem.

Our approach is similar in concept to work described in (Ranganathan & Dellaert 2004) and (Ranganathan & Dellaert 2006). The weighted partial world models we maintain in our inference technique has some similarity to the concept of a probabilistic topological map, as defined by these authors. In both our technique and theirs, a multi-hypothesis, topological space is maintained. The distinguishing difference is that, while we only apply a ranking heuristic function, they use odometry measurements to assign relative probabilities to each of the potential world models.

Problem Specification

We describe the problem of topological mapping in terms of the inference of an undirected, un-weighted graph in which each vertex is given a label corresponding to the degree of the node. The vertices of this graph correspond to distinguishable places in the world and the edges correspond to connecting bidirectional paths. As the graph is traversed by the robot, it is able to sense the label of its current vertex and additionally is able to enumerate the edges of the place in a systematic way, (*e.g.* clockwise), relative to the edge by which it entered.

We refer to the edge by which the robot enters a place as a *reference edge*. The edge selected for the next move can be specified in relation to this reference edge. We define the transition (or motion) function δ as follows: $\delta(v_i, e_{i,j}, r) = v_j$ which means leave vertex v_i by the edge that is r edges (*e.g.* clockwise) after the reference edge $e_{i,j}$, and this takes us to vertex v_j. By recording its motions the robot is capable of retracing any previously taken trajectory since: if $\delta(v_i, e_{l,i}, r) = v_j$ and $\delta(v_j, e_{i,j}, s) = v_k$ then $\delta(v_j, e_{j,k}, -s) = v_i$ (Figure 1).

During each step of the exploration process, the robot records the label (degree) of its current topological node. As this exploration process continues an exploration tree is constructed, the full version of which contains a single world model for every consistent correspondence among all previously visited topological nodes. Each *level* of this exploration tree will be based on the information obtained from the traversal of a potentially unexplored edge. At any step t, each of the maintained hypotheses in the tree is consistent with the observational data collected up to that point. For all but trivially small observation sequences, the size of the complete tree quickly becomes intractable to maintain. The goal of this work is to manage the growth of the exploration tree so that only those world models that appear of relatively high likelihood are retained.

Exploration Strategies

Breath-First Traversal (BFT)

Here, for completeness, we briefly describe the original exploration strategy considered in (Dudek, Freedman, & Hadjres 1996). The strategy processes new edges in a FIFO manner, based on a breadth-first traversal. BFT may be represented as a tree where the root is the starting vertex v_s (where the robot starts the exploration), and a level i in the tree contains the i^{th} neighbors of v_s. The i^{th} neighbors of a vertex v are all vertices terminating distinct paths of length i which originate from node v. A single vertex u may be present many times as an i^{th} neighbour of v provided $(i > 1)$. (In the case of $i = 1$, this is only possible if multiple edges are allowed between the same vertices; *i.e.* we are exploring a multigraph.) BFT (breadth-first traversal) will visit all the vertices in a finite graph after at most depth d, d being the diameter of the graph.

Breadth-First Ears Traversal (BFET)

For our purposes, an good exploration strategy will limit, as much as possible, the number of world hypotheses that need to be considered. Of the three types of errors originally identified by Dudek *et al.*, it is possible to show inconsistent the second and third variety; MIS-CORRESPONDENCE and

[1] Occam's Razor is the principle enunciated by William of Occam that the simplest explanation is the best.

NEW-LOOKS-OLD. The first type, OLD-LOOKS-NEW, in which the current location is assumed to be a new node, can only be suspected by considering the implausibility of the world model suggested. We can do no better than this since there is no method of detection for errors of type OLD-LOOKS-NEW. The strength of the original BFT exploration strategy is its guarantee of eventual coverage given a finite world, however, it appears that strategies employing more passes through the potentially previously explored areas can help reveal correspondence errors of the second and third type better than BFT.

We first present a deterministic exploration strategy called breadth-first ears traversal (BFET) that, like BFT, is guaranteed of eventually visiting all vertices (and edges) of a finite world. In the next section we will describe a simple stochastic variant.

BFET works as follows. Embedded within the original BFT is a sub-exploration strategy that attempts to traverse each *ear*[2] leading from the current vertex v. For each potential ear leading from the vertex v, the robot explores the path p_1 beginning with v in one direction (*e.g.* clockwise) for some number of steps (until, for example, a node with the same degree as v is encountered). The robot then backtracks and explores the path p_2 beginning with v in the opposite direction (*e.g.* counter-clockwise) for the same number of steps. This process is continued with larger and larger sets of steps taken in both directions until the degree trace for the path taken in two directions matches up; *i.e.* path p_1 visits its vertices in the reverse order of those in p_2. This process is guaranteed to terminate given a finite graph since there is a bound on both the largest cycle in the graph and the number of cycles that any nodes can belong to.

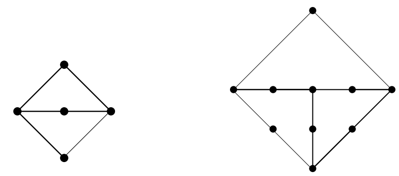
Once the two paths taken by the robot in its attempted exploration of the ear match up there is at least the potential for these two paths to actually represent visits of the same vertices is opposite order. Therefore, in the exploration tree, there must now exist a model of the world which reflects the fact that we have found a cycle leading from and back to the node we are currently investigating. Of course, there will also be other models, which, we suggest, are less likely.

Loop-Based Exploration (LBE)

Essentially BFET works by eliminating inconsistent models through the re-visiting of previously explored vertices in a cyclic manner. Our loop-based exploration strategy (LBE) attempts to capture the spirit of this approach.

LBE works as follows. If the robot is currently visiting a vertex of degree three or higher, then it selects with a probability p the first edge, $r = 1$, from the incoming reference edge for its next traversal (*e.g.* the first clockwise). Otherwise, it takes with probability $(1 - p)$, the second edge, $r = 2$, from the incoming reference edge (*e.g.* the second clockwise). If the current vertex is of degree two, then it selects the edge that is not the reference edge, and if the edge is of degree one, then it backtracks.

[2] Any connected planar graph can be decomposed into a set of cycles which are called ears.



(a) (b)

Figure 2: Examples of closed graphs which could explain an endless sequence of observations recording the visiting of alternate vertices of degree 2 and 3.

If a relatively large value of p is selected, this algorithm has the effect of visiting cycles in the graph one at a time, and having much the same effect on the exploration tree as the BFET algorithm for each of cycle examined. The larger the value of p, the better, on average we explore a particular cycle, but this comes at the cost of the average coverage time for the graph. Although LBE can not guaranteed coverage of a finite graph, we will show that given a good choice for p, in practice this strategy performs as well or better than the more complex BFET strategy.

Heuristic Weighted Search

In this section we describe a beam-style search algorithm which bounds the number of hypotheses maintained at each step of the exploration process based on heuristic evaluation function. We assume that the simplest models capable of explaining the observed data are the best ones and rank them accordingly. The principle, known as Occam's razor, states, "if presented with a choice between indifferent alternatives, then one ought to select the simplest one." The concept is a common theme in computer science and underlies a number of approaches in AI; *e.g.* hypothesis selection in decision trees and Bayesian classifiers.

We define a simple hypothesis as one with as few vertices as possible and, for tie breaking purposes, one with as few singly connected or *dangling* edges as possible; *i.e* minimal number of edges leading to unexplored areas. The second factor rewards those models which are approaching a *closed* model of the environment and ultimately assume that the entire region has been explored.

For example, consider a situation in which the robot has observed the node degrees: $(2, 3, 2, 3, 2, 3, 2, 3, \dots)$ while following an arbitrary exploration strategy. We must surmise that we are in a cycle of some multiple of length two, or that our world contains a large component in which each adjacent topological node alternates between degree two and three. If we have done enough exploration to suggest that we should have covered the entire environment, then we might suspect a world that looks like one of the ones depicted in Figure 2. In most applications, there is probably some prior knowledge that can be exploited to give a rough idea of the size of the region being explored, and therefore, some guess of the probability of having achieved coverage of the area in question when using a given exploration strategy.

(a) (b)

Figure 3: Examples of graphs solved previously in a.) (Dudek *et al.* 1993) and b.) (Dudek, Freedman, & Hadjres 1996). Each of these graphs were solved by our approach using LBE ($p = 0.99$) in less than half a second with $N = 1$; *i.e.* only one model was maintained throughout the exploration process (which was the correct one).

At each traversal of an edge during the exploration process, we first enumerate the new models that can be generated from each of the currently maintained world hypotheses, and we then rank them using our heuristic function. The top N of these models are then selected for maintenance and the rest are discarded. This approach can be considered conceptually similar to a particle filter. Instead of a motion model, we enumerate every possible option leading from the currently maintained particles, and instead of a probabilistic weighting and re-sampling, we cull all but the top proportion of the new particles (assign them a weight of 1 or 0) based on their ranking according to the heuristic function.

This approach allows online exploration, but risks throwing away the correct solution. Off-line variants could run the same algorithm repeatedly on the same observational sequence but employing an iteratively larger value for N until a believable solution was obtained.

Discussion of Results

We examined our approach to topological mapping in this problem domain through a number experiments conducted in simulation. Our simulation tool takes as input: 1.) an undirected graph representing the world to explore; 2.) the exploration strategy employed by the robot; 3.) the number of observations to gather; and 4.) the number of world hypotheses N to maintain. The simulator then determines if the robot, after its exploration, maintains in its world hypothesis space a graph that is isomorphically equivalent to the input graph (and its ranking in our hypothesis space). The graphs considered were randomly generated planar graphs produced by selecting a connected sub-graph of the Delaunay triangulation of a set of random points.

For medium sized, sparse graphs, our particle-filter style approach with an adequate number of maintained hypotheses was generally successful at retaining the correct solution in its exploration tree by the time coverage of the graph was achieved. Figure 4 shows an example of a successful outcome on a 10 node graph. Figure 3 compares our approach to graphs considered in previous work. The difficulty of the problem increased with the density and size of the graph, and the better performance of the new exploration strategies was more apparent under these circumstances. For example, Figure 6 shows a comparison of the different exploration strategies over ten node and thirty node graphs of various densities. Although the ranking results are not shown in these experiments, generally the correct graph was the first ranked model among those retained once coverage was achieved. Interestingly, the stochastic LBE exploration with a large enough value assigned to p, performed as good or better than the BFET strategy. For these experiments, a typical graph was usually solved (or not) in the order of a few minutes on a 2.2GHz P4 with 1.00 GB of RAM using un-optimized matlab code. Running time was dependent both on the number of hypotheses maintained, and the cover time of the graph which depended on the exploration strategy (Figure 9).

If not enough models were maintained throughout the exploration process, (the value assigned to N), then the chances of discarding the true solution increased (Figure 7). However, for small graphs, good results could be obtained using LBE and BFET with just a handful of models. By increasing the number of models maintained, it was possible to correctly infer quite large graphs (Figure 5).

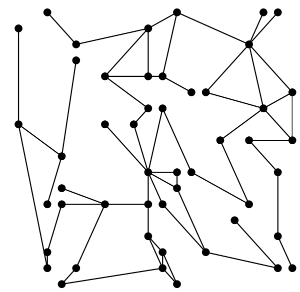


Figure 5: Example of a 50 node graph with an edge to node ratio of 1.2 that was solved by our approach in less than an hour. the correct graph was maintained by the algorithm (with $n = 1000$) as the first ranking model from the point of coverage onwards. LBE was used as the exploration strategy ($p = 0.99$) and achieved coverage at step 3918.

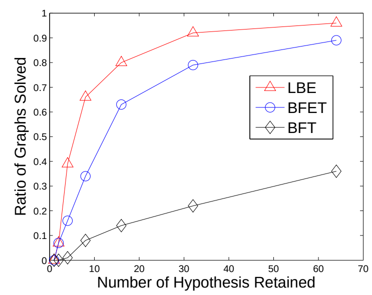
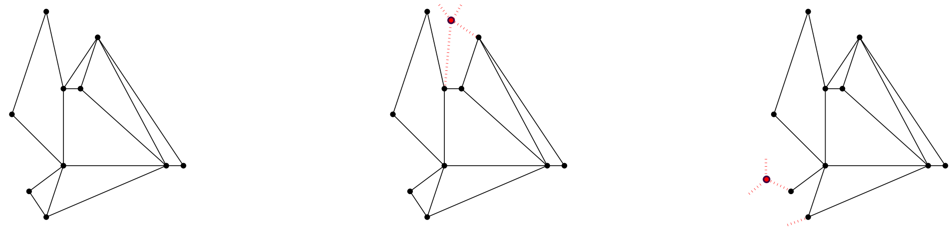
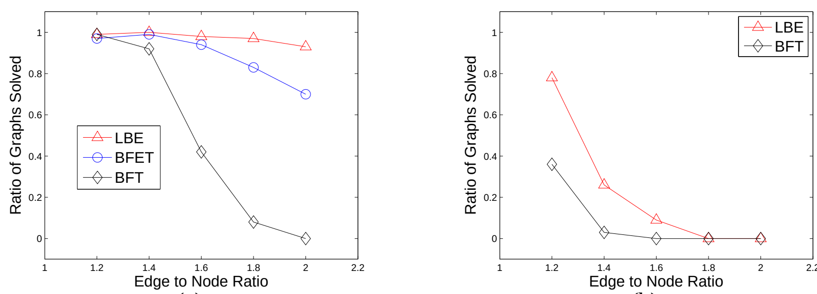


Figure 7: Ratio of graphs solved for different numbers of hypotheses maintained by the algorithm (value of N). Results obtained from 100 trials of 10 node graphs with an edge to node ratio of 1.6. For LBE, the parameter p was assigned a value of 0.99.



(a) (b) (c)

Figure 4: Example of the top three ranking world models, from left to right, inferred by the algorithm with $N = 20$ after running the BFET exploration strategy for 1000 steps on a 12 node graph with an edge to node ratio of 1.6. (Actual coverage was achieved at step 284.) The first ranking model is the correct one. Incorrect edges shown in dotted red.



(a) (b)

Figure 6: Ratio of graphs for which the true solution was retained in the hypothesis space after the exploration strategy under consideration reached edge coverage of the graph. Results were obtained from 100 trials at each edge density for graphs of size: a.) 10 nodes; and b.) 30 nodes. In this experiment 100 hypotheses were maintained by the mapping algorithm ($N = 100$). For LBE, the parameter p was assigned a value of 0.99. (BFET results were unobtainable for the larger graphs because of its poor cover time.)

The *distribution* of the size of the hypotheses generated by the various exploration algorithms reveals that the newer strategies are better at discriminating among the smaller sized models, presumably by showing inconsistent errors of the MIS-CORRESPONDENCE and NEW-LOOKS-OLD types. For example, BFET quickly generates many hypotheses, a few of which are small and have stayed consistent through much exploration, and many which are in relation quite large and therefore less believable (Figure 8). Table 1 reveals the differences in the mean hypothesis size obtained over a number of trials for the different exploration strategies.

Although the BFET algorithm is guaranteed to cover a finite region, its cover time in practice was relatively poor (Figure 9). Unfortunately, this makes its use difficult for environments which are suspected to be large, since the probability of coverage would be low even after considerable exploration. In the environments we consider here, the LBE strategy does much better in practice, even with an aggressive value of p.

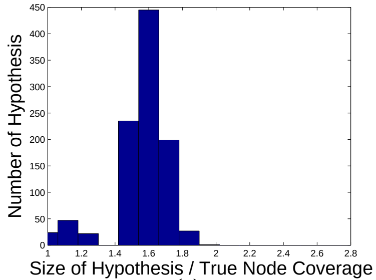
Strategy	Mean Node Coverage	Normalized Model Size
BFT	8.48 +/-(1.11)	1.22 +/-(0.19)
BFET	6.86 +/-(1.78)	1.67 +/-(0.30)
LBE ($p = 0.95$)	5.57 +/-(2.46)	2.15 +/-(0.57)
LBE ($p = 0.99$)	4.33 +/-(1.86)	2.56 +/-(0.79)

Table 1: Mean and standard deviation for coverage and model size normalized by coverage for the first 1000 hypotheses generated by the different exploration strategies. Results obtained from 100 trials on random 10 node graphs with an edge to node density of 1.6.

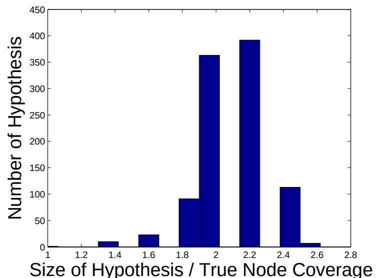
Conclusion and Future Work

In this paper we have considered the topological mapping problem given a robot with extremely limited sensory capabilities. We have shown that even in the case of highly ambiguous, non-unique topological 'signatures' it possible for such a robot to infer a set of hypotheses for its environment that likely includes the true model. Our approach combines an exploration strategy that attempts to eliminate inconsistent models with a beam style search that bounds the number of models maintained at each step based on the principle of Occam's razor.

In future work, we would like to look at handling more realistic sensory data. For example, incorporating additional, but still relatively poor, sensory data such as range only



(a)



(b)

Figure 8: Distribution of the first 1000 hypotheses generated for a.) the BFT exploration strategy and b.) the BFET exploration strategy. The result was obtained from a typical run of the algorithm on a 10 node graph with an edge to node density of 1.6. BFT covered 7 of the 10 nodes in this time, while BFET covered only 5.

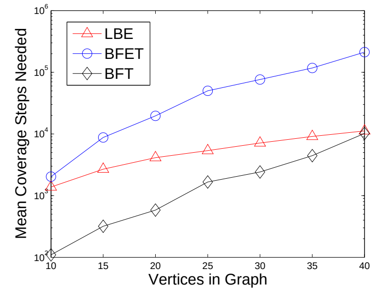


Figure 9: Average number of steps required for edge coverage of the graph for the different exploration strategies. Note the log scale for the vertical axis. Average was taken over 100 trials using an edge density of 1.6. For LBE, the parameter p was assigned a value of 0.99.

odometry. Additionally, we would like to consider the effect of sensor errors; *i.e* missing or spurious observations. It's possible that these aims could by accomplished by shifting our heuristic based evaluation method to a probabilistic one. Such an approach could weigh the relative likelihood of the maintained models at any time based on previously calibrated measurement models and some prior over potential environments.

References

Choset, H., and Nagatani, K. 2001. Topological simultaneous localization and mapping (SLAM): toward exact localization without explicit localization. *IEEE Transactions on Robotics and Automation* 17(2):125 – 137.

Dudek, G.; Jenkin, M.; Milios, E.; and Wilkes, D. 1993. Map validation and self-location in a graph-like world. In *International Joint Conference on Artificial Intelligence*.

Dudek, G.; Jenkin, M.; Milios, E.; and Wilkes, D. 1997. Map validation and robot self-location in a graph-like world. *Robotics and Autonomous Systems* 22(2):159–178.

Dudek, G.; Freedman, P.; and Hadjres, S. 1996. Mapping in unknown graph-like worlds. *Journal of Robotic Systems* 13(8):539–559.

Giguere, P.; Dudek, G.; Prahacs; and Saunderson, S. 2006. Environment identification for a running robot using inerial and actuator cues. In *Proceedings of Robotics: Science and Systems*.

Kuipers, B., and Beeson, P. 2002. Bootstrap learning for place recognition. In *AAAI National Conference on Artificial Intelligence*.

Kuipers, B., and Byun, Y. T. 1991. A robot exploration and mapping strategy based on a semantic hierarchy of spatial representations. *Journal of Robotics and Autonomous Systems* 8:47–63.

Martinelli, A.; Tomatis, N.; and Siegwart, R. 2005. Some results on SLAM and the closing the loop problem. In *IEEE/RSJ Intenational Conference on Intelligent Robots and Systems*.

Newman, P., and Ho, K. 2005. SLAM-loop closing with visually salient features. In *IEEE Intl. Conf. on Robotics and Automation*.

Ranganathan, A., and Dellaert, F. 2004. Inference in the space of topological maps: an MCMC-based approach. In *Intenational Conference on Intelligent Robots and Systems*, 1518–1523.

Ranganathan, A., and Dellaert, F. 2006. A rao-blackwellized particle filter for topological mapping. In *Proc. of the International Conference on Robotics and Automation*, 810–817.

Rekleitis, I. M.; Dujmović, V.; and Dudek, G. 1999. Efficient topological exploration. In *Proceedings of International Conference in Robotics and Automation*.

Sala, P. L.; Sim, R.; Shokoufadeh, A.; and Dickinson, S. J. 2005. Landmark selection for vision-based navigation. *IEEE Transactions on Robotics and Automation*.

Se, S.; Lowe, D.; and Little, J. 2001. Vision-based mobile robot localization and mapping using scale-invariant features. In *IEEE Intl. Conf. on Robotics and Automation*.

Shatkay, H., and Kaelbling, L. P. 1997. Learning topological maps with weak local odometric information. In *International Joint Conference on Artificial Intelligence*, 920–929.