COMP 558: Assignment 1
Available: Thursday, January 26th, 2012
Due Date: Tuesday, February 7th, 2012, before midnight, via webct.

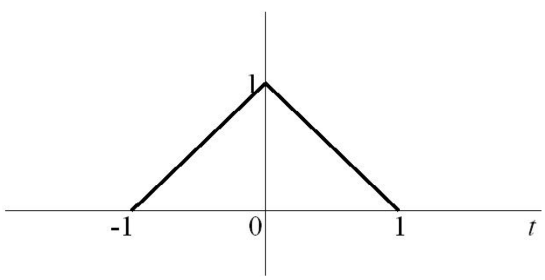
Notes: You are encouraged to become familiar with the **MATLAB** environment which is currently installed on Unix machines in the CS labs. Using Matlab will greatly simplify the experimental parts of this assignment. I expect everyone to submit original work. Your completed solutions should be submitted in electronic form (PDF) via your webct account using the submission option for assignment 1.

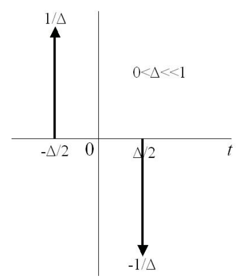
Question 1: Convolutions and Fourier Transforms in 1D (50%)

The goal of this question is for you to develop an intuition for the convolution process and to derive some basic properties of Fourier transforms.

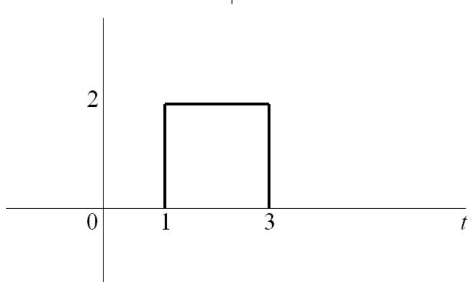
Part 1 (15%) In each row, convolve the signal on the left with the one on the right. You should be able to perform these operations by hand (and not by using software). Provide a sketch of the resulting convolutions, showing the various steps. In each case be sure to label the significant time and intensity values. (5% each)

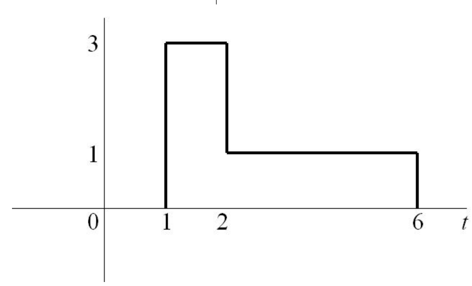
(i)



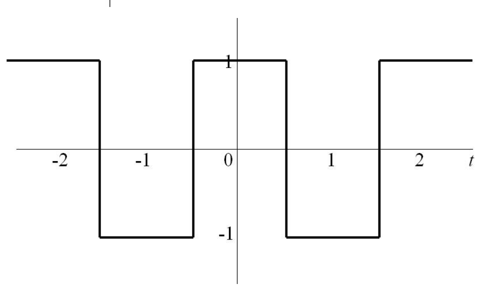


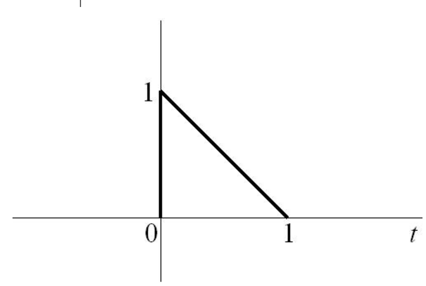
(ii)





(iii)





Part 2 (35%) You are now asked to manipulate the definition of the convolution integral and of the Fourier transform in order to derive some useful mathematical properties. Recall that the convolution of two 1D signals $f(t)$ and $g(t)$ can be obtained by

$$[f * g](t) = \int_{\mathbb{R}} f(t-\tau)g(\tau)d\tau,$$

and that the Fourier transform $\hat{F}(\omega)$ of $f(t)$ can be obtained by

$$\hat{F}(\omega) = \int_{\mathbb{R}} f(t)e^{-j\omega t}dt.$$

Compute the Fourier transform of the following expressions. Try to simplify your answer as much as possible. (7% each)

a) $\delta(t - t_0)$

b) $af(t) + bg(t)$

c) $[f * g](t)$

d) $f(st + t_0)$, where s is a constant

e) What is the relationship between the Fourier transforms of $f(st+t_0)$, $f(t)$ and $\delta(t-t_0)$?

Question 2: Convolutions and Fourier transforms in 2D (50%)

The goal of this question is to investigate the convolution and spectrum decomposition of a 2D signal (an image in this case) as a means to achieve specific image processing goals. Consider the two angiogram images located at
`http://www.cim.mcgill.ca/~epiuze/?page=COMP558`.

a) Convolve[1] the first image with the Laplacian of a Gaussian:

$$H(x,y) = \nabla^2 \left(e^{-\frac{x^2+y^2}{2\sigma^2}} \right)$$

for $\sigma = 0, 1, 2, 4$. Show the output for the different values of σ. As explained in class, it might be easier to express the Laplacian of a 2D Gaussian using polar coordinates. (15%)

b) How would you describe the effect of such a filter on images? In other words, explain how this convolution process can be used for edge detection. (5%)

[1] Since we are dealing with a discrete signal (the image), the convolution needs to be performed in a per-pixel basis. You can represent the filter as an image by computing the value of $H(x,y)$ at each pixel. The filter image should be large enough to contain the filter in its entirety.

c) Experiment with other values of σ. How does the choice of σ affect the output? What is the σ for which you observe that this effect is the most pronounced for the angiogram images? (5%)

d) Now compute and show the Fourier transforms of the original image, its rotated version, and of the filter H, using Matlab functions. (10%)

e) Use a property of Fourier transforms to perform the convolution in a) in the Fourier domain and comment on the result. (5%)

f) Comment on the relative amplitude and phase spectra of the Fourier transform of the original image, of the second one, which is a rotated copy of it. What do you observe? For what computer vision problem(s) could this property be useful? (10%)