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Limited Communication, Multi-Robot Team Based Coverage

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Abstract— **This paper presents an algorithm for the complete coverage of free space by a team of mobile robots. Our approach is based on a single robot coverage algorithm, which divides the target two-dimensional space into regions called cells, each of which can be covered with simple back-and-forth motions; the decomposition of free space in a collection of such cells is known as the Boustrophedon decomposition. Single robot coverage is achieved by ensuring that the robot visits every cell. The new multi-robot coverage algorithm uses the same planar cell-based decomposition as the single robot approach, but provides extensions to handle how teams of robots cover a single cell and how teams are allocated among cells. This method allows planning to occur in a two-dimensional configuration space for a team of N robots. The robots operate under the restriction that communication between two robots is available only when they are within line of sight of each other.**

I. INTRODUCTION

The task of covering the free space is common to many applications. Automated humanitarian de-mining, lawn-mowing, and vacuum cleaning all require the robot to pass an end-effector over a designated area. Many of those applications require complete coverage. Complete coverage planning for a mobile robot guarantees that a robot's detector area, such as the robot's footprint or the range of a metal detector, passes over all reachable points in the target environment.

Using multiple robots can accelerate the process of coverage, thereby improving efficiency, which is evaluated in terms of area covered over time. Ideally we would like to maximize this quantity, but optimal coverage is impossible because the robots have no prior knowledge of the workspace; thus it is always possible to generate an antagonistic placement of obstacles in the environment that would cause any algorithm to have suboptimal performance. However we seek to reduce *repeat coverage*, which is defined as any robot covering previously covered space. Any reduction in repeat coverage increases the performance, as more robots cover simultaneously new space.

Central in the multi-robot approach is the issue of communication. When unrestricted communication is available, then the robots can disperse through the environment and proceed to cover different areas in parallel, constantly updating each other on their progress. When communication is restricted to close proximity [1] or line of sight i.e., communication is available only when they have an unobstructed line of sight between them, the robots have to remain together in order to avoid covering the same area multiple times.

We examine the problem of multi-robot coverage path planning for a team of robots with limited communication. Information sharing between the robots is restricted to line-of-sight communication (Fig. 1). Minimizing *repeat coverage* is further complicated by the lack of information exchange. In earlier work [1] some robots covered repeatedly the same area due to lack of communication resulting in no improvement in efficiency. The proposed algorithm is based on prior single robot coverage methods that guarantee complete coverage. To achieve provable completeness, most single robot coverage planners use a cellular decomposition of the environment. Exact cellular decompositions represent the free configuration space by dividing it into non-overlapping cells such that adjacent cells share a common boundary, the interior of each cell intersects no other cell, and the union of all the cells covers the free space. Covering each cell is simple, and provably complete coverage results from ensuring that the robot visits every cell.

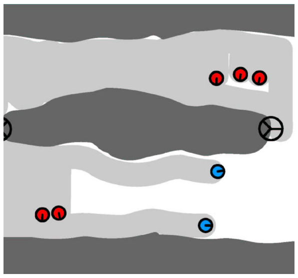


Fig. 1. Seven robots covering around an obstacle.

Our algorithm is based on a sensor-based multi-robot coverage approach, which covers an unknown space while simultaneously constructing a cellular decomposition, which in turn is used to guarantee complete coverage. A robot team consists of all robots within line-of-sight to each other. Each distinct team maintains its own internal representation of the world, which is shared and updated whenever two teams come within line-of-sight of each other and join into one team. Each robot has knowledge of its position and heading with respect to a global coordinate frame.

This paper is divided into the following sections: Section II discusses related work and Section III presents some necessary terminology related to the Boustrophedon cellular decomposition. Section IV describes our multi-robot coverage algorithm. Section V discusses the advantages and disadvantages of our cell coverage technique. Section VI contains experimental results from a variety of environments. Finally, section VII presents our conclusions.

II. RELATED WORK

Previous work can be grouped using two different criteria: communication ability, and determinism. First we examine deterministic approaches that guarantee complete coverage and then we present a brief overview of a variety of techniques which use a stochastic approach (sometimes referred to as swarm, or biologically inspired, or behaviour-based techniques) to solve the multi-coverage problem.

Our work takes root in the Boustrophedon decomposition [2], which is an exact cellular decomposition where each cell can be covered with simple back-and-forth motions. The cells are defined by sweeping a slice [3] (a one-dimensional line) through the configuration space and noting where the connectivity of the slice changes in the free configuration space. These connectivity changes occur at critical points. A method that uses simple sonar range sensors to detect critical points was introduced in [4]. Using this method, the robot can simultaneously cover an unknown space while looking for critical points to ensure complete coverage.

Butler *et al.* have developed a cooperative sensor-based coverage algorithm [5] based on the single robot algorithm. The basic concept of this algorithm is that cooperation and coverage are algorithmically decoupled. This means that a coverage algorithm for a single robot can be extended to a cooperative setting. To produce cooperative coverage, an overseer algorithm is added to the single robot algorithm, which takes incoming data from other robots and integrates it into the cellular decomposition. In their approach unrestricted communication is assumed among the robots. It can be shown that the overseer indeed performs this operation in such a way that coverage can continue under the direction of the single robot algorithm without the algorithm even knowing that cooperation occurred.

Early work by Kurabayashi *et al.* proposed an off-line planning algorithm for sweeping a known area with the ability to plan for relocating objects. The algorithm acts in two stages: first the complete path is planned off-line and the area is divided among robots [6], then the location of the movable object is examined and the optimal relocation path is estimated [7]. Experimental results show an improvement in the efficiency due to smart relocation of movable objects. Recently, Latimer *et al.* [1] employed an algorithm based on the single robot cellular decomposition approach. Dispersing the robots through the environment was emphasized and encouraged to allow parallel coverage with finer granularity [5], [1]. However, because this approach only allowed communication between robots in physical contact with each other, many robots still ended up covering the same space.

Tao and How [8] use a decentralized approach to select cells in a grid world to be covered. They use a limited motion and sensor model and assume global communication. Negotiation between agents is used in order to facilitate cooperation. More recently, Luo and Yang [9] use a neural network to represent the environment. Each cell/neuron corresponds to a cell in the occupancy grid and the activity in each neuron represents the belief that the cell is occupied, unknown, or covered. They demonstrate their approach for two collaborating robots covering an environment in simulation.

Ichikawa and Hara [10] proposed a multi-robot coverage behaviour that emerges from simple obstacle avoidance with large number of robots. Their approach is simple but does not guarantee full coverage and the same area is covered multiple times. Wagner *et al.* [11] use robots that employ traces to mark the covered areas in a biologically inspired approach termed ant-robotics. The environment is represented as a graph where each node is a cell in a grid-world. Communication among agents is done implicitly via the trace of pheromones each robot leaves. Another biologically inspired approach was used by Bruemmer *et al.* [12] for covering an area using a swarm of small robots. The complete coverage was determined by a human observer for different swarm sizes. More recently, Batalin and Sukhatme [13] proposed two behaviour-based algorithms for multi-robot coverage. The communication is limited to visual contact and the robots disperse through the environment to ensure maximum coverage with no guarantee for completeness.

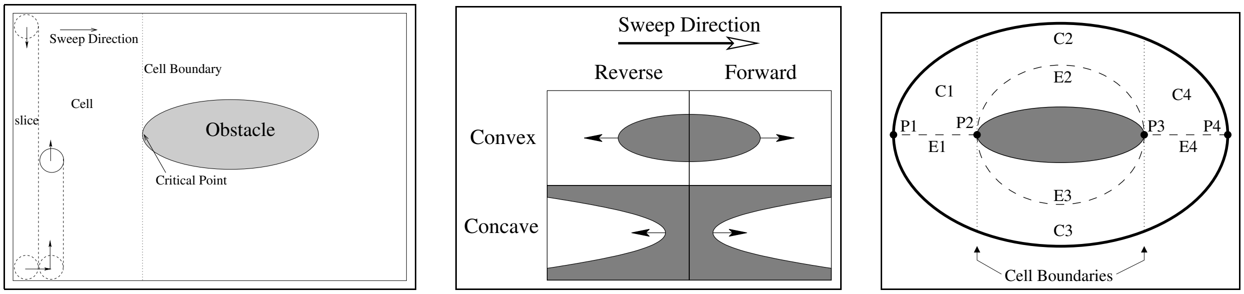
III. TERMINOLOGY

To better describe the multi-robot coverage task, we borrow the following terms from single robot coverage - *slice*, *cell*, *sweep direction*, and *critical point* (see Fig. 2a). A *slice* is a subsection of a *cell* covered by a single, in our case vertical, motion. A *cell* is a region defined by the Boustrophedon decomposition where slice connectivity is constant. *Sweep direction* refers to the direction the slice is swept. Lastly, a *critical point* represents a point on an obstacle which causes a change in the slice connectivity.

Critical points are further divided into four categories (see Fig. 2b) based on sweep direction and convexity/concavity of the critical points. The sweep direction is relative to a robot, and can change during the process of coverage. A critical point is characterized as convex/concave if it is introduced by the convex/concave region of an obstacle (i.e., the convexity of the point the slice contacts the obstacle). See Fig. 2b for an illustration of convex and concave critical points. Therefore, a forward convex critical point (FCV) can be a reverse convex critical point (RCV) if the sweep direction reverses. Concave critical points are also known as terminating critical points. It is worth noting that no forward concave critical points are ever encountered as by definition the sweep direction guides robots away from them.

We also borrow the concept of a Reeb graph [14], [15], a graph representation of the target environment where the nodes are the critical points and the edges are the cells (Fig. 2c). Due to the nature of the Boustrophedon decomposition, all concave critical points are connected to exactly one cell, i.e., a node of degree one in the Reeb graph. Similarly, all convex critical points are connect to exactly three cells i.e. a node of degree three in the Reeb graph.

A feature that is useful for adapting the single robot coverage algorithm to multi-robot coverage is that we can have two robots tracking endpoints of a slice as it is swept through the free space and when they lose the line of sight



(a) (b) (c)

Fig. 2. (a) The terms borrowed from single robot coverage with one robot and one obstacle in the target environment are presented. The robot is performing coverage with simple back-and-forth motions. (b) The four types of critical points, based on concavity and the surface normal vector parallel to the sweep direction. Forward indicates parallel sweep and normal vector directions, whereas reverse indicates opposite directions. Note that the shaded areas are obstacles and the arrows represent the normal vectors. (c) A simple Reeb graph is overlaid on top of a simple elliptical world with one obstacle. P1-P4 are critical points which represent graph nodes. E1-E4 represent edges which directly map to cells C1-C4.

connection to each other we can then conclude the existence of a critical point. We refer to these two robots as *explorers* to distinguish them from the rest of the team that just covers the free space (termed *coverers*). It is worth noting that prior work in single robot coverage actually had to rely on sophisticated obstacle detecting motion strategies to determine the location of critical points; now because multiple robots are used simple wall following allows for critical point detection.

For the team based coverage the following terminology is used: *union cell*, *top cell*, *bottom cell*, *complete* and *incomplete critical points*. See Fig. 3 for a display of these definitions. The *union* cell is defined as the cell pointed to by the normal vector at a convex critical point. Without loss of generality we consider a horizontal sweep direction. The other two cells associated with a convex critical point are relative to the global reference frame and are defined as the *top* (higher y-coordinate) and *bottom* (lower y-coordinate) cells respectively. Note that cell naming is relative to a critical point, therefore a cell may have different names with respect to different critical points. A critical point is characterized as *complete* if all of the cells adjacent to it are completely covered; otherwise, the critical point is referred to as *incomplete*.

IV. ALGORITHM DESCRIPTION

All robots start in a horizontal formation with perfect positioning in a global frame of reference; see for example the five robots that started at the upper left corner of Fig. 4. We consider the case where only line-of-sight communication is provided. Although the world is not known *a priori*, we do assume a static environment.

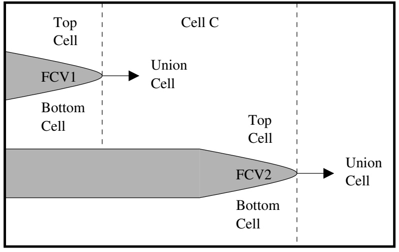


Fig. 3. Depicts cell naming adjacent to two forward convex critical points (FCV1, FCV2). The *union cell* is pointed at by the critical point normal and the *top cell* and *bottom cell* are above and below the critical point respectively. The dashed lines represent cell boundaries. Note that Cell C is both the union cell of FCV1 and the top cell of FCV2.

The algorithm is based on two main ideas. First, during the coverage of a single cell, the boundaries of the cell are covered by two robots (explorers) that follow the top and the bottom boundary, with the same lateral speed, until they are no longer within line of sight of each other. The two explorers use a break in the line of sight to detect critical points and thus determine the termination of the cell. Using the line of sight as an extended sensor has been successfully used already by a number of different authors [16], [17], [18].

Second, in order to maintain the cohesiveness of the team and to avoid redundant coverage our approach allows the team of robots to divide in two sub-teams only once. When an obstacle induces a convex critical point that terminates the current cell and introduces two new cells (see for example, the point P_2 in Fig. 2c), the team of robots is allowed to split into two sub-teams. The two sub-teams cover only the cells adjacent to the obstacle that caused the split until the two sub-teams meet again and rejoin into a single team. Each of the two sub-teams requires at least two robots for top and bottom cell-boundary exploration. Thus, the algorithm we present requires a minimum of four robots.

The team or divided sub-teams cover the cells, one cell at a time, and the Reeb graph determines which cell to cover. Initially, the robots cover the starting cell as one large team (*Cover a Single Cell*), simultaneously creating a cellular decomposition and the corresponding Reeb graph. After each cell is covered, the team proceeds to the closest critical point with uncovered cells . If no such critical point exists, the space has been fully covered and the algorithm terminates.

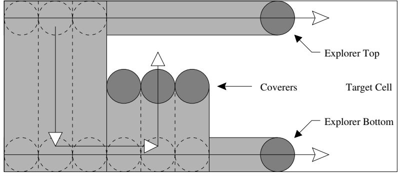


Fig. 4. Illustrates the *explorer/coverer* approach, where two robots *explorers* outline the top and bottom boundaries while the remaining robots (*coverers*) execute simple back-and-forth coverage.

As we saw earlier, the number of cells connected to a critical point must be exactly one (concave) or three (convex). Every known critical point is adjacent to at least one covered cell. Therefore, the target critical point, which is incomplete, is convex and has one or two uncovered cells attached to it. If the number of uncovered cells at a target critical point is one, the team invokes the procedure to cover a single cell (Cover Single Cell), and then the process repeats.

If the target critical point is forward convex (FCV), the entire team proceeds to cover the union cell, and the process repeats. However, if the target critical point is reverse convex (RCV), the team splits into two sub-teams and starts covering the cells adjacent to the obstacle that introduced the critical point. One sub-team covers cells in a clockwise fashion around the obstacle. Conversely, the other sub-team covers cells in a counter-clockwise fashion. This is the only case where a robot team splits into sub-teams. The encircling process further guarantees the two sub-teams will meet again (*Rejoin*).

After rejoining, the two sub-teams update their internal representations (Reeb graphs). Then the unified team travels to the closest incomplete critical point according to their merged Reeb graphs and the process repeats. This continues until no incomplete critical points exist, which implies all cells have been covered. The following sections describe the major parts of the algorithm.

A. Cover a Single Cell

Cover a Single Cell is the fundamental primitive behaviour of our algorithm. It extends the normal back-and-forth sweeping technique (see Fig. 2a) from single robot coverage by designating the first two robots to enter the cell as *explorers*. Their roles are to detect critical points, cover the boundaries of the cell, and to facilitate rejoining.

First, we present some terminology (see Fig. 4). *Target cell* refers to the current cell being covered. *Explorer-top* (*Etop*) refers to the first robot to enter the target cell, *Explorer-bottom* (*Ebottom*) refers to the second robot. *Coverers* is the collective term that addresses any remaining robots.

Upon entering the target cell, the explorers move (wall-follow) in unison such that the two robots maintain the same x-coordinate, thus guaranteeing line-of-sight communication within the cell. *Etop* and *Ebottom* wall-follow along the top and bottom cell boundaries respectively. Simultaneously, the coverers move in as space permits and begin simple back-and-forth covering (top of Fig. 4). The explorers continue wall-following until one of two situations occurs: Either the line-of-sight is broken; thus, a convex critical point is detected (Fig. 5a,5b). Or, the explorers approach each other; which results in the detection of a reverse concave critical point (terminating), see Fig. 5c.

In the first situation the line-of-sight is used as a form of ranged critical point detection. If one of the robots traveled opposite to the sweep direction, a forward convex critical point was detected. Otherwise, a reverse convex critical point was detected (Fig. 5b). In both situations, the explorers retrace their steps until line-of-sight is once again established. *Ebottom* then proceeds to travel vertically upward until it joins with *Etop*. Note that in both situations a robot will pass by the detected critical point, allowing its position to be recorded.

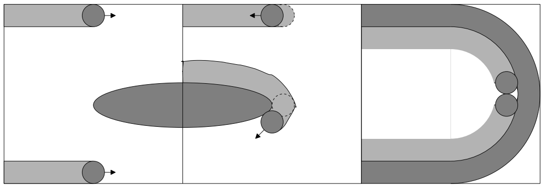

Reverse Convex Critical Point
(a)
Forward Convex Critical Point
(b)
Reverse Concave Critical Point
(c)

Fig. 5. The two environmental possibilities *explorers* encounter. (a),(b) For convex critical points, losing line-of-sight signals the end of a cell. (c) Two explorers approaching each other reveals a reverse concave critical point.

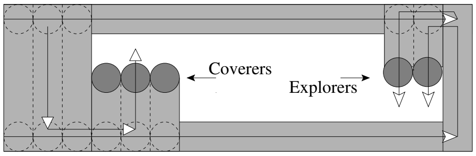


Fig. 6. Shows *explorers* covering opposite the sweep direction after encountering a critical point.

Once together, the explorers proceed to cover the cell with simple back-and-forth motions (Fig. 6), sweeping opposite to the sweep direction. In this manner, the coverers and explorers are moving toward each other, ensuring they will meet.

In the second situation, where the two explorers approach each other, *Etop* backs off and allows *Ebottom* to wall-follow the remaining unknown surface. If a reverse concave critical point is found, it is recorded and the explorers pair up to cover in the opposite direction as in the previous situation [1].

B. Encircle

Every time an obstacle introduces a reverse convex critical point, there are exactly two cells to be covered. Our algorithm covers the cells that are adjacent to that obstacle by splitting the team of robots into two sub-teams. The overall goal of the encircle stage is to take advantage of the finer granularity of smaller teams by splitting a larger team, while ensuring that no cell is going to be covered twice because of the lack of communication/coordination between sub-teams (line-of-sight limitation).

Executing an encircle action splits the team into two sub-teams. (Fig. 7) Provided that each sub-team has a minimum of two robots, any partition is acceptable. A reasonable heuristic is to allocate robots proportional to the distance from the critical point to the top and bottom boundaries.

Sub-team Clockwise (TCW) and sub-team Counter-Clockwise (TCCW) refer to the sub-teams that cover cells in opposite directions around an obstacle. In both cases, if the target cell is already covered, sub-teams travel to the other

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[1] It is worth noting that there is special case that the remaining surface revealed is an opening to a traversable corridor (see Fig. 8). In such case, both robots would move inside the narrow corridor one after the other and continue exploring as soon the space is wide enough or find a terminating critical point.

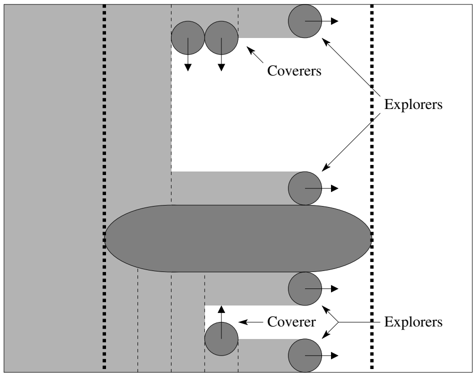

Fig. 7. Shows two sub-teams encircling an obstacle. The thick dashed lines represent cell boundaries and the thin dashed lines represent slice boundaries. Note that four robots were dispatched to the taller top cell (Cell A), while only three robots were assigned to the shorter bottom cell (Cell B).

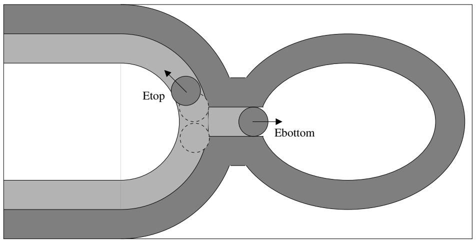

Fig. 8. Illustrates the second situation, when the remaining surface reveals a corridor where only one explorer can fit.

critical point [2] without covering (executing the procedure *Go to Other Critical Point*), and the algorithm continues. The two sub-teams cover around the obstacle in opposite directions until one of the following two terminating conditions are met:

- Case 1: The sub-teams encounter each other (establish line-of-sight communication) within a cell.
- Case 2: One sub-team encounters a concave (terminating) critical point. This terminates one branch of the Reeb graph and the sub-team backtracks.

In Case 1, the two sub-teams rejoin then proceed to jointly finish covering the remainder of the current cell. In Case 2, the terminated sub-team travels through the cell(s) in the direction opposite to its original direction (clockwise if counter-clockwise and vice versa), counting all encountered cells as covered.

It is important to note that the terminated sub-team does not cover before rejoining the other sub-team. Instead it follows the boundary of the obstacle being encircled. Similar to Case 1, once line-of-sight is established and the two sub-teams rejoin, they cooperatively cover the target cell and conclude the encircle process. Next we are going to discuss briefly the *Go Clockwise* (GCW)/ *Go Counterclockwise* (GCCW) procedures that guide the two sub-teams around the obstacle.

GCW and GCCW are deterministic algorithms that return the next target cell. The two sub-teams split at a reverse critical point to cover the cells adjacent to the obstacle that introduced the critical point, moving in opposite directions (clockwise, counterclockwise). The team moving clockwise (GCW) follows a right hand rule and every time selects the adjacent cell that has the obstacle to the right (facing toward the sweep direction). For example in Fig. 9 the team going clockwise after covering cell A proceeds to cover cell B, the one that keeps the obstacle at the right side with respect to the sweep direction. If the team continues and has to cover the cells below the obstacle, for example cell D, the sweep direction is now from right to left and thus the obstacle is still on the right hand side. Ignoring the terminating conditions the team moving clockwise in Fig. 14 it would cover the cells A,. . . , F in order. The team moving counterclockwise follows a left hand rule respectively. The two sub-teams continue covering cells until one of the terminating conditions that we saw earlier occur. When the two teams establish line of sight they rejoin to a single team.

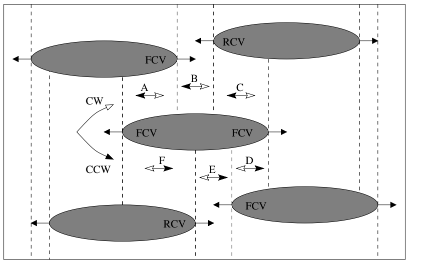

Fig. 9. Illustrates the clockwise and counter-clockwise paths taken during the encircle action. The double-headed arrows represent the sweep direction, white for clockwise, black for counter-clockwise. The single black arrows represent critical point normal vectors and forward convex (FCV) / reverse convex (RCV) critical points are labeled appropriately for a clockwise path. Cell boundaries are represented by vertical dashed lines. A clockwise path would travel through the cells A through F alphabetically, while a counter-clockwise path would travel through the cells in the reverse order [F,. . . , A].

V. EXPLORER/COVERER DISCUSSION

There are clear benefits to an explorer/coverer split approach within a cell. Covering both the top and bottom boundaries simultaneously ensures complete coverage of the boundaries without the need for elaborate motion strategies (e.g. reverse wall-following) required in prior work. Line-of-sight communication allows for ranged critical point detection. Furthermore, in the case of two sub-teams covering from different ends of the same cell, the explorers would enable both sub-teams to rejoin earlier, saving significant amounts of time and minimizing repeat coverage.

Outlining the region also allows coverers, more space to turn. This space permits robots to take wider, more gradual turns, thus increasing the maximum acceptable speed. In other words, a gradual turn can be executed safely at a higher speed than a sharp turn. An appropriate analogy would be outlining regions before filling them in a coloring book.

Additional savings may be possible depending on robot configuration. For example, with homogeneous circular robots and the explorer/coverer technique, the coverer robots need not even touch the obstacle to guarantee complete coverage. They only need to come within a robot radius of the obstacle to wall-follow. This makes sense, since our niches extend only a radius from the obstacle and the explorers already covered a full robot diameter.

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[2] Note that all cells have exactly two bounding critical points by the nature of the Boustrophedon decomposition.

As for disadvantages, coverers will still generate repeat coverage by overlapping with some region previously covered by the explorers. Furthermore, the explorer/coverer approach requires line-of-sight communication and introduces a considerable amount of complexity. However, we believe the advantages are well worth the cost of such complexity.

VI. EXPERIMENTAL RESULTS

Experiments are currently conducted for a variety of environments using the robotic simulation package Player/Stage [19], [20] and for different numbers of robots. Next we present an illustrative example for our approach. Seven robots start together at the lower left corner of an environment, two of them start exploring, while the remaining five start covering the free space (Fig. 10a). When the first obstacle interrupts the line of sight between the two explorers, the existence of a critical point is deduced (Fig. 10b) and the explorer bottom moves toward the explorer top in order accurately record the position of the critical point (Fig. 10c). After the first cell is fully covered, the robots split into two teams. The top cell is covered by a team of three robots, two explorers and one coverer, while the bottom cell is assigned to a team of four robots two explorers and two coverers (Fig. 10d,e). While the explorer bottom follows the bottom boundary of the top cell the direction of motion is reversed due to a forward convex critical point. The two explorers mark the end of the top cell and join the one coverer in covering the remaining area (Fig.10e). Next the top team moves in the bottom cell where they rejoin with the bottom team and complete the coverage of the bottom cell (Fig. 10f). Finally the unified team proceeds to cover the final cell. It is worth noting that as the two explorers meet they detect the final critical point (reverse concave) in Fig. 10h. Fig. 10i presents the final step of the completely covered environment.

VII. CONCLUSIONS

In this paper we described a new multi-robot coverage algorithm. Our approach is based upon the Boustrophedon decomposition in order to guarantee complete coverage. Information sharing was restricted to line-of-sight communication in an unknown environment. Under these constraints, we contributed algorithmic multi-robot solutions for both single cell coverage and Reeb graph traversal while trying to minimize repeat coverage.

The communication ability is central in multi-robot collaboration tasks. In multi-robot coverage, lack of communication traditionally resulted in repeat coverage that reduced the efficiency of the approach. Maintaining the cohesiveness of the team by allowing only minimal splitting we greatly reduced repeat coverage. The performance of the proposed algorithm depends on the average cell length. If in most cells the team does not "fit" then the remaining robots idle, a form of dynamic repeat coverage.

For future work, techniques to split more than once at any given time are being investigated. More splitting theoretically provides finer granularity, but without a guarantee to rejoin, sub-teams are susceptible to repeat cell coverage.

VIII. ACKNOLEDGEMENTS

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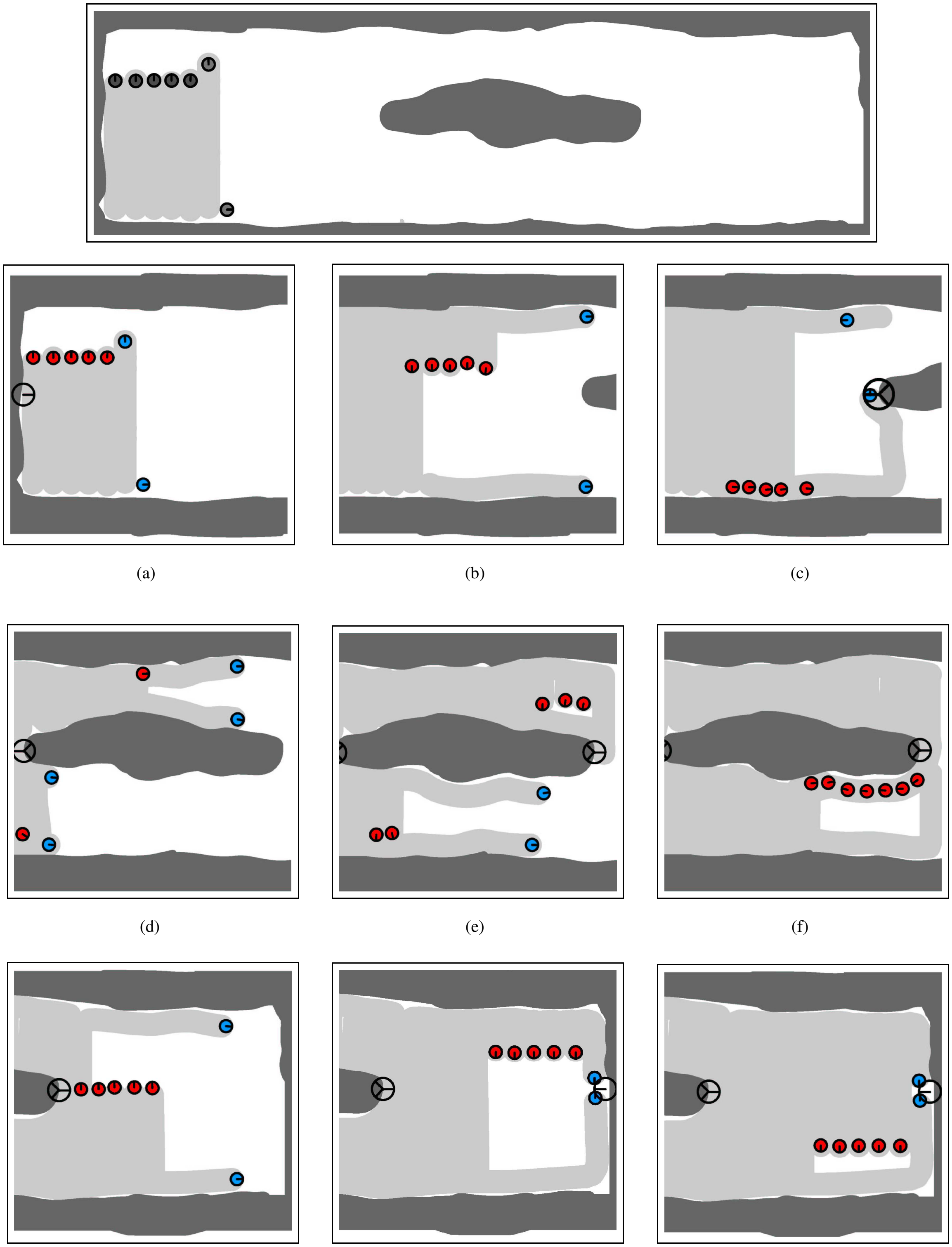

(a) (b) (c)

(d) (e) (f)

(g) (h) (i)

Fig. 10. Formation based coverage of an unknown/unstructured environment: Top figure the complete environment. (a) The Robots start covering and exploring. (b)The two explorers just before line of sight is interrupted. (c) Explorer bottom discovers the reverse convex critical point. (d) Top team has two 3 robots bottom has four (only 3 visible) (e) Explorer bottom of the top team discovered forward convex critical point (reversal of x direction) and joins top explorer in coverage task. (f) Rejoining of the two teams to finish covering the bottom cell. (g) One team, explorers and coverers are covering the final cell. (h) The two explorers meet, thus discovering a reverse concave critical point, no more cells to be covered. (i) Completed coverage.