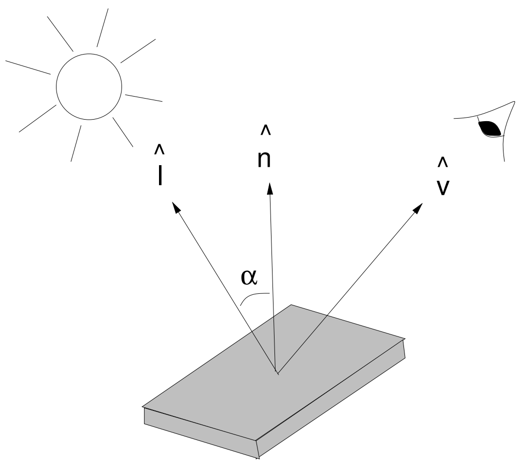
Goal: To recover surface depth from shading in a single image.



$\hat{l}$: light source direction

$\hat{n}$: normal to a surface patch

$\hat{v}$: viewing direction

α: angle between $\hat{n}$ and $\hat{l}$

For a lambertian surface:

$$E(x, y) = \rho\lambda\hat{l} \cdot \hat{n}, \tag{9}$$

where $E(x, y)$ is the image intensity, λ is the incident illumination, ρ is the albedo of the surface.

Let $\hat{l}$ coincide with $\hat{v}$:

$$E(x, y) = \frac{1}{\sqrt{1 + z_x^2 + z_y^2}}, \tag{10}$$

where $z(x, y)$ is the depth of the surface.

$z(x, y)$ must be recovered from $E(x, y)$ and initial conditions.

Proposed Solution: Evolve a curve such that it tracks the height contours of $z(x, y)$. [Kimmel *et al.*, IJCV95]

Height climbed while progressing a distance $|\Delta C|$ in the direction $\hat{n}$ in the (x, y) plane is given by $|\Delta C| = |\Delta z| \cot(\alpha)$.

Let z denote time in the course of evolution, *i.e.*, $z = t$. Since $E = \rho\lambda \cos(\alpha)$, we have

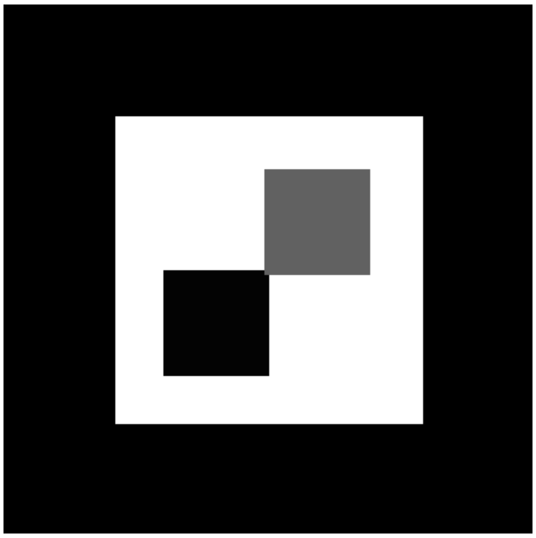
$$\left|\frac{\Delta C}{\Delta t}\right| = cot(\alpha) = \frac{E/\rho\lambda}{\sqrt{1 - (E/\rho\lambda)^2}}. \tag{11}$$

The curve evolution equation is:

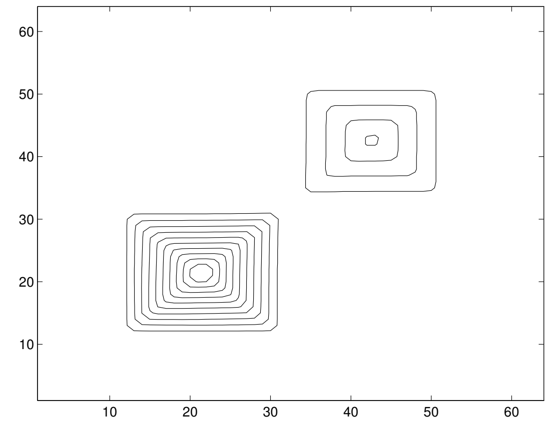
$$\begin{cases} \frac{\partial \mathcal{C}}{\partial t} & = \frac{E/\rho\lambda}{\sqrt{1-E^2/(\rho\lambda)^2}} . \widehat{n}, \\ \mathcal{C}(s,0) & = \mathcal{C}_0(s). \end{cases} \tag{12}$$

where the initial curve, $\mathcal{C}_0$, is a known height contour of z.

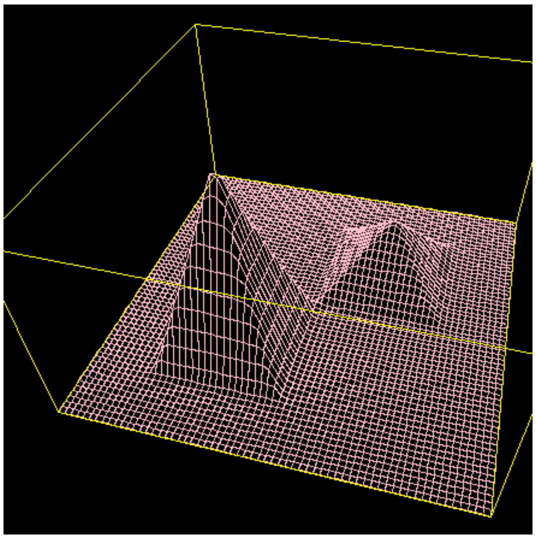
Examples - Pyramids



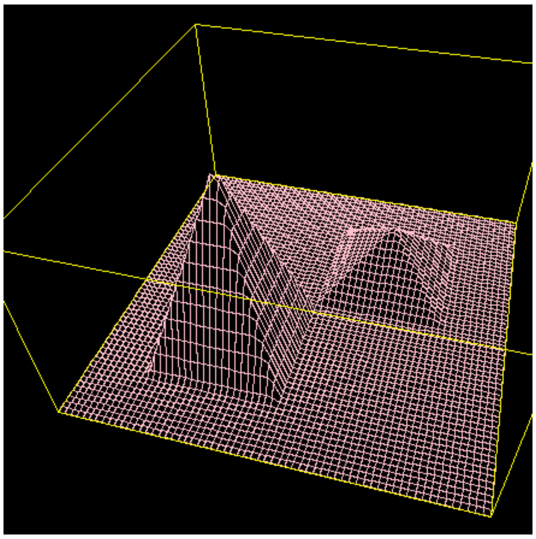
shaded image



equal height contours

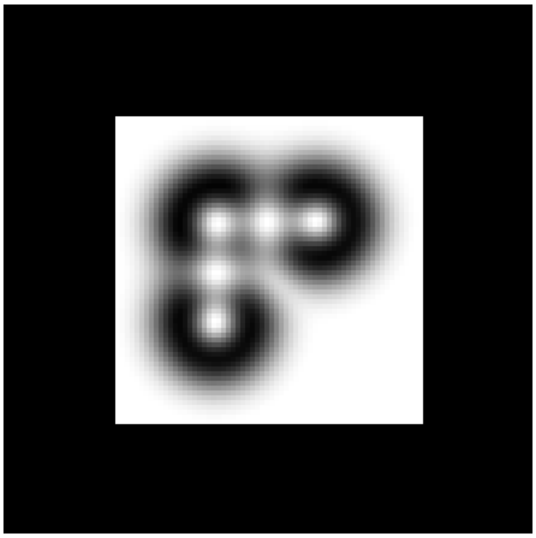


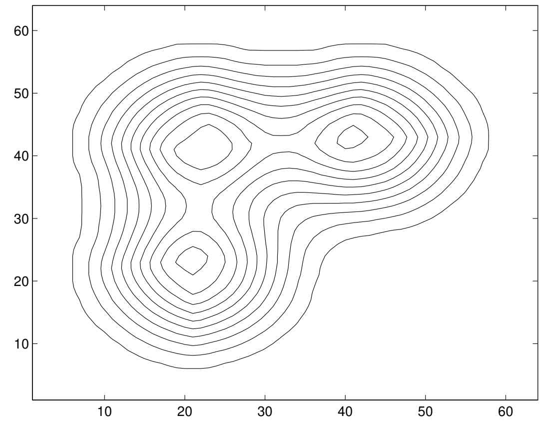
numerical solution



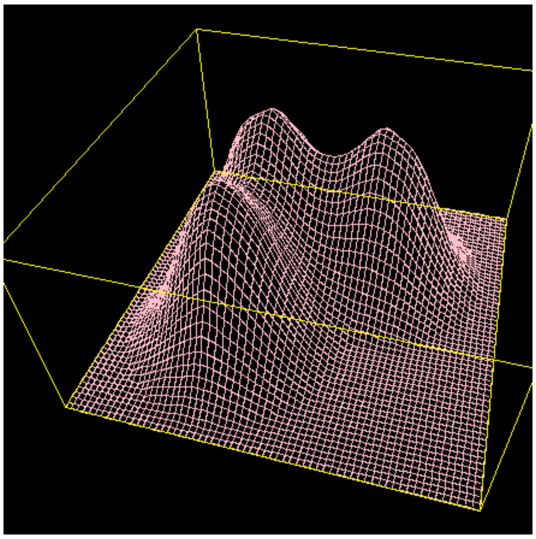
true surface

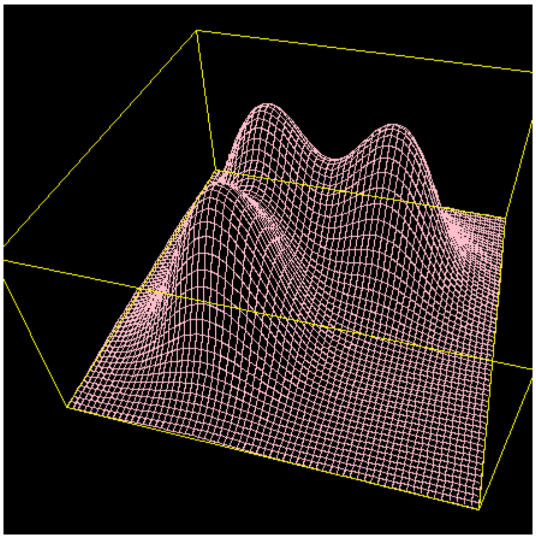
Examples - Three Mountains





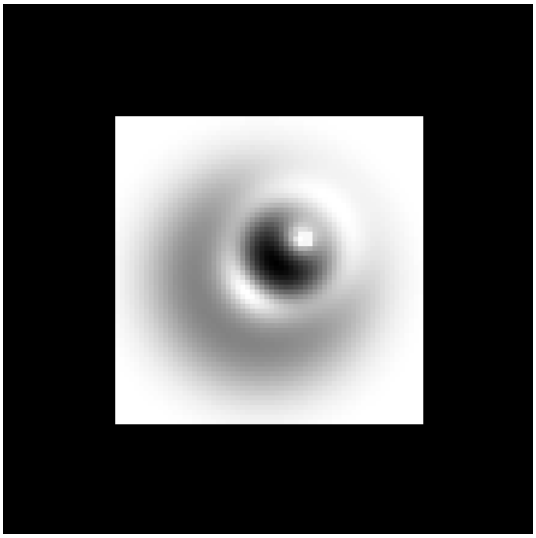
shaded image   equal height contours



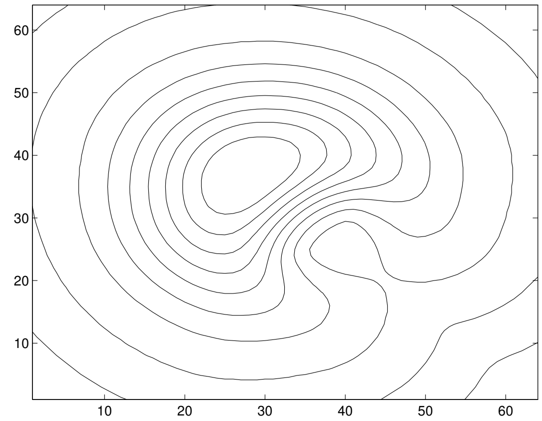


numerical solution   true surface

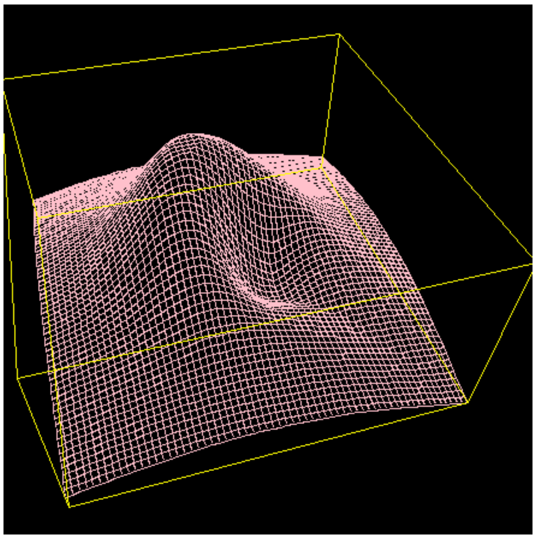
Examples - Volcano



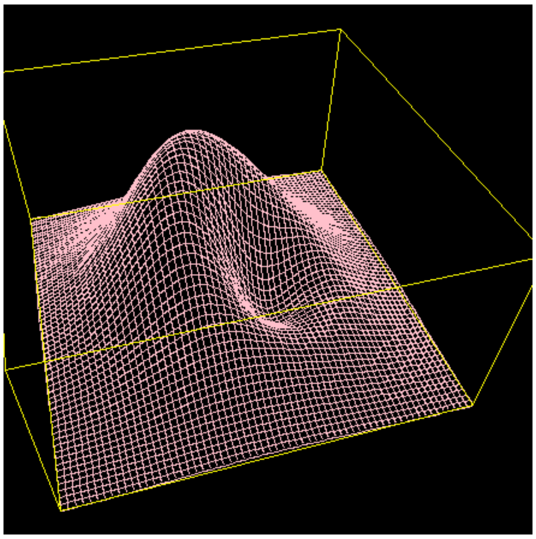
shaded image



equal height contours



numerical solution



true surface