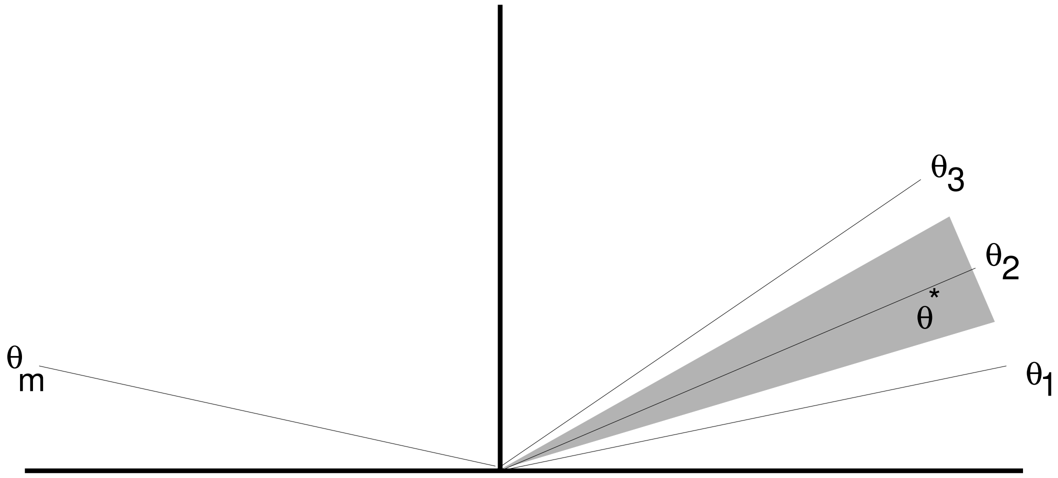
Goal: To recover the trace, tangent and curvature fields.

Tangent field: $(x(s), y(s)) \to (x'(s), y'(s))$

Curvature field: $(x(s), y(s)) \to (x''(s), y''(s))$

Discretization of Orientation:
θ_λ is the discrete orientation, $\lambda = 1, ..., m$



$\theta_\lambda - \pi/2m \leq \theta^* \leq \theta_\lambda + \pi/2m$

Let the certainty of tangent with orientation θ_λ at position (x_i, y_i) be:

$p_i(\lambda) \in [0, 1]$ for $i = 1, ..., n; \lambda = 1, ..., m$

The orientation certainty vector is given by:

$\widehat{p}_i = [p_i(1), p_i(2), ..., p_i(m)]$

Associate to each orientation vector element $p_i(\lambda)$ a discrete measure of curvature $\kappa_i(\lambda)$.

Trace of Curve?

Singularities?

Hierarchies

Constraints

TRACE POINTS

TANGENTS

CURVATURES

CURVATURE CONSISTENCIES

Quantization

fine

coarse

Stage 1: Measurement
Convolution with linear operators to obtain initial tangent estimates at each position and orientation.

$$G(x, y) = LSF(x).e^{-y^2/\sigma_y^2}$$
$$LSF(x) = e^{-x^2/\sigma_1^2} - Be^{-x^2/\sigma_2^2} + Ce^{-x^2/\sigma_3^2}$$

Stage 2: Interpretation
Threshold to find strongest convolutions?

Tangent λ is cocircular to tangent $\lambda^{'}$ iff
$\Gamma(\theta, \theta_t) = \Gamma(\theta_t, \theta^{'})$
for some $\theta, \theta^{'}, \theta_t$.

Range of θ is $(\theta_\lambda - \epsilon/2, \theta_\lambda + \epsilon/2)$
Range of $\theta^{'}$ is $(\theta_{\lambda^{'}} - \epsilon/2, \theta_{\lambda^{'}} + \epsilon/2)$
Range of θ_t is $(\theta_{ij} - \alpha, \theta_{ij} + \alpha)$

Discrete Cocircularity Condition:

$|\Gamma(\theta_\lambda, \theta_{ij}) - \Gamma(\theta_{ij}, \theta_{\lambda^{'}})| < \epsilon + 2\alpha$

Measurement stage consists of convolutions against "line detectors".

With $\theta_{\lambda i}$ the orientation of the operator at position (x_i, y_i), the **normalized** convolutions

$$\{p_i(\lambda), i = 1, ..., n; \lambda = 1, ..., m\}, 0 \le p_i(\lambda) \le 1$$

provide an estimate of the "confidence" in tangent λ at position i.

Cocircularity support for tangent λ at position i:

$$s_i(\lambda) = \sum_{j=1}^{n} \sum_{\lambda'=1}^{m} r_{ij}(\lambda, \lambda^{'}) p_j(\lambda^{'})$$

where $r_{ij}(\lambda, \lambda^{'}) = c_{ij}(\lambda, \lambda^{'})$,
the cocircularity coefficient.

Partition the neighborhood support set about tangent A into a discrete set of curvature classes $\mathcal{K}_k(A), k = 1, ..., K$.

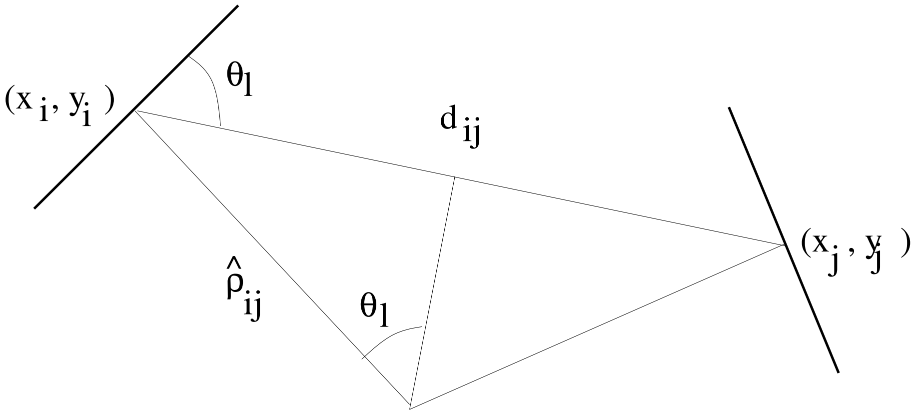
If tangent A is cocircular to B and A is cocircular to C and B, C belong to the **same curvature class** with respect to A, then B is cocircular to C.

Revised cocircularity support function:

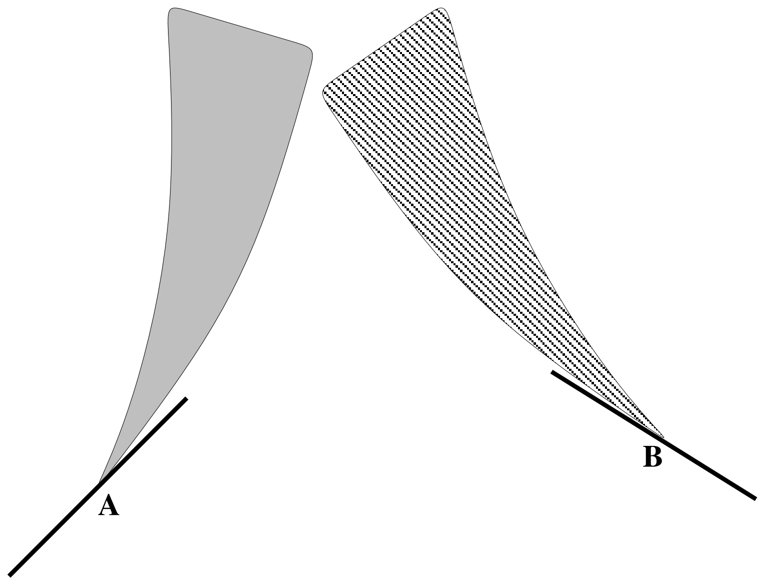
$$s_i(\lambda) = \max_{k=1,K} \sum_{j=1}^{n} \sum_{\lambda'=1}^{m} r_{ij}^k(\lambda, \lambda') p_j(\lambda')$$

where $r_{ij}(\lambda, \lambda') = c_{ij}(\lambda, \lambda').K_{ij}^k(\lambda, \lambda')$.

$$
\begin{aligned}
K^k_{ij}(\lambda,\lambda^{'}) &= 1 \quad \text{if} \quad \rho^k_{\mathsf{min}} \leq \widehat{\rho}_{ij}(\lambda) \leq \rho^k_{\mathsf{max}} \\
&= 0 \quad \text{otherwise.}
\end{aligned}
$$



$$\widehat{\rho}_{ij}(\lambda) = \frac{d_{ij}}{2Sin(|\Gamma(\theta_l,\theta_{ij})|)}$$



Modify the consistency coefficients:

$$r_{ij}^{kk'}(\lambda, \lambda') = c_{ij}(\lambda, \lambda') K_{ij}^{k}(\lambda, \lambda') C_{ij}^{kk'}(\lambda, \lambda')$$

$C_{ij}^{kk'}(\lambda, \lambda') = 1$ if curvature class k of λ is "consistent with curvature class k' of λ' at j;

$C_{ij}^{kk'}(\lambda, \lambda') = 0$ otherwise.

$$A(p) = \sum_{i=1}^{n} s_i(\lambda)p_i(\lambda)$$

The $p_i(\lambda)$'s provide a measure of which tangents are chosen.

The $s_i(\lambda)$'s indicate how mutually consistent they are.

Idea is to iteratively update the $p_i(\lambda)$'s in order to maximize the average local support.