Trace Inference

Goal: To recover the trace, tangent and curvature fields.

Tangent field: \( (x(s), y(s)) \rightarrow (x'(s), y'(s)) \)

Curvature field: \( (x(s), y(s)) \rightarrow (x''(s), y''(s)) \)

Discretization of Orientation:
\( \theta_\lambda \) is the discrete orientation, \( \lambda = 1, ..., m \)

\[ \theta_\lambda - \pi/2m \leq \theta^* \leq \theta_\lambda + \pi/2m \]
Let the certainty of tangent with orientation $\theta_\lambda$ at position $(x_i, y_i)$ be:

$p_i(\lambda) \in [0, 1]$ for $i = 1, \ldots, n; \lambda = 1, \ldots, m$

The orientation certainty vector is given by:

$\hat{p}_i = [p_i(1), p_i(2), \ldots, p_i(m)]$

Associate to each orientation vector element $p_i(\lambda)$ a discrete measure of curvature $\kappa_i(\lambda)$.

Trace of Curve?

Singularities?
Hierarchies

Constraints

 TRACE POINTS

 TANGENTS

 CURVATURES

 CURVATURE CONSISTENCIES

Quantization

 fine

 coarse
Two Stages

Stage 1: Measurement
Convolution with linear operators to obtain initial tangent estimates at each position and orientation.

\[ G(x, y) = LSF(x) \cdot e^{-y^2 / \sigma_y^2} \]
\[ LSF(x) = e^{-x^2 / \sigma_1^2} - Be^{-x^2 / \sigma_2^2} + Ce^{-x^2 / \sigma_3^2} \]

Stage 2: Interpretation
Threshold to find strongest convolutions?
Discrete Cocircularity

Tangent $\lambda$ is cocircular to tangent $\lambda'$ iff
$\Gamma(\theta, \theta_t) = \Gamma(\theta_t, \theta')$
for some $\theta, \theta', \theta_t$.

Range of $\theta$ is $(\theta_\lambda - \epsilon/2, \theta_\lambda + \epsilon/2)$
Range of $\theta'$ is $(\theta_\lambda' - \epsilon/2, \theta_\lambda' + \epsilon/2)$
Range of $\theta_t$ is $(\theta_{ij} - \alpha, \theta_{ij} + \alpha)$

Discrete Cocircularity Condition:

$|\Gamma(\theta_\lambda, \theta_{ij}) - \Gamma(\theta_{ij}, \theta_\lambda')| < \epsilon + 2\alpha$
Cocircularity support

Measurement stage consists of convolutions against “line detectors”.

With $\theta_{\lambda_i}$ the orientation of the operator at position $(x_i, y_i)$, the normalized convolutions

$$\{p_i(\lambda), i = 1, \ldots, n; \lambda = 1, \ldots, m\}, 0 \leq p_i(\lambda) \leq 1$$

provide an estimate of the “confidence” in tangent $\lambda$ at position $i$.

Cocircularity support for tangent $\lambda$ at position $i$:

$$s_i(\lambda) = \sum_{j=1}^{n} \sum_{\lambda' = 1}^{m} r_{ij}(\lambda, \lambda') p_j(\lambda')$$

where $r_{ij}(\lambda, \lambda') = c_{ij}(\lambda, \lambda')$, the cocircularity coefficient.
Curvature Classes

Partition the neighborhood support set about tangent A into a discrete set of curvature classes $\mathcal{K}_k(A), k = 1, ..., K.$

If tangent A is cocircular to B and A is cocircular to C and B, C belong to the same curvature class with respect to A, then B is cocircular to C.

Revised cocircularity support function:

$$s_i(\lambda) = \max_{k=1,K} \sum_{j=1}^{n} \sum_{\lambda' = 1}^{m} r_{ij}^k(\lambda, \lambda') p_j(\lambda')$$

where $r_{ij}(\lambda, \lambda') = c_{ij}(\lambda, \lambda').K_{ij}^k(\lambda, \lambda').$
Partitioning Function

\( K_{ij}^k(\lambda, \lambda') = 1 \) if \( \rho_{\min}^k \leq \hat{\rho}_{ij}(\lambda) \leq \rho_{\max}^k \)

\( = 0 \) otherwise.

\[ \hat{\rho}_{ij}(\lambda) = \frac{d_{ij}}{2\sin(|\Gamma(\theta_l, \theta_{ij})|)} \]
Curvature Consistency

Modify the consistency coefficients:

\[ r_{i,j}^{k,k'}(\lambda, \lambda') = c_{i,j}(\lambda, \lambda') K_{i,j}^k(\lambda, \lambda') C_{i,j}^{k,k'}(\lambda, \lambda') \]

\( C_{i,j}^{k,k'}(\lambda, \lambda') = 1 \) if curvature class \( k \) of \( \lambda \) is "consistent with curvature class \( k' \) of \( \lambda' \) at \( j \);

\( C_{i,j}^{k,k'}(\lambda, \lambda') = 0 \) otherwise.
Average Local Support

\[ A(p) = \sum_{i=1}^{n} s_i(\lambda) p_i(\lambda) \]

The \( p_i(\lambda)'s \) provide a measure of which tangents are chosen.

The \( s_i(\lambda)'s \) indicate how mutually consistent they are.

Idea is to iteratively update the \( p_i(\lambda)'s \) in order to maximize the average local support.