

# Multi-Robot Simultaneous Localization and Uncertainty Reduction on Maps (MR-SLURM)

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**Abstract**—This paper presents two strategies for simultaneous localization and uncertainty reduction on maps for a team of robots. The proposed strategies differentiate between homogeneous and heterogeneous multi-robot teams assigning different roles based on risk and/or capabilities. We apply the proposed algorithms to the Robot-Camera Sensor Network localization problem, where a team of robots moves through an environment equipped with a camera sensor network. Each robot uses its own pose estimate to localize every camera it encounters. This is achieved by using the camera observation of the robot to extract a 6-DoF transformation between the camera and the robot. The inverted transformation places the camera in the robots global frame of reference, and map merging among the multiple robots places the cameras and the team of robots in a common frame of reference. At the core of the estimation, an extended Kalman filter algorithm is used to estimate the joint pose of robots and cameras. Experimental results from realistic simulations are presented that validate the proposed strategies.

## I. INTRODUCTION

The exploration of unknown environments is one of the fundamental problems in robotics with multiple applications. During exploration, there are two opposing objectives. On one hand, the task should be completed as fast as possible, guiding the robot(s) to always seek new areas to explore. On the other hand, when robots move into new territory their positional uncertainty increases, and consequently, the resulting map becomes inaccurate. As such, there is a need to return to already explored territory, refine the robot's pose estimate, and reduce the uncertainty in the produced map. Introducing many robots into the exploration and mapping task increases the efficiency as the task can be broken down and each robot completes only one part of the task. In addition, the overall robustness increases, as in the case of a robot malfunction, the other robots can take over and complete the task. However the complexity of the solution increases as the robots have to coordinate with each other.

In this paper we address the problem of simultaneously localizing a team of mobile robots and mapping a camera sensor network [1]. The team moves inside an environment equipped with a several cameras, connected in the same network as the team of mobile robots; see Fig. 1. The goal is to localize all the cameras in a global frame of reference, and to maintain the pose estimate for each mobile robot in the team as accurately as possible. This variant of the mapping problem eliminates the data association concerns which, though relevant, are not at the center of the decision between exploration and relocalization. During exploration we assume that the robots are in constant communication.

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Fig. 1. A sample office environment with the Generalized Voronoi Graph (GVG) displayed, and five robots ( $R_1, \dots, R_5$ ). Without loss of generality we assume the cameras are located at the junctions (vertices) of the GVG.

In general, an indoor environment equipped with a camera sensor network, capable of communicating with a team of robots, typically also provides inter-robot communication infrastructure. As the team moves through the environment it uses the robot's pose estimate, every time a robot encounters a camera, in order to place the camera in the common frame of reference of the robots. By maintaining an ongoing estimate of each robot's pose, the robots can probabilistically estimate, (and update) the position of any sensor that observes them, given the appropriate motion and measurement models. Furthermore, the environment is represented as a graph, where vertices are locations that can be seen by the cameras and the edges represent accessible paths between these areas; the cameras have no overlapping fields of view.

In the proposed exploration framework, where uncertainty reduction is one of the fundamental concerns, different choices are made in order to balance accuracy and efficiency. In general, any robot exploring new territory and producing a map makes the following decisions:

- **Choice between exploring new territory or relocalizing:** After each step, select with probability  $p_{explore}$  if the robot should explore and with probability  $1 - p_{explore}$  if the robot will relocalize; a strategy known as  $\epsilon$ -greedy[2], [3].
- **Select Destination**
  - *Explore:* If the current location is a frontier, move to new territory. Otherwise, select the closest frontier territory. In the proposed work, during exploration of new territory the ear-based exploration strategy [4] is used to facilitate frequent loop-closures.
  - *Relocalize:* Select as destination the node with the highest uncertainty [3].
- **Path plan to destination:** An A\* path-planning algorithm considering a combination of distance and uncertainty reduction as the cost function is utilized [5].

In other words, any multi-robot exploration strategy needs to address several different components. At the core of the proposed system is an EKF-based state estimation algorithm

that computes the pose of every robot together with the pose of the cameras in a common frame of reference. At each step, for each robot, a planner makes first the choice of where to go next: explore a new area or relocalize by visiting a known part of the world; and then plans a path to the selected goal. Both the goal selection and the final path to the goal are computed using an A\* planning algorithm with a cost function that combines the distance travelled with the projected uncertainty variation from such action; the A\* uses the known map to estimate the expected uncertainty of an action as described in [5].

One key distinction among different multi-robot approaches is based on the composition of the team. When all the robots are similar, in terms of cost and capabilities, then each robot can exchange roles with any other robot if it is appropriate. In contrast when the robots operate in dangerous environments robots with different capabilities are deployed. For example, in a search and rescue, or in a contamination identification scenario, the less capable, therefore cheaper robots, are used to explore unknown territory thus risking damage [6], while the more capable/expensive robots operate inside the explored area providing support. The main goal of the more capable robots is to improve the overall accuracy of the map. In this paper we present two strategies depending on the variation of the robot capabilities, heterogeneous versus homogeneous robot teams. The focus is on the heterogeneous team with the homogeneous team performance presented for comparison. Another way to describe the difference between the two scenarios is to consider exploration and relocalization as role assignments. In homogeneous teams, employing  $\epsilon$ -greedy strategy, the role assignment is dynamic. In contrast, in heterogeneous teams the role assignment is static, depending on the capabilities of each robot.

This paper also presents an extension of the map merging algorithm [7] from the 2D case,  $[x, y, yaw]$ , to 6 DoF,  $[x, y, z, roll, pitch, yaw]$ , for a team of robots of arbitrary size. Multiple observations of the same landmark by different robots are fused together in a systematic manner using the extended Kalman filter formulation.

## II. RELATED WORK

Several different strategies have been proposed for the problem of exploration. Frontier based exploration was extended to multiple robots in [8]. Zlot *et al.* [9] proposed a market based scheme for exchanging exploration tasks among the robots. In both cases the goal was to improve the efficiency of the exploration with no concern for the quality of the map. An alternative market based scheme was proposed by Gerkey *et al.* [10]. Wurm *et al.* [11] used a frontier based exploration to select destinations which were then allocated to the robots using the Hungarian method of task assignment. Earlier work [12] considered the gain in information by exploring a new area and the cost to reach the above mentioned area. A greedy approach to exploration was proposed in [13] where the information gain from different actions was explicitly modelled. The greedy approach was also used in [14] where the task of mapping was considered under the assumption of perfect localization. The Centibot

project was a DARPA sponsored program that demonstrated the deployment of one hundred robots. In [15] the authors proposed merging the information of the robot team members into shared maps, and then generate tasks using a topological decomposition of the environment. Semantic information has also been used to identify locations and facilitate exploration of multi-robot teams [16]. More recently, the problem of selecting goals was formalized as a decentralized Markov Decision Process [17], the goal is to minimize interference among the robots. All of the above methods are concerned with efficiency, that is, minimizing the total distance travelled by the robot-team with minimal concerns on the accuracy of the resulting map. Theoretical exploration algorithms on graphs have been studied extensively [18] but they do not address the positional uncertainty accumulated during edge traversals. In a related topic Basilico and Amigoni [19] proposed a strategy for exploring an unknown space with efficiency as the main motivation. Their choice is based on the Multi-Criteria Decision Making theory, and focuses on maximizing the perceived information.

During exploration, different criteria can be used to select the next frontier node from where to explore unknown territory: the closest node, or a random node, or the node with maximum or with minimum uncertainty; [3] presents a study of the effect the above choices have on the performance and the accuracy for the single robot case.

In [20] two measures from information theory, the trace or the determinant of the covariance matrix, provided an estimate for the overall uncertainty of the system. Very recently, Carrillo *et al.* [21] proposed a new method for quantifying the uncertainty using the D-opt criterion as the metric. In this paper we use the more traditional metric of the trace of the covariance matrix [22], [23];  $E = \sqrt{\text{trace}(P)}$ . Where  $P$  is the state covariance composed of the pose of the robot and the poses of all the discovered cameras. The smaller the trace  $E$  the more accurate the estimate. We are in the process of comparing the two metrics; however, such a comparison is beyond the scope of this paper.

The trace of the covariance was also used in [24] in a weighted linear combination of uncertainty and distance for a path  $p$ ; see (1).

$$C(p) = \omega_d \text{length}(p) + \omega_u \text{trace}(P(p)) \quad (1)$$

In the cost function  $C(p)$ , the trace of the covariance matrix  $P$  is an approximation of the uncertainty in the map, while the length of the path is another contributing factor on evaluating the quality of path  $p$ . The same formulation has been used more recently also in [5], [25], [26].

### A. Hybrid Robot/Camera Sensor Network Localization

As mentioned earlier we extend the cooperative robot-camera sensor network localization paradigm [26] to multi-robot teams. For the sake of completeness a small overview of the single robot case approach is presented next. A robot, equipped with a unique detectable target, travels through an environment which is equipped with a camera sensor network. When the robot ( $R$ ) is detected by a camera ( $C_j$ ) the

Camera-Robot transformation  $T_{C_j}^R$  is inverted and the pose of the robot is used to localize the camera in a common frame of reference. An EKF based scheme is used to propagate uncertainty between the camera locations using the motion model of the robot. In the multi-robot case, each robot maintains their own state  $\mathbf{x}_i$ , see (2), which consists of the position and orientation of the robot together with the map of the camera poses:

$$\mathbf{x}_i = [\mathbf{x}^{R_i}, \mathbf{x}_{C_1}, \dots, \mathbf{x}_{C_N}]^T \quad (2)$$

where  $\mathbf{x}^{R_i} = [x_t^i, y_t^i, \theta_t^i]$  is the position and orientation of the robot in 2D;  $\mathbf{x}_{C_j} = [x_t^j, y_t^j, z_t^j, \theta_t^j, \phi_t^j, \psi_t^j]^T$  is the pose of the camera in 3D;  $x_t^i, y_t^i, z_t^i$  represents the position of the  $i^{\text{th}}$  camera, and  $\theta_t^i, \phi_t^i, \psi_t^i$  are the Euler angles roll, pitch, yaw respectively.

The covariance matrix  $P_i$ , see 3, can be decomposed in several blocks:

$$\mathbf{P}_i = \begin{bmatrix} \mathbf{P}_{R_i R_i} & \mathbf{P}_{R_i C_1} & \dots & \mathbf{P}_{R_i C_{N_i}} \\ \mathbf{P}_{C_1 R_i} & \mathbf{P}_{C_1 C_1} & \dots & \mathbf{P}_{C_1 C_{N_i}} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{P}_{C_{N_i} R_i} & \mathbf{P}_{C_{N_i} C_1} & \dots & \mathbf{P}_{C_{N_i} C_{N_i}} \end{bmatrix} \quad (3)$$

where  $\mathbf{P}_{R_i R_i}$  is the robot's  $i$  uncertainty covariance;  $\mathbf{P}_{C_j C_j}$  is the uncertainty covariance of the camera's  $j$  pose. The cross correlation between robot  $i$  and camera  $j$  is represented by  $\mathbf{P}_{R_i C_j}$  and the cross correlation between two cameras is represented by the matrix  $\mathbf{P}_{C_i C_j}$ . The cameras are represented in the covariance matrix in the order they were discovered.

### III. MULTI-ROBOT EXPLORATION

In this paper we expand the Simultaneous Localization and Uncertainty Reduction on Maps (SLURM) strategy from single robots [4] to teams of robots. Central to our approach is the satisfaction of the conflicting goals of efficiency and accuracy. Depending on the homogeneity of the robots capabilities two strategies are proposed. When every robot has the same capabilities, the robots spread through the environment and each robot operates inside its own space, using a Voronoi decomposition of the explored space. When the team comprises of robots with different qualities, especially in the presence of hazardous conditions, the more capable robots refine the map staying in the safe explored area, while the least capable/expensive robots drive through the unknown and explore the environment. In both cases the team selectively explores and relocalizes in a balanced manner that ensures a high quality map as the end result.

Different exploration strategies can be employed by each individual robot. At any point each robot has knowledge of a partial map, in our case in the form of a sub-graph, and a set of destinations, in our case a set of vertices, termed frontiers [8] which provide access to the unexplored part of the environment. At the core of each strategy is how to select the next frontier node from where to continue the exploration. In this work we utilize the ear-based exploration [27]. The proposed strategy works as following: the robots explore the environment one cycle at the time, for each robot. In

the graph model used in this work, after reaching a frontier node each robot takes the first edge clockwise, and continues selecting the next edge clockwise until the robot returns to an already explored vertex. During edge traversals the odometric error is propagated using an extended Kalman filter (EKF). The pose of the mapped camera, in the already explored vertex, is used in the EKF update step. This strategy deliberately ensures loop closure regularly, by returning to explored areas, thus reducing the map uncertainty at regular intervals.

Another important consideration relates to the communication capabilities of the robot team and the frequency at which they exchange information. The more often the robots exchange information, the more accurate the results will be, however, frequent updates will also place a heavy computational load on the system. In this work we consider each trajectory for exploring a new part or for refining the map as an atomic action and only after it is completed the robots exchange information and merge their maps. The next subsections describe the two approaches in detail.

#### A. Homogeneous team – Voronoi Task Decomposition

When all robots have similar capabilities, the explored part of the map is partitioned using the Voronoi decomposition on the geodesic distance of the graph. In other words, each robot considers as candidates for exploration or relocalization all the nodes that are closest to it than to any other robot. This strategy minimizes the travelling distance by guiding the robots to visit frontier nodes at nearby locations, and letting the distant nodes to be handled by robots closer to them. This area allocation is dynamic and varies as robots move to new areas. In addition if a robot has no frontier nodes inside its allocated area, then it either uses its time to refine the map by relocalizing or after a few attempts, it travels to the closest frontier node. It is worth noting that the uncertainty reduction spreads to the rest of the map during the merge map operation, see section IV, due to the cross-correlation terms of the covariance matrix.

Figure 2 presents two annotated time snapshots of the exploration of an environment by three robots. In Fig. 2a the robots are placed at a frontier node, and an arrow points to the selected frontier edge each robot has selected. In Fig. 2b, each robot has explored an ear and has return to a known node, effectively performing loop closure. The

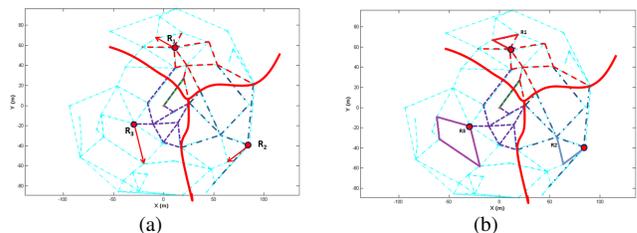


Fig. 2. Homogeneous team exploration. (a) The three robots are at a different frontier vertex each. The arrow points to the frontier edge (unexplored edge) they will follow next. (b) Each robot performed ear-based exploration adding a new loop in the map. The hand drawn solid lines indicate the Voronoi boundaries assigning the explored vertices to the individual robots based on proximity.

hand-drawn lines indicate an approximation of the Voronoi decomposition.

### B. Heterogeneous team – Explorers and Map Refiners

When the exploration of unknown environments is complicated by the presence of hazardous conditions, it is worth considering an alternative strategy to minimize the cost from potential robot failures. The main idea is to use a heterogeneous team in which inexpensive robots, termed explorers, with limited sensing and computing capabilities are used to explore into the unknown territory, risking encounters with hazardous conditions, while expensive robots, termed map-refiners, with accurate localization capabilities traverse through the safe, explored space [6].

Figure 3 presents an annotated example of the heterogeneous exploration strategy in action. In Fig. 3a there are three explorer robots positioned at different frontier nodes, and two map-refiners that are going to stay inside the explored world. In Fig. 3b each of the three explorers has followed an ear in the environment until returning to an already explored node. The two refiners traverse the known world in order to reduce the uncertainty. The robots continue exploring and refining the map in parallel, until there are no unexplored edges left.

### C. Exploration versus Exploitation

An important component of the proposed approach is the allocation of resources to the task of map refinement. In addition to relocalizing as the robots traverse known territory on their way to unexplored areas, the robots deliberately select paths that will take them through the known map of the environment with the express purpose of reducing the overall map uncertainty. In order to generate better paths which balance the distance and uncertainty objectives we have to combine them into a single cost function. Unfortunately, the two are not compatible; in other words, they do not share common units for comparison. Equation 1 is used to combine distance and uncertainty at a common cost function. Choosing the weighting factors  $\omega_d$  and  $\omega_u$  represents the compromise between distance travelled and mapping uncertainty or *efficiency* versus *accuracy*. We use a flexible approach based on varying one intrinsic parameter; the contribution of each quantity is normalized by a rough estimate of its maximum possible value. When each quantity is normalized, a single free parameter  $\alpha$  in the range  $[0, 1]$  is used to specify the participation of each factor. Based on

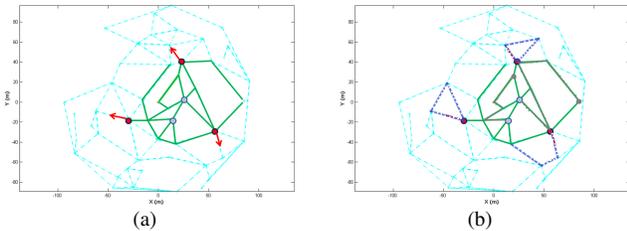


Fig. 3. Heterogeneous team exploration: Explorers and Map Refiners. (a) The arrows indicate the chosen frontier edge. (b) Each explorer performed an ear-based exploration, adding a loop in the graph. The two map refiners travelled inside the known graph (solid lines).

this approach, the weights used in the proposed cost function are:

$$\omega_d = \frac{\alpha}{\max distance} \quad , \quad \omega_u = \frac{1 - \alpha}{\max uncertainty} \quad (4)$$

By setting  $\alpha$  low during map refinement the robot selects the path that should result to the best uncertainty reduction. During exploration  $\alpha$  is set close to one, resulting to short/efficient exploration paths.

In the heterogeneous algorithm, the robots have distinct roles of exploring and refining the map. In the homogeneous case though each robot has to make the decision, in each round, if it should go and explore new territory or if it should plan a path that will improve map accuracy. We employ the  $\epsilon$ -greedy strategy in which: with probability  $P_{explore}$  the robot moves to a new area, and with probability  $1 - P_{explore}$  the robot plans a path through the explored area.

$$\mathbf{P}_{ij} = \begin{bmatrix} \mathbf{P}_{R_i R_i} & \mathbf{P}_{R_i C_1} & \dots & \mathbf{P}_{R_i C_{N_i}} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{P}_{C_1 R_i} & \mathbf{P}_{C_1 C_1} & \dots & \mathbf{P}_{C_1 C_{N_i}} & \mathbf{0} & \dots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{P}_{C_{N_i} R_i} & \mathbf{P}_{C_{N_i} C_1} & \dots & \mathbf{P}_{C_{N_i} C_{N_i}} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} & \mathbf{P}_{R_j R_j} & \dots & \mathbf{P}_{R_j C_{N_j}} \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} & \mathbf{P}_{C_1 R_j} & \dots & \mathbf{P}_{C_1 C_{N_j}} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} & \mathbf{P}_{C_{N_j} R_j} & \dots & \mathbf{P}_{C_{N_j} C_{N_j}} \end{bmatrix} \quad (5)$$

### IV. MAP MERGING

In order to merge the maps the robots employ an augmented version of the algorithm proposed by Zhou and Roumeliotis [28]. In particular the pose of each camera in our map is equivalent to the landmarks in [7], however, each camera pose is represented by six variables,  $x_C = [x, y, z, \theta, \phi, \psi]^T$ . Currently the robots synchronize in pairs. As such, for every two robots,  $R_i$  and  $R_j$  an update operation is performed for every camera mapped by both robots. First we join the covariance matrices of the two robots ( $\mathbf{P}_i$  and  $\mathbf{P}_j$ ; see (3)) in a joint covariance matrix  $\mathbf{P}_{ij}$ ; see (5). Where the cross-correlation terms between the two robots are set to zero.  $N_i$  ( $N_j$ ) is the number of cameras seen by robot  $R_i$  ( $R_j$ ) respectively. As discussed earlier the data association problem is addressed by the unique id of each camera. However, the cameras are visited (discovered) in different order by each robot and thus, their placement inside the state vector and the covariance matrix is different.

The cameras that have been observed by both robots provide an implicit measurement since their global coordinates are unique and their difference is equal to zero. If  $\hat{\mathbf{x}}_{C_k}^i$  ( $\mathbf{x}_{C_k}^i$ ) is the estimate (actual pose) of the  $k^{th}$  camera as observed by robot  $R_i$  and  $\hat{\mathbf{x}}_{C_k}^j$  ( $\mathbf{x}_{C_k}^j$ ) is the estimate (actual pose) of the same camera as observed by robot  $R_j$ , then the implicit measurement from the dual observation of the  $k^{th}$  camera is:

$$\mathbf{Z}_k = \mathbf{x}_{C_k}^i - \mathbf{x}_{C_k}^j = \mathbf{0}_{6 \times 1} \quad (6)$$

$$\hat{\mathbf{Z}}_k = \hat{\mathbf{x}}_{C_k}^i - \hat{\mathbf{x}}_{C_k}^j \quad (7)$$

and the residual is:

$$r_k = Z_k - \hat{Z}_k = -\hat{Z}_k \quad (8)$$

The observation matrix  $H$  for this measurement is:

$$H_k = [\mathbf{0}_{6 \times 3} \mathbf{0}_{6 \times 6(m-1)} I_{6 \times 6} \mathbf{0}_{6 \times 6(N_i-m)} \mathbf{0}_{6 \times 6(n-1)} - I_{6 \times 6} \mathbf{0}_{6 \times 6(N_j-n)}] \quad (9)$$

where  $m$  is the position of the  $k^{\text{th}}$  camera in the state vector of robot  $R_i$  and  $n$  is the position of the  $k^{\text{th}}$  camera in the state vector of robot  $R_j$ .  $N_i$  ( $N_j$ ) is the number of cameras in the state vector of robot  $R_i$  ( $R_j$ ).  $\mathbf{0}_{m \times n}$  is a  $m$  by  $n$  matrix of zeros

The update using the mutual observation of the  $k^{\text{th}}$  camera is performed using the standard Kalman filter equations [29]. After the update the second appearance of the  $k^{\text{th}}$  camera is removed, both from the state vector and from the covariance matrix, together with the cross-correlation terms. The above describe update is performed sequentially for every camera that is observed by both robots. In robot teams of more than two robots the merge operation is performed as following. First, a merge is performed between the first and the second robot ( $R_1$  and  $R_2$ ), then the result is used in a merge with the third robot, and so on. After the  $k^{\text{th}}$  successive merge, the state and covariance are populated with the robots  $R_1$  to  $R_{k+1}$  and all the unique landmarks observed by the  $k+1$  robots; see (10) for a block depiction of the covariance.

$$\mathbf{P}_{1-k} = \begin{bmatrix} \mathbf{P}_{RL_1} & \mathbf{P}_{RL_1RL_2} & \cdots & \mathbf{P}_{RL_1RL_{k+1}} \\ \mathbf{P}_{RL_2RL_1} & \mathbf{P}_{RL_2} & \cdots & \mathbf{P}_{RL_2RL_{k+1}} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{P}_{RL_{k+1}RL_1} & \mathbf{P}_{RL_{k+1}RL_2} & \cdots & \mathbf{P}_{RL_{k+1}} \end{bmatrix} \quad (10)$$

where  $\mathbf{P}_{RL_i}$  contains the state of robot  $i$  and the landmarks associated with it, the structure for the covariance matrix of a single robot with its observed landmarks is presented in (3). When all the maps are merged, then the final result contains information for each robot and for all the landmarks without any duplicates. It is worth noting that the merge operation is computationally expensive, and we are currently investigate options that will improve the efficiency of the operation.

Figure 4 illustrates the merge map operation between two robots; Fig. 4(a) presents the map of robot  $R_1$  before the merge, Fig. 4(b) presents the map of robot  $R_2$ , and Fig. 4(c) presents the updated map containing all the mapped cameras (landmarks). V. EXPERIMENTAL RESULTS

Several experiments were performed in simulation using realistic parameters collected during our previous experiments [30]. In particular we have tested the proposed algorithms for environments with different number of vertices, and varying graph densities. It is worth noting that when the environments have denser connectivity, the frequency of loop-closures increases resulting in reduced uncertainty accumulation. In addition, as the number of robots increases so does the number of multiple observations of the same landmark.

Figure 5 presents uncertainty accumulation and distance travelled results for the two proposed strategies, for a varying

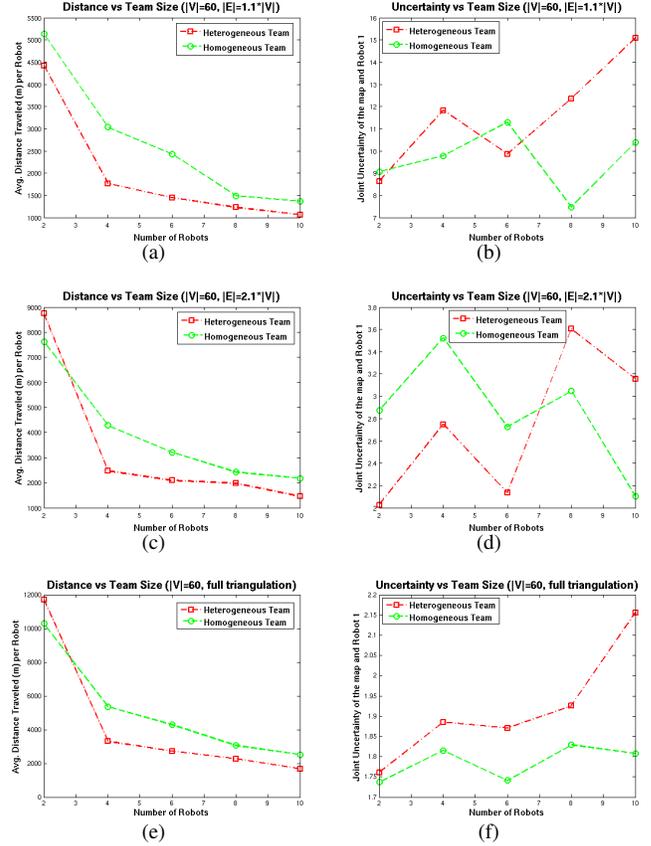


Fig. 5. A comparison between the homogeneous team strategy and the heterogeneous team strategy. Each graph presents average over five trials. The first column presents the average distance travelled per robot; the second column presents the map uncertainty as described by the square root of the trace of the covariance. In all experiments we used a world with sixty cameras; with varying graph density. (a,b) presents results from a sparse graph (almost a spanning tree) where the number of edges was 1.1 times the number of vertices. (c,d) a fairly dense graph, the number of edges is equal to 2.1 times the number of vertices. (e,f) a complete triangulation.

number of robots and for graphs with different densities. In order to ensure the fairness of the comparison between the two techniques, we set up the probability  $P_{\text{explore}}$  in the homogeneous case to be 70% and in the case of a heterogeneous team, two in three robots (66%) are explorers. In all the experiments we used the same number of vertices with varying map density. The map density is given as a function between the number of vertices and the number of edges. In Fig. 5(a,b) the structure of the environment is similar to a spanning tree ( $|V| = 1.1 * |E|$ ); where  $|V|$  is the number of vertices and  $|E|$  the number of edges. In Fig. 5(c,d), there were twice as many edges as vertices, and finally, in Fig. 5(e,f) a complete triangulation was used. The first column presents the average distance travelled per robot, averaged over five trials. The second column presents a measure of the accumulated uncertainty in the form of the square root of the trace of the map covariance.

As can be seen in Fig. 5, the heterogeneous team has systematically lower distance travelled without suffering a significant loss in accuracy. It is worth noting that when the graph is dense the uncertainty variation is negligible. This is due to the fact that in a complete triangulation, each

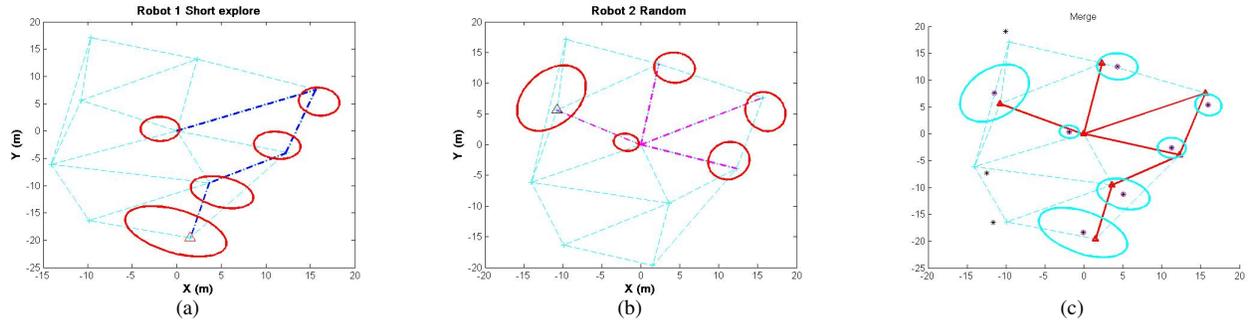


Fig. 4. Map merging of two robots. After an initial exploration each robot has a partial map with high uncertainty. (a) Robot 1 performed depth first exploration; (b) Robot 2 performed breadth first exploration; (c) the joint map after the merge.

robot traverses at most two edges and closes the loop; as such it is nearly-irrelevant if there will be any time spend explicitly refining the map. In addition a denser map means more edges to be traversed while covering the same number of landmarks. Therefore, the uncertainty accumulation is insignificant.

## VI. CONCLUSIONS

In this paper we presented two new strategies for team-based exploration. The differentiating factor is the homogeneity of the robots capabilities.

The choice between exploring new territory or refining the existing map is at the center of the proposed methodology. The desired accuracy of the map, often dictated by the intended application, is what guides the selection of the exploration strategy. In addition, in dangerous environment, ensuring the safety of the most expensive/capable robots will enable deployment of robot teams in search and rescue scenarios.

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