

# An Improved Interpretation of Depletion Approximation in p–n-Junctions

B. Mazhari and A. Mahajan

**Abstract**—The conventional treatment of depletion approximation in p–n-junctions often leaves a student with an erroneous impression that the approximation essentially involves complete neglect of the fraction of the space charge region (SCR) where charge density makes a transition to zero. This paper describes a simple analytical model for clarifying the relationship of depletion approximation to the SCR.

**Index Terms**—Capacitance, electric field, p–n-junctions, semiconductor device modeling.

## I. INTRODUCTION

A p–n-junction is an integral part of a number of semiconductor devices, including diodes, bipolar junction transistors (BJTs), and field-effect transistors (FETs). A good understanding of p–n-junction diodes is thus essential for understanding the operation of all these semiconductor devices. One of the most important factors that affects the characteristics of p–n-junctions is the presence of the space charge region (SCR) close to the junction. Almost all the p–n-junction characteristics, including current versus voltage, capacitance, and breakdown, are affected by this SCR. As a result, a treatment of p–n-junctions often begins with an analysis of SCR [1]–[8] obtained through a solution of Poisson’s equation. To simplify the solution of Poisson’s equation, one commonly assumes that the SCR has very few electrons and holes, in other words, is depleted of mobile carriers [1]–[8]. As a result of this depletion approximation, the solution of Poisson’s equation becomes straightforward, at least for uniformly doped abrupt junctions, and leads to well-known expressions for various physical quantities, including electric field, depletion widths, junction capacitance, etc.

Although the limitations of depletion approximation are well recognized and several improved models have been proposed to correct it [9]–[11], depletion approximation itself is often misunderstood. Fig. 1(a) shows a typical charge density profile in SCR, and Fig. 1(b) shows the charge density profile under depletion approximation. A comparison of the two figures can easily lead to the mistaken view that depletion region refers only to region-I in Fig. 1(a), which is depleted of carriers. It can also lead to the conclusion that results obtained using depletion approximation can be justified only if the length of the transition region is much smaller than the depletion width. An improved interpretation of depletion approximation, however, is that it involves not so much the neglect of the transition region itself, but

only its approximation by a region of constant charge density, as illustrated in Fig. 2. Although this view of depletion approximation can be implicitly seen in the figures of charge density profile shown in many textbooks, the incorrect view described previously often forms in a student’s mind because this picture is not explicitly articulated. In the next section, the authors present a simple analytical model that can serve as a useful pedagogical aid in clarifying the nature of depletion approximation.

## II. ANALYTICAL MODEL

Consider a symmetrical p–n-junction with uniform and identical doping in n- and p-regions. The charge density profile in the transition region of SCR has a complicated form that makes analytical modeling difficult. As a result, an SCR with linearly varying charge density in the transition region is considered with the perspective that an understanding of the relationship of depletion assumption with respect to this simplified charge density profile would also lead to a better understanding of the relationship of depletion approximation with respect to actual SCR.

Fig. 3(a) shows a simplified charge density profile with a linearly varying charge density profile in the transition region of fixed width  $L_T$  assumed to be a few debye lengths [3], [4] in magnitude. A relationship between depletion width and the charge density profile shown in Fig. 3(a) can be obtained through solution of Poisson’s equation for each of these charge density profiles. For the charge density profile shown in Fig. 3(a), Poisson’s equation can be solved to obtain the following expressions for the electric field:

$$E(x) = -\frac{q}{\epsilon_s} N_D \times \left( x_o + \frac{L_T}{2} - x \right), \quad \text{for } 0 < x < x_o. \quad (1)$$

$$E(x) = -\frac{q}{\epsilon_s} N_D \times \left[ \left( x_o + \frac{L_T}{2} - x \right) + \frac{(x - x_o)^2}{2L_T} \right], \quad \text{for } x_o < x < L_T + x_o. \quad (2)$$

A similar set of expressions can be written for the p-side of the junction as well. Integration of the electric field with appropriate boundary conditions gives

$$V_{bi} - V_a = \frac{qN_D}{\epsilon_s} \times \left( x_o^2 + x_o L_T + \frac{L_T^2}{3} \right) \quad (3)$$

where  $V_{bi}$  and  $V_a$  are the built-in and applied voltages, respectively. The solution of Poisson’s equation under depletion approximation, on the other hand, gives

$$V_{bi} - V_a = \frac{qN_D}{\epsilon_s} \times \frac{W^2}{4}. \quad (4)$$

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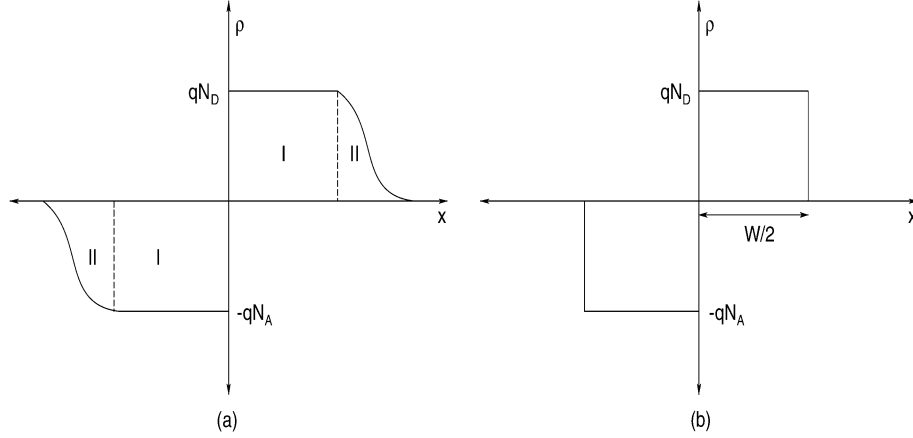


Fig. 1. Schematic diagram showing (a) the charge density profile in a p-n-junction and (b) the depletion approximation to the charge density profile.

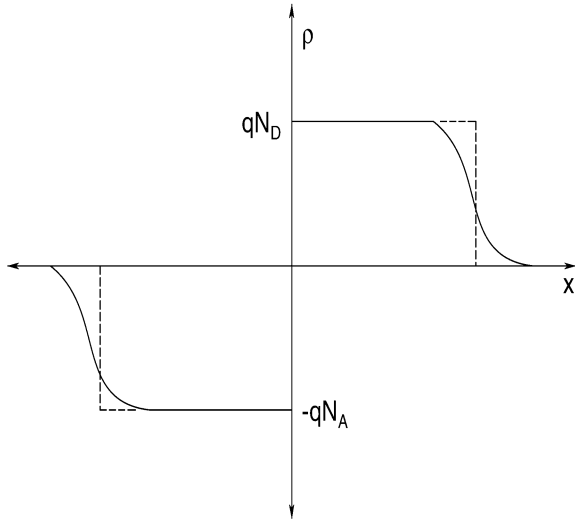


Fig. 2. Schematic diagram showing the real nature of depletion approximation (dashed line) with respect to the actual charge density profile.

Comparison of (3) and (4) gives

$$\frac{W}{2} = (x_o + 0.5L_T) \times \sqrt{\left(1 + \frac{1}{12} \left(\frac{L_T}{x_o + 0.5L_T}\right)^2\right)}. \quad (5)$$

For  $L_T/W \leq 1$ , a condition that can be shown to be valid up to moderate forward biases, (5) can be simplified to

$$\frac{W}{2} \cong (x_o + 0.5L_T). \quad (6)$$

Equation (6) reveals that the depletion edge lies at the center of the transition region as shown in Fig. 3(b). This result clearly shows that the depletion approximation does not neglect the transition region but only approximates it by a region of constant charge density. The insight that depletion edge lies at the center of transition region leads to an interesting observation that although depletion approximation commits an error by overestimating charge in one part of the SCR [region A in Fig. 3(b)], it also commits an equal but opposite error by underestimating charge in another part (region B) of the transition region. This observation has important implications for physical parameters

that depend on the integrated charge density because the mutual cancellation of errors serves to increase the accuracy of analytical results. As an example, consider the maximum electric field within the junction. For the charge density profile shown in Fig. 3(a), the following expression using (1) and (2) can be obtained:

$$E_{\max.} = -\frac{q}{\epsilon_s} N_D \times \left(x_o + \frac{L_T}{2}\right). \quad (7)$$

Under depletion approximation, the expression for the electric field can be expressed as

$$E_{\max.} = -\frac{q}{\epsilon_s} N_D \times \frac{W}{2}. \quad (8)$$

Use of (6) shows that the expression for the maximum electric field under depletion approximation is the same as that obtained with a charge density profile having a linearly varying transition region. Similarly, consider the junction capacitance. For the charge density profile shown in Fig. 3(a), an expression for junction capacitance can be obtained by noting that the magnitude of total charge on the n- or the p-side can be expressed as

$$Q_J = qN_D \times (x_o + 0.5L_T). \quad (9)$$

Equation (3) together with (9) can be used to obtain an expression for junction capacitance

$$C_J = -\frac{\partial Q_J}{\partial V_a} = \frac{\epsilon_s}{2(x_o + 0.5L_T)}. \quad (10)$$

The expression for junction capacitance under depletion approximation, on the other hand, can be written as

$$C_J = \frac{\epsilon_s}{W}. \quad (11)$$

In this case also, use of (6) shows that the expression for junction capacitance under depletion approximation is the same as that obtained with the simplified charge density profile shown in Fig. 3(a). Although this process of cancellation of errors results in improvement in accuracy of results obtained using depletion approximation for some parameters, it works satisfactorily only for moderate values of forward bias and for relatively lightly doped p-n-junctions.

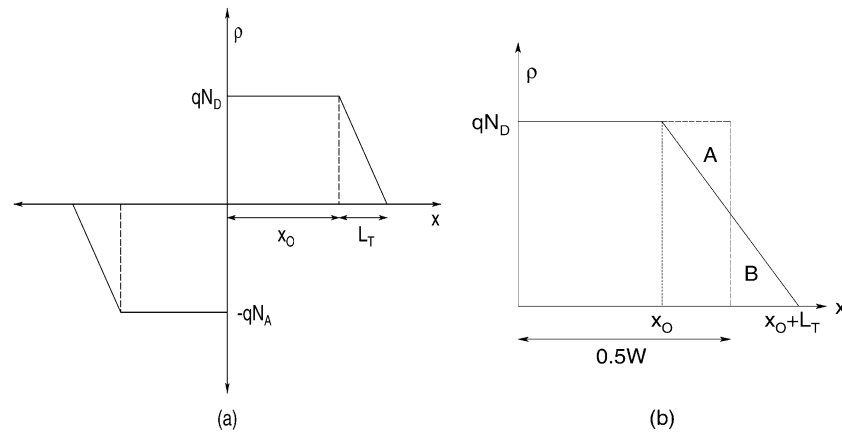


Fig. 3. (a) Schematic diagram showing the simplified charge density profile in which the transition region is modeled as a linearly varying charge density region of fixed width  $L_T$ . (b) Schematic diagram showing the relationship of depletion width to the simplified charge density profile.

### III. CONCLUSION

To summarize, a simple analytical model has been presented that can serve as a useful aid in clarifying that the depletion approximation in p–n junctions does not involve neglect of the transition region itself, but only its approximation by a region of constant charge density. The accuracy of analytical results obtained through depletion approximation for physical parameters, such as maximum electric field and junction capacitance, is improved through a process of a mutual cancellation of errors.

### REFERENCES

- [1] W. Shockley, *Electrons and Holes in Semiconductors*. Princeton, NJ: Van Nostrand, 1950.
- [2] J. Lindmayer and C. Y. Wrigley, *Fundamentals of Semiconductor Devices*. Princeton, NJ: Van Nostrand, 1965.
- [3] S. M. Sze, *Physics of Semiconductor Devices*, 2nd ed. New York: Wiley, 1981.
- [4] R. S. Muller and T. I. Kamins, *Device Electronics for Integrated Circuits*. New York: Wiley, 1986.
- [5] C. T. Sah, *Fundamentals of Solid-State Electronics*. River Edge, NJ: World Scientific, 1991.
- [6] B. G. Streetman, *Solid State Electronic Devices*, 3rd ed. Englewood Cliffs, NJ: Prentice-Hall, 1994.
- [7] D. J. Roulston, *An Introduction to the Physics of Semiconductor Devices*. New York: Oxford Univ. Press, 1999.

- [8] J. Singh, *Semiconductor Devices—Basic Principles*. New York: Wiley, 2001.
- [9] B. R. Chawla and H. K. Gummel, "Transition region capacitance of diffused p–n junctions," *IEEE Trans. Electron Devices*, vol. 18, no. 3, pp. 178–195, Mar. 1971.
- [10] J. J. Liou and A. Lindholm, "Forward-voltage capacitance and thickness of p–n junction capacitance," *IEEE Trans. Electron Devices*, vol. 34, no. 7, pp. 1571–1579, Jul. 1987.
- [11] A. Haggag and K. Hess, "Analytical theory of semiconductor p–n junctions and the transition between depletion and quasi-neutral region," *IEEE Trans. Electron Devices*, vol. 47, no. 8, pp. 1624–1629, Aug. 2000.

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