Final Project Report: Perceptual Illusions arising from Dot Displays

Sandra Skaff

School of Computer Science
McGill University
ID no: 0048097
email: sandra@cim.mcgill.ca

April 28, 2003

Abstract

Certain arrangements of dots immediately group into contours. These contours are then grouped by the visual system into larger arrangements to form different shapes depending on the dot spacing. Closely spaced arrangements give rise to subjective effects. These effects are termed illusions. These illusions vary from apparent brightness to subjective edges and occlusions. The change in percept is constant up to a dot to space ratio of about 1:5 after which the figure is usually perceived illusion-free. This inter-dependence of dot size and spacing is called the *size/spacing constraint.* In this paper, we shall investigate how the dot spacing affects the perception of the illusion keeping the dot size constant. The size/spacing constraint was investigated on black and white images. We shall extend this idea to include images of different colors in order to observe their effects on the perception of illusions. Two cases are presented: (i) one in which blue dots were rendered on a slightly darker blue background and (ii) one in which randomly colored dots were rendered on a white background. Psychophysical experiments indicate that the perception of the illusion in the two close blue colors case is the same as before. However, subjects find it more difficult to perceive the illusion when the dots are randomly colored because it is more difficult to group the dots into contours in this case.

1 Introduction

One of the most striking characteristics of human vision is the human retina. It is amazing how dramatically our visual experience differs from our retinal image. Retinal images are formed on the back of our eyeballs, upside down; the optic nerve fibers send information to the brain. Our perception is closely related to surfaces and objects in the real world rather than our retinal images. However, sometimes our visual system deceives us in the sense that we tend to see what is not there. In other words, given the structure of our visual system, this latter tries to make explanations of the image in front of it, so that we perceive what we call illusory phenomena.

What is a perceptual illusion? What is a dot display? How does the perception of an illusion vary with a dot display? How do different colors of the dot display influence the perception of the illusion? The answers to these questions and many more will be tackled in this paper in which we shall describe the psychophysical experiments that were implemented, and interpret the data collected from human subjects who performed these experiments.

A combination of neurophysiology and psychology is essential for the explanation of perceptual phenomena and hence illusions. To start with the basic neurophysiology of the brain [4], the image is formed on the retina after the output from each eye is conveyed to the retina by the ganglion cells. A large fraction of the optic nerve fibers synapse deep in the brain at the lateral geniculate nuclei (LGN). The retinal and the LGN cells have concentric center-surround receptive fields. They are primarily concerned with making a comparison between the light level in one small area of the visual scene and the average illumination of the immediate surround. The LGN cells then send their axons to the primary visual cortex, layer IV. The cells in the visual cortex can be divided into two groups. The first group are "simple" cells and they behave as though they received their input from center-surround, circularly symmetric receptive fields. The second major group of orientation specific neurons are the more numerous "complex" cells. They are less particular about the exact position of the line than the simple cells, and they are affected by the orientation of the stimulus.

1.1 Gestalt Psychology

Gestalt psychology was co-founded by Max Wertheimer [10]. We note here that the word Gestalt is a German name. Wertheimer provides many laws of perceptual organization which are described in [10]. An important one for our purposes is "good" continuation. In designing a pattern, for example, one should have a feeling how successive parts should follow one another [10]. In other words, one knows how "inner coherence" is to be achieved. It can be described by the following example. If two tree branches are crossing each other in a form similar to an X (see Figure 1), and we were to figure out from which bottom branch part ('c' or 'd') a top branch part ('b') came from, we would know the correct answer right away by good continuation. In this case, we would know that 'b' is continued from 'c' and not from 'd'.

Some other laws of Gestalt psychology that are of interest for this paper are the "factor of proximity" and the "factor of similarity". The factor of proximity is the tendency of close parts to band together. For example, in Figure 2 (from [10]), the row will be perceived as

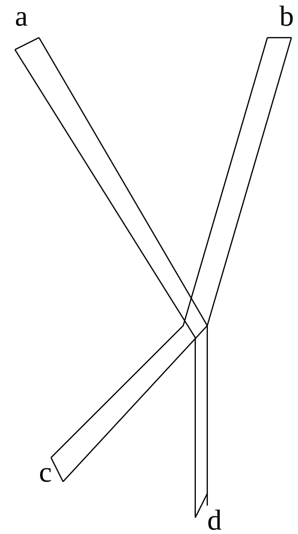


Figure 1: A high-level inference example: the tree branch.

ab/cd and not as *a/bc/de*. This means that every two dots which are close to each other are perceived together, and any other theoretical grouping will not be perceived. The factor

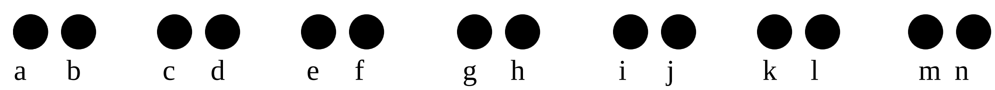


Figure 2: A horizontal line of dots from which the factor of proximity is evident.

of similarity can be defined as the tendency of like parts to band together. For example, in Figure 3 (from [10]), we tend to perceive every two circles together and every two dots together. Another cue, in addition to shape, that helps us recognize objects is color. If the

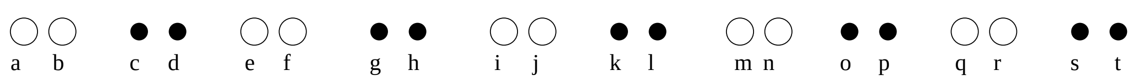


Figure 3: A horizontal line of dots from which the factor of similarity is evident.

color of dots *b* and *c* are made similar in Figure 2 (for a colored version please refer to the weblink [8], to the figure 'gest-prox-color.jpg'), then some people might have the tendency to perceive the first part of the row as *a/bc/de* instead of the original grouping of *ab/cd*.

We come to develop our second extension to Zucker and Davis's experimentations [11] in which random colors of dots are rendered on a white background. This idea falls upon the laws of organization or grouping of Gestalt pychology discussed above. One of the laws of perceptual organization is the factor of similarity, and since the dots have the same color and the same shape, they tend to be grouped together into lines. However, if the dots are randomly colored as is the case in our experiments, the visual system might be impeded in its attempt to group the dots into lines and consequently into some form or shape. This reduces the chances of perceiving the illusion, especially at sparse dot spacings.

1.2 Size/Spacing Constraint

Zucker and Davis [11] answer the question of how the *size/spacing constraint* influences our perception of visual illusions given a dot display. The insight of this idea comes from the fact that our visual systems are adept at organizing dots into contours. This is because we tend to group dots according to the spatial proximity and similarity factors of Gestalt psychology (as discussed in Section 1.1). According to Zucker and Davis [11], many factors come into the picture upon analyzing gropuing: the size of the dots, the spacing between them, and the configuration of the dots. For a fixed size of dots, if a set of dots are overlapping, then we see a solid line. If the spacing between these same dots is less than a critical spacing, then the virtual line is perceptually equivalent to a solid line. However, if the spacing between the dots is greater than a critical spacing, then the virtual line perceived is not perceptually equivalent to a solid line. Then Zucker and Davis [11] state that since the spacing depends on the size of the dots, they infer the criterion, the *size/spacing constraint*. We shall explain this latter idea by the following example. Imagine a set of dots of equal size that has spacing between the dots less than the critical spacing. This means that this set is perceptually equivalent to a solid line. If the size of each of the dots is decreased gradually, we shall come to a point where the line is no longer perceived or the set of dots is no longer grouped into a perceptual line even though the spacing between the dots is held constant. This implies that the critical spacing depends on the size of the dots, which explains the size/spacing constraint. It is obvious that the dots, now smaller, should be brought closer to each other i.e. the critical spacing should be decreased when the dot sizes are decreased. However, the dots cannot be made too small or too large. If the diameter of the dots is less than 0.5 min or greater than 10 min, then the illusion disappears due to the inability of the visual system to group the dots into a line. The best size for the dots is about 1 min in diameter. We should note that distances are expressed in terms of visual angle (min), in order for the values to be independent from the distance of the subject to the monitor. The notion of a visual angle is explained in the Appendix.

Grouping processes in [11] are classified into two categories: single and complex. A single process is based on the factor of good continuation, for example, from the laws of Gestalt pyschology (explained in Section 1.1). It is a high-level process because one look at an object is sufficient so that it becomes clear to the visual system. An example is that of the crossing tree branches that was explained in Section 1.1. By looking at the object, the person can find out which top branch comes from which bottom branch. On the other hand, a complex process involves different levels each with its own properties, starting from the stimulus array, one process taking over from another. An example is the trace inference process [5] in which the lowest level is orientation selection or the inference of locally oriented entities (tangents) at particular retinotopic locations. Then the higher levels are thresholding these tangents and retaining those with high support taking into account the co-circularity condition, first, the curvature consistency, second, and the lateral maxima, third. Finally, relaxation labelling is implemented to retain one tangent at each desired location. In short, according to Zucker and Davis [11], solid and densely dotted contours arise from highly constrained physical events; therefore high level or single processing is sufficient for such events. However, sparsely dotted contours arise from complex physical situations such as viewing a contour through a periodic occlusion; therefore, the type of processing required is not based on image measurements, but is more complex.

The authors in [11] demonstrate the existence of different processes by providing three examples. The first one is the Necker Cube, which is named after the crystallographer Louis Albert Necker who lived in the 1880's. As can be observed from Figure 4 (from [11]), a cube can be perceived in all three parts, a, b, and c, despite the variation in the spacing between the dots. This means that a high-level process is involoved during the perception process, and the visual system is perceiving the edges rather than the dots. The next example is

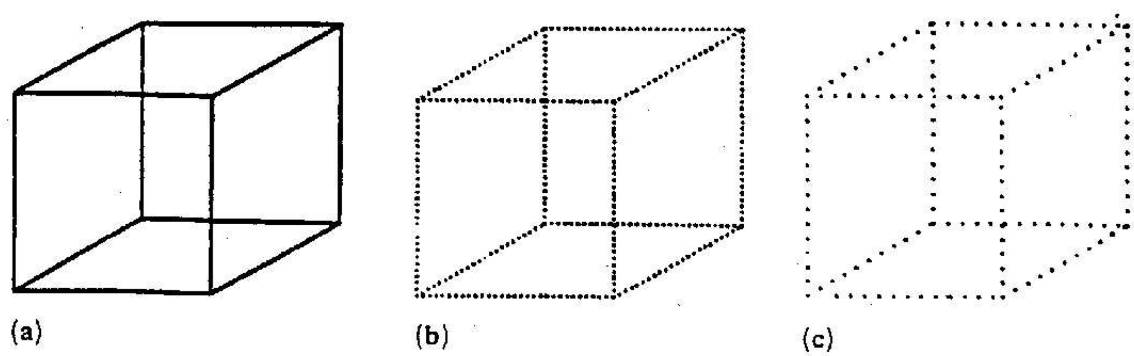


Figure 4: The Necker Cube drawn with: (a) solid lines, (b) densely dotted lines, and (c) sparsely dotted lines

the Kanizsa Edge which can be seen in Figure 5 (from [11]). The subjective edge differs as a function of the dot density. When the dots are dense enough, apparent occlusion and apparent brightness are perceived. In the last example, the Sun Illusion is perceived, which is shown in Figure 6 (from [11]). If the spacing between the dots is small enough, in this case, apparent brightness is perceived in the center and an edge or a contour defining the "sun". The latter two illusions are examples of a complex process because the visual system has to infer or group the appropriate lines from the dots before perceiving the shape and then the illusion. If the lines cannot be perceived due to the large density of dots, then the appropriate shape cannot be perceived, and subsequently, the illusion would not be perceived.

Zucker and Davis carry out a sequence of experiments [11]. They record the responses of subjects upon perceiving the sun illusion figure only (Figure 6) both on computer and on paper. They performed experimentation for the Kanizsa edge (Figure 5) except that they say that their results are informal. Their results indicated that the observers tended to see the illusion for dot:space ratios less than 1:4, whereas for ratios of 1:6 or more they did not, similar to the sun illusion. We shall describe their experiment with the sun illusion display

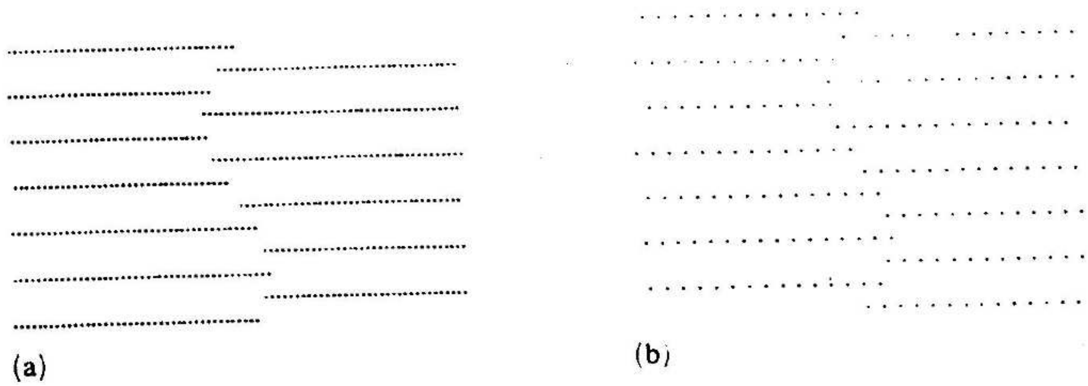


Figure 5: The Kanizsa Edge drawn with: (a) densely dotted lines and (b) sparsely dotted lines

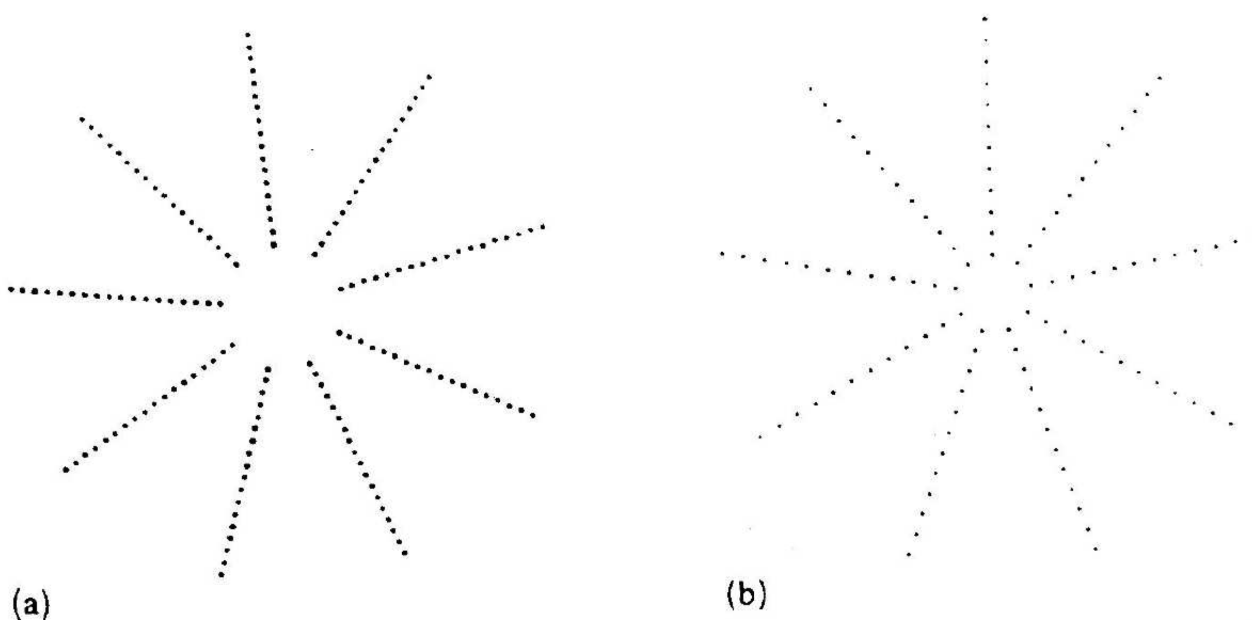


Figure 6: The Sun Illusion drawn with: (a) densely dotted lines and (b) sparsely dotted lines

on the computer, not on paper, because it is relevant to our purposes. The dot:space ratio, 1:N, means that given a diameter of 1 min, the space between two dots is N. Images of the sun illusion were generated on a BBN Computer Corporation Bitgraph display. The illusion was presented with black points on a white background. Subjects sat with their heads either 51 cm or 102 cm from the screen. The different spacings between the dots displayed were 1, 2, 3, 4, 5, and 6, and the subjects briefly viewed each image five times, in random order, in a single session. Three subjects did the experiment and the results were recorded. The results indicated that the observers tended to see the illusion for dot:space ratios less than 1:4, whereas for ratios of 1:6 or more they did not. We shall call this ratio range (1:4 to 1:6) at which an observer stops seeing an illusion the "threshold." We note that Zucker and Davis do not use the term "threshold" in [11]. The data show an abrupt change from the illusion being present below the threshold to an illusion being absent above the threshold.

Finally, Zucker and Davis [11] discuss how the lines are inferred from the dots. The first step in this process is the inference of the structure of the contour or the subjective edges in such a way to localize the endpoints. Next, given the trace or some points through which the trace passes, and with the use of the first derivatives which are the tangents, the contour can be recovered by integration. This is sort of a chicken-and-egg problem here because the contour is assumed to be given from step one while it is what is being computed in step two. Two cases are considered in this case: when the dots are sparse and when they are dense. When the dots are sparse, every two points are connected by a line tangent and then other schemes such as distance between points are needed to determine whether the pair should be connected. When the dots are dense, early orientation selection is used to group a set of dots into a line. This basically means that as the dots are grouped into contours, the tangents to these contours are in such a way that fires certain receptive fields of the brain (Section 1). In this case, the dots have enough energy to match the receptive fields unlike the sparse dots. Evidence was obtained from receptive field simple cells in the striate cortex that their responses saturate with 3% contrast for inducing stimuli. This means that the dots must cover enough of the receptive field so that the excitatory response is at least 3% [11]. This tells how far apart two dots can be before the subjective perceptual line will no longer be visible.

1.3 Lightness Perception and Visual Illusions

We extend Zucker and Davis's experiments [11] to include colored images. In this section, we shall investigate the effect of two very similar colors in a dot display on the perception of illusion. We shall start with some background on lightness perception. Adelson researches on lightness perception and illusions and provides some nice examples in [1]. We shall start with some definitions that will make our discussion clearer. *Illumimance* is the amount of light incident on a surface. *Reflectance* is the proportion of incident light that is reflected from a surface. *Luminance* is the amount of visible light that comes to the eye from a surface. These are all physical quantities and can be measured by physical devices. Now Adelson comes to define the problem of lightness constancy [1]. An illuminance image, $E(x, y)$, and a reflectance image, $R(x, y)$, are multiplied to produce a luminance image, $L(x, y)$:

$$L(x, y) = E(x, y)R(x, y) \quad (1)$$

An observer is given L at each pixel, and attempts to determine the E and the R that were multiplied to make it. This problem is clearly ill-posed; however humans perform quite well at it. This means that illuminance and reflectance are not arbitrary functions. Let us consider the following example. Patches q and r in the block in Figure 7 (from [1]) have the same illuminance; however, they have different reflectances and different luminances. If a square of certain illumination is placed between two squares of light illuminantion, it

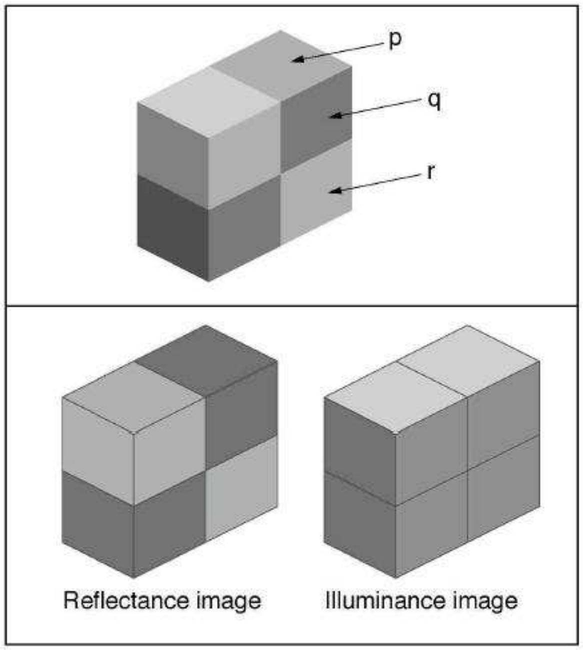


Figure 7: A checker block with patches of different illuminances.

appears darker than when it is placed between two squares of darker illumination (Figure 8, from [1]). This can be explained by observing Figure 9 (from [1]). The influence of a Ψ-junction can propagate along the contours that meet at the Ψ. This is explained in [7]. This effect is also evident in the famous Mach band illusion [3]. The basic idea is adjacent bands of decreasing lightness are perceived. The physical intensity of light measured would be decreasing from a light to a darker band. However, the perceptual intensity distribution has a bump in the light band and a dip in a dark band, if we are looking across from the light band to the darker band. This is due to the fact that the receptors in the brain that are responding to the light band are intensely stimulated. Therefore, they would send a large amount of inhibition toward the receptors that are responding to the dark band. We assume that if the hue is added to the lightness, these properties do not change. The proof is that what was discussed above pertains to humans' lightness constancy, and since humans exhibit

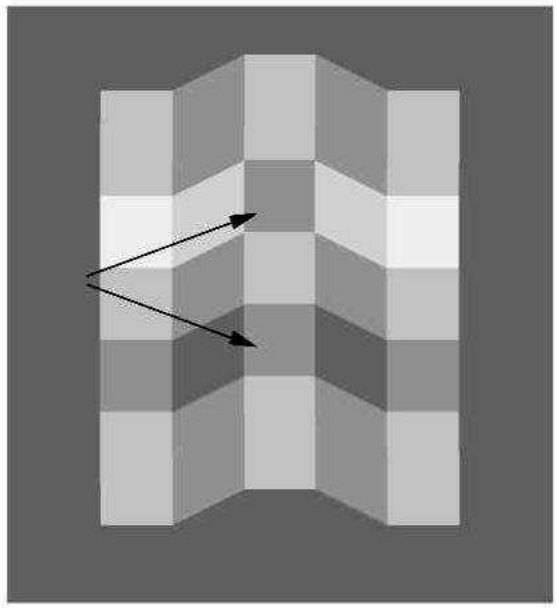


Figure 8: The same square between two sets of patches of different light illuminations.

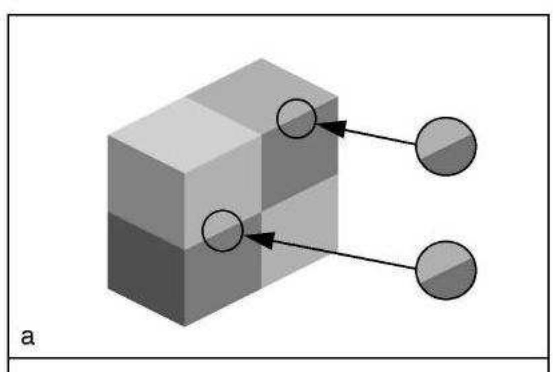


Figure 9: The junction between two patches of different illuminations.

color constancy under different lighting conditions as well, then our assumption should be valid.

Using what we have said above, we can explain why Zucker and Davis [11] used a black dot display on a white background. Black and white provide maximum color contrast between junctions where the dots and the background meet, which makes it crystal clear for the subject as to where the dots are. This also makes him/her able to make out the lines and thus a shape from the dots perceptually and subsequently perceive an illusion. In this paper, we extend Zucker and Davis's dot displays in [11] by observing the effect of color on human perception of illusions. Using Adelson's idea [1] we try to make the color contrast between the dots (black) and the background (white) less obvious. In other words, we shall make the subject look at two very close colors of the dots and the background. We shall show that the subject still perceives the illusion based on Adelson's idea. Since the two objects of different colors are next to each other, and the dots are lighter than the background, the subject tends to perceive the dots as being lighter than their original color due to the adjacent darker background. On the other hand, he/she perceives the background as being darker than its original color due to the adjacent lighter dots. This means that the subject will still perceive the illusion with a threshold very close to the original threshold (see the definition of threshold in Section 1.2).

2 Psychophysical Experiments

An experiment is essentially a two-step process. First, the dot displays were rendered on the screen using the C language and the graphic libraries, OpenGL, GLUT, and Glu [9]. OpenGL was used because it is the premier environment for developing portable, interactive 2D and 3D graphics applications. Since its introduction in 1992, OpenGL has become the industry's most widely used and supported 2D and 3D graphics application programming interface (API), bringing thousands of applications to a wide variety of computer platforms. GLUT is the OpenGL Utility Toolkit, a window system independent toolkit for writing OpenGL programs. It implements a simple windowing application programming interface (API) for OpenGL. GLU is the OpenGL Utility Library. After the dots were rendered on the screen, the next step is engaging subjects to perform the experiments and recording the results. This section describes the details of the experimentation process of both the sun illusion and the Kanizsa edge illusion.

The experimentation process for both illusions was similar. The subject sat with his/her head at a distance of 50 cm from the monitor, similar to what was done in [11]. Images of the display were rendered on a screen of a Linux computer of 1280x1024 pixels monitor resolution. Before starting with the experiment, the subject was trained in order to get aquainted with the buttons, so he was presented with some images and he had to press the appropriate button. The number of images used in one session was 60, as 12 different spacings between the dots were considered (0, 2, ..., 11), and each image was viewed 5 times briefly. The subject had to indicate whether an illusion was seen or not. A session was made up of a set of images for both illusions (sun illusion and Kanizsa edge) of one set of colors. The colors that were used for the dots/background were black/white, blue/blue, and colored/white.

In the experiments, the subject had the choice of the following buttons:

"0" if he/she perceived no illusion

"1" if he/she perceived an illusion

In the case of black/white and blue/blue images, seven subjects performed the experiment, three females and four males. They are all graduate students from the Centre for Intelligent Machines (CIM). They did not know when they were supposed to perceive illusions and when they were not supposed to. After looking at the results, we can say that some of them perceived the illusion in the right range, that is below the threshold set by Zucker and Davis. However, a couple of subjects always perceived an illusion in either case or in both cases as we shall discuss in Section 3. Therefore, the perfect results that Zucker and David depict are a bit unrealistic, and so too is there claim that the perception of the illusion changes abruptly after the threshold. From the seven subjects who performed the first two experimental sessions, three subjects performed the third experimental session (colored/white). The results are explained in Section 3.

3 Experimental Results

In this section, we shall describe the experimental results obtained from the experiments performed, which were described in Section 2. The first set of experiments involved the display of black dots on a white background as was carried out by Zucker and Davis [11], and from these results we were able to find the threshold (see our definition of threshold in Section 1.2) for each subject and then an overall threshold. These results are described in Section 3.2. The second set of experiments involved the display of blue dots on a slightly different blue background. The threshold computed is close to the one computed in the previous case for both the sun illusion, which is discussed in Section 3.2.1 and the Kanizsa edge illusion, which is discussed in Section 3.2.2. The final set of experiments involved the display of a random coloring of the dots on a white background. The results are depicted in Sections 3.4.1 and 3.4.2 for the sun illusion and Kanizsa edge illusion respectively. However, before starting with the above analyses, we shall start by analyzing the responses of a particular subject in a particular experiment.

3.1 Subject Responses Analyses

In a psychophysical experiment, different subjects with different backgrounds perform an experiment. However, sometimes a subject might not be concentrating and might be pressing buttons randomly just to finish and get through the experiment. If the data of such a subject is included in the experimental data, it would more likely tend to mislead the results than imply something meaningful. A small analysis can help us tell whether the subject data is good enough to be included in the overall data set or not. It is based on signal detection theory [2]. Below a certain threshold there is a signal, which in our case is an 'illusion', and above it there is no signal, which in our case is 'no illusion'. If there is an illusion, and the subject hit '1', then the response is classified as a "hit". If the subject hit '0', then the response is classified as a "miss". If there is no illusion and the subject hit '1', then the response is classified as a "false alarm". If the subject hit '0', then the response is classified as a "correct rejection". Then the hit rate, hr, and the false alarm rate, far, can be computed using the following formulae:

$$hr = \frac{\#hits}{\#hits + \#misses}$$
$$far = \frac{\#false\_alarms}{\#false\_alarms + \#correct\_rejections} \tag{2}$$

The idea in Equation 2 is that given a particular threshold, the hit rate and the false alarm rate of each subject can be computed. These rates will give the performance of this subject at perceiving the illusion. A subject with a high hit rate and a low false alarm rate performed best at detecting the illusion and vice versa. The further away his/her hit rate and false alarm rate, the better the subject performed at perceiving the illusion. An ideal hit rate would be 1 and an ideal false alarm rate would be 0.

The maximum number of hits (or false alarms) at a distance which is below (or above) the threshold is 5 because there are 5 images of each case, then at each distance the correct and incorrect percentages are:

$$\%(corr) = \frac{\#hits}{5}$$
$$\%(incorr) = \frac{\#false\_alarms}{5} \tag{3}$$

3.2 Threshold Detection

In this section, the threshold or spacing of dots above which the illusion is not perceived is located. The results of the session in which the black/white images were rendered in front of the subject were used as was performed in [11]. We shall consider both the sun illusion and the Kanizsa edge illusion cases.

3.2.1 Threshold Detection: The Sun Illusion

In this case, seven subjects performed the experiment, and they had to press buttons "0" or "1" as appropriate (described in Section 2). According to Zucker and Davis [11], an illusion is present if the subject perceives the brightness in the center of the sun or the contour that defines the sun. Therefore, we consider that the subject saw an illusion in the same cases.

The average performace of the subjects in this experiment is depicted in Figure 10, in which the average percentage of correct trials of all seven subjects is plotted as a function of the number of spaces between the dots. It can be seen from the figure that the subjects did

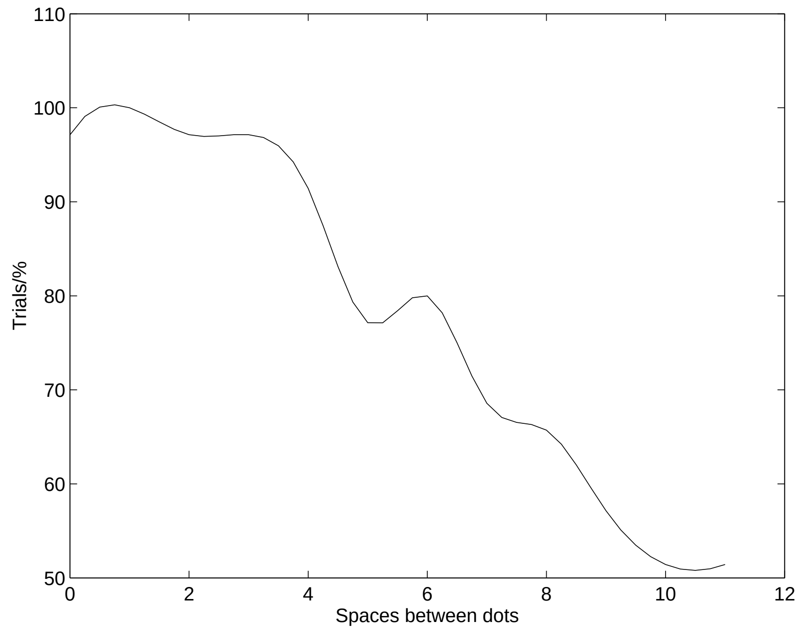


Figure 10: The average percentage of trials during which all the subjects saw the sun illusion, as a function of the number of spaces between dots (in units of min).

a very good job at detecting the illusion when the spaces between the dots was very small and less than 4. After that their detection degraded; therefore, we can consider that the threshold of stopping to perceive illusions in this case is 4-5, which is similar to Zucker and Davis's findings in [11], in which they get 4-6.

The Performance of Our Subjects Taking into account what has been discussed in Section 3.1, we shall analyze the performace of our seven subjects at perceiving the sun illusion. We first display the hit rate and false alarm rate for each of them in Table 1, given that the threshold is 5. As we can see, subject one performed the best in the experiment.

Subject Number	Hit Rate	False Alarm Rate
One	0.8800	0.0571
Two	1.0000	0.4000
Three	1.0000	0.7143
Four	0.9600	0.5143
Five	0.9200	0.8286
Six	1.0000	1.0000
Seven	1.0000	1.0000

Table 1: The hit rate and false alarm rate for each of the subjects for the sun illusion experimental part in black and white, taking into account that the threshold is 5.

The percentages of correct and incorrect trials for this subject are plotted in Figure 11 (see Equation 3). The subject had a high correct percentage below the threshold, and then it

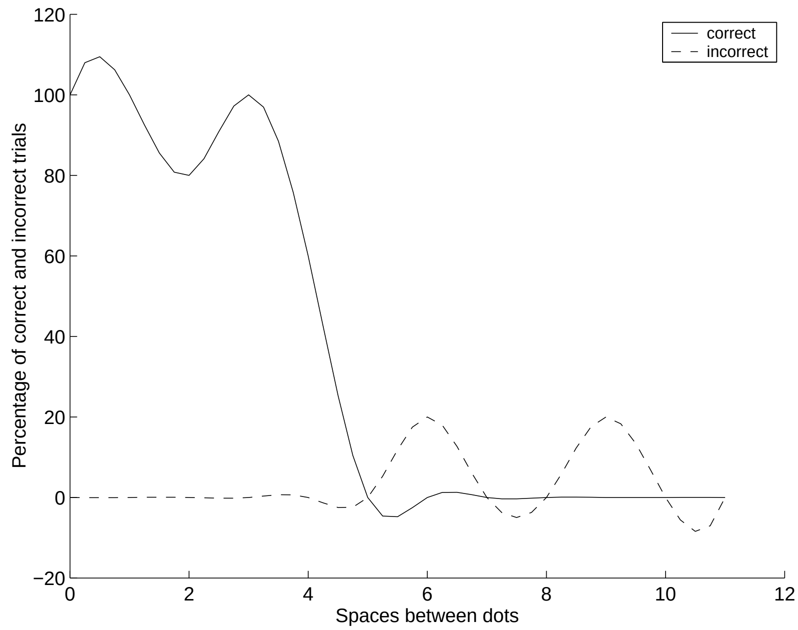


Figure 11: The percentage of correct and incorrect trials during which the subject saw the sun illusion, as a function of the number of spaces between dots (in units of min).

decreased above the threshold, which was expected. As for the false alarm rate, it remained close to zero for almost all dot spacings which is very good. Figure 12 shows the hit rate and the false alarm rate for subject seven who did not perform well on the experiment. This subject has equal hit rates and false alarm rates of 1, which means that he/she claimed to perceive an illusion in all images. Below the threshold it is a hit to see an illusion at all times, and since there are no misses, the hit rate is $\#hits/\#hits = 1$; on the other hand, above the threshold, it is a false alarm to perceive an illusion, and since there are no correct rejections, the false alarm rate is $\#false\_alarms/\#false\_alarms = 1$ (see Equation 2). This also explains why the percentage of correct trials are very high below the threshold and low above the threshold; on the other hand, the percentage of incorrect trials are very low below the threshold and very high above the threshold (see Equation 3). Such outliers can be excluded from the subject data in order not to mislead the results; however, they may also be retained as noise in the data. We can see that excluding outliers such as subjects six and seven improves the overall correct percentage of trials if we compare Figure 13 to Figure 10. In particular, the correct percentage of trials drops down to 30% instead of 50% in Figure 10 when the distance between the dots becomes very large and the illusion becomes very inevident. In this paper, we do not remove any outliers for our analysis, hence we retain all outliers as noise is inevitable in the real world. We should note, however, that this may tend to degrade the results.

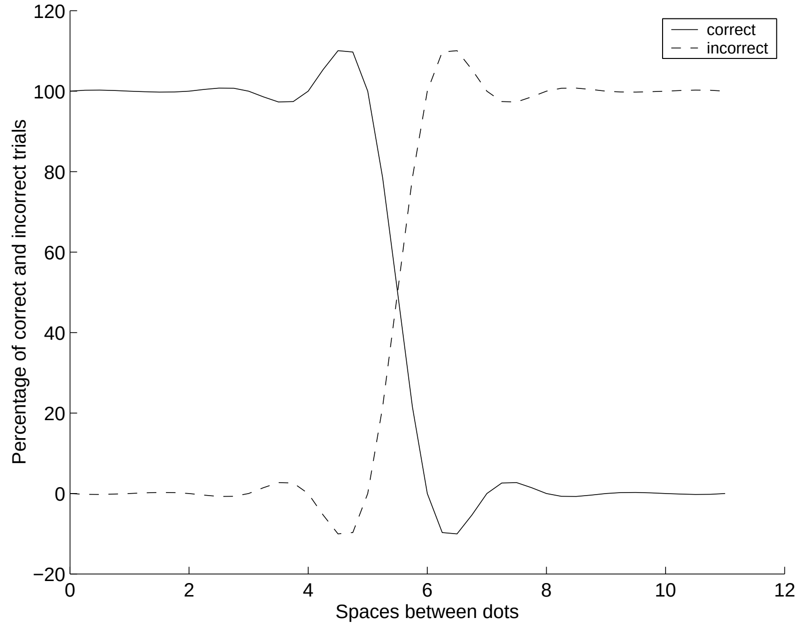


Figure 12: The percentage of correct and incorrect trials during which the subject saw the sun illusion, as a function of the number of spaces between dots (in units of min).

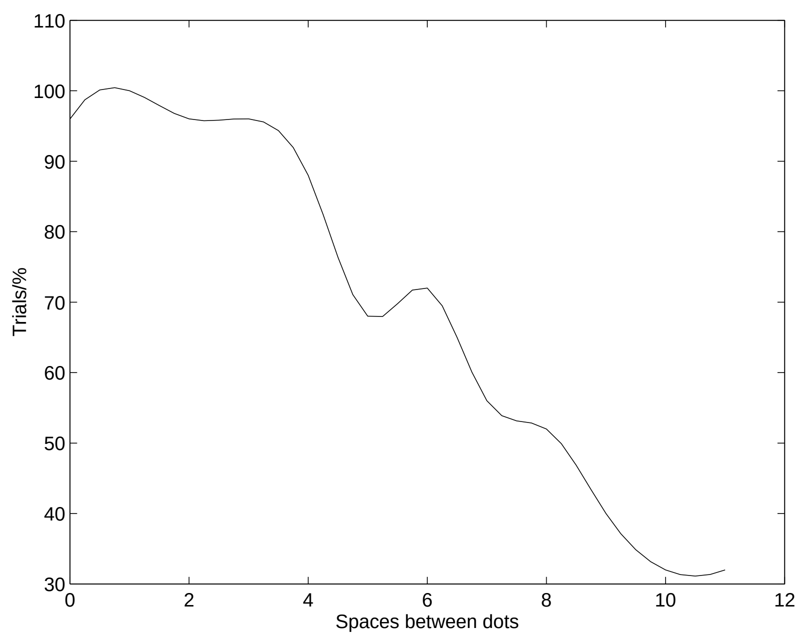


Figure 13: The average percentage of trials during which all the subjects saw the sun illusion, as a function of the number of spaces between dots (in units of min), excluding the outliers.

3.2.2 Threshold Computation: The Kanizsa Edge Illusion

In this case, seven subjects performed the experiment as well, and they had to press buttons "0" or "1" as appropriate (described in Section 2). According to Zucker and Davis [11], an illusion is present if the subject perceives the edge in the middle of the two sets of lines or if there is an occlusion. In this case, an occlusion means that one set of lines tends to cover another. The average performace of the subjects in this experiment is depicted in Figure 14, in which the average percentage of correct trials of all seven subjects is plotted as a function of the number of spaces between the dots. It can be seen from the figure that the subjects

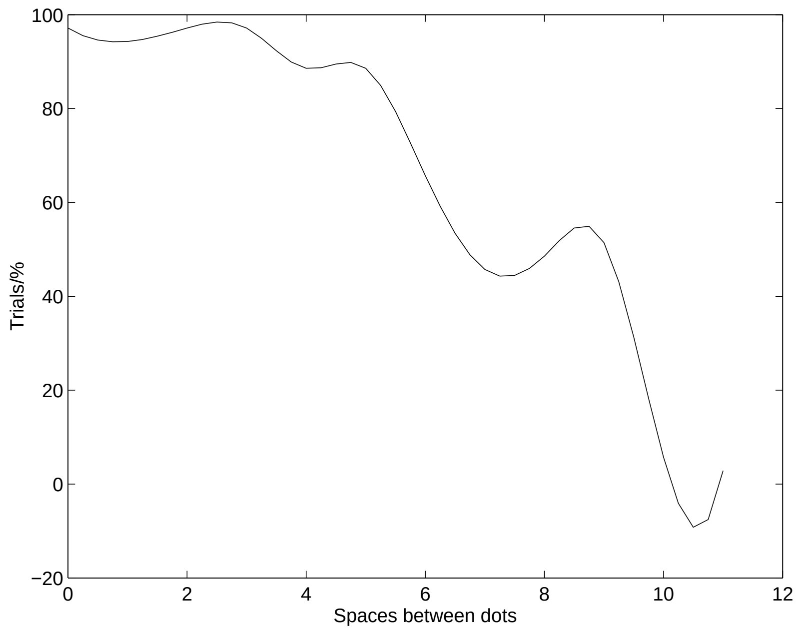


Figure 14: The average percentage of trials during which all the subjects saw the Kanizsa edge illusion, as a function of the number of spaces between dots (in units of min).

did a very good job at detecting the illusion when the spaces between the dots was very small and less than 5. After that their detection degraded; therefore, we can consider that the threshold of stopping to perceive illusions in this case is 5-6, which is similar to Zucker and Davis's findings in [11], in which they get 4-6.

The Performance of Our Subjects Taking into account what has been discussed in Section 3.1, we shall analyze the performace of our seven subjects in perceiving the Kanizsa edge illusion. We first display the hit rate and false alarm rate for each of them in Table 2, given that the threshold is 6. As we can see, subject one performed the best in this experiment as well. The percentage of correct and incorrect trials for this subject are plotted in Figure 15 (see Equation 3). Subject seven performed the worst recording the highest false alarm rate. Figure 16 shows the performance of this subject (see Section 3.2.1 for explanation).

Subject Number	Hit Rate	False Alarm Rate
One	0.8000	0.0333
Two	1.0000	0.3333
Three	1.0000	0.5000
Four	0.7667	0.1667
Five	1.0000	0.2667
Six	1.0000	0.5667
Seven	1.0000	0.7000

Table 2: The hit rate and false alarm rate for each of the subjects for the Kanizsa edge illusion experimental part in black and white, taking into account that the threshold is 6.

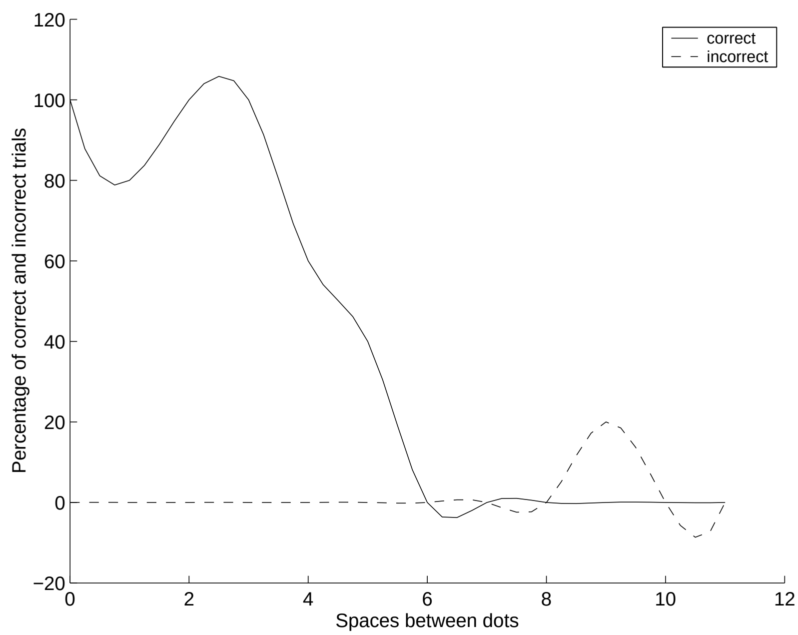


Figure 15: The percentage of correct and incorrect trials during which the subject saw the Kanizsa edge illusion, as a function of the number of spaces between dots (in units of min).

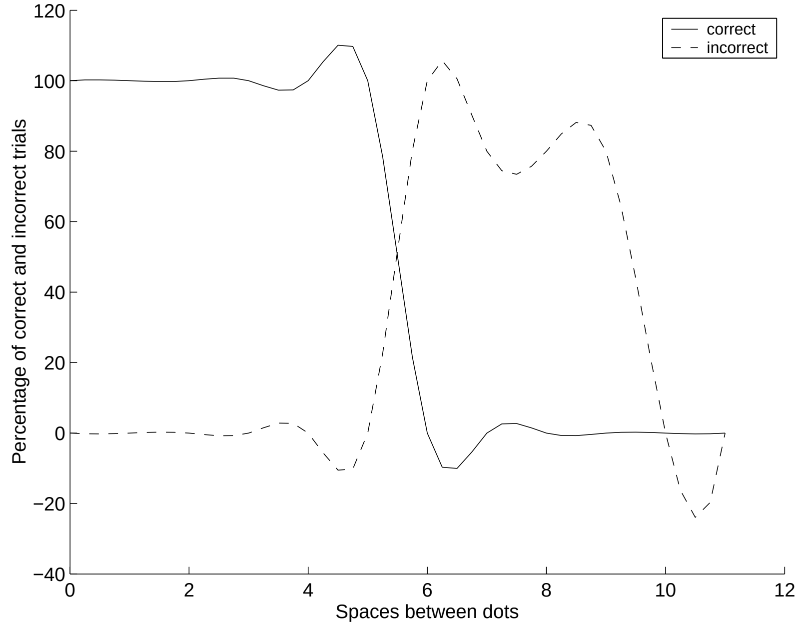


Figure 16: The percentage of correct and incorrect trials during which the subject saw the Kanizsa edge illusion, as a function of the number of spaces between dots (in units of min).

3.3 Illuminance Invariance

Illuminance is the amount of light incident on a surface. We have discussed in Section 1.3 how the visual system corrects for the prevailing illumination. Taking a particular example, two blue colors which are very similar, the visual system tends to perceive the lighter one as being lighter blue and the darker one as being darker blue. Therefore, if blue dots are plotted on a slightly darker blue background, this color correction scheme will make the visual system perceive more contrast and thus make out the edges or the junctions as discussed in Section 1.3. After making out the edges, the dots will become clearer to the observer or the subject, and he/she can see the shape. Figure 17 shows the average performance of all seven subjects at perceiving the sun illusion. Upon comparison of the performance of the subjects in this case to their performance in the case of black dots on a white screen, it can be noted that their performance on average is more or less the same. The threshold at which the "Trials %" starts to fall off in the figure is around 4 which is similar to that in Figure 10.

Another illusion the seven subjects are asked to perceive is the Kanizsa edge. Figure 18 shows the average performance of these subjects at this task. Comparing this plot to the one of the black and white dots (Figure 14), we can see that it might seem better than the black and white scenario. This might be due to the fact that in the attempt of the visual system to make out the edges between the different blue colors, it will tend to "pass" conceptually through many different contrasts for the two blue colors in order to find the best contrast that will make out the edges. While perceiving these different contrasts, it would tend to perceive illusions.

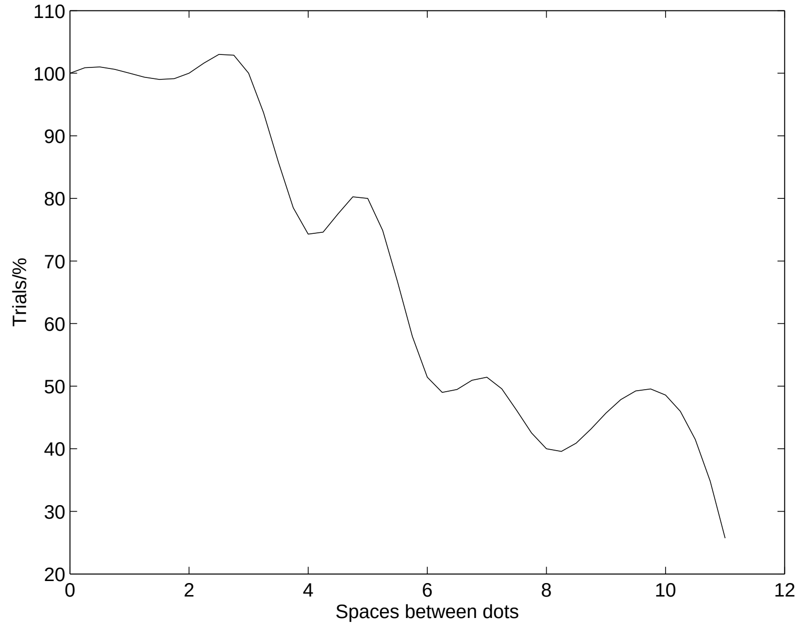


Figure 17: The average percentage of trials during which all subjects saw the sun illusion, as a function of the number of spaces between dots (in units of dot diameter).

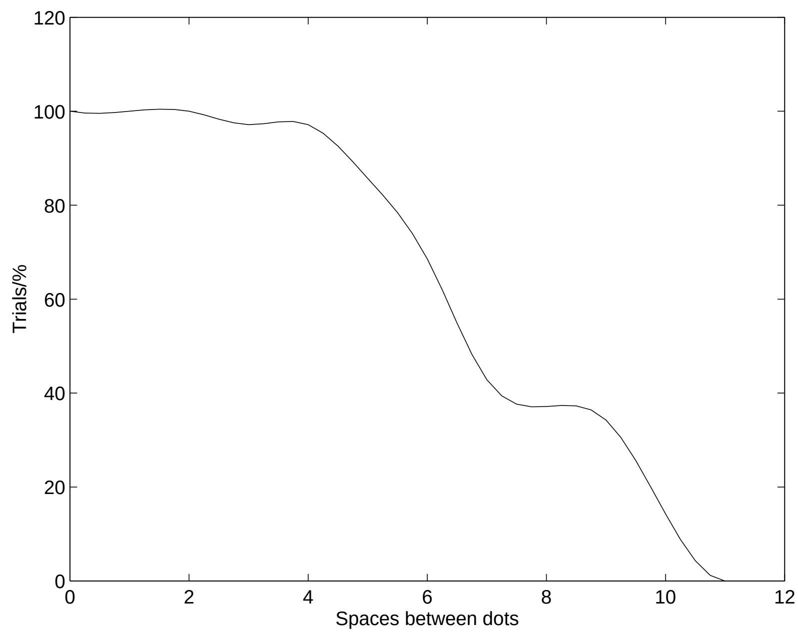


Figure 18: The average percentage of trials during which all subjects saw the Kanizsa edge illusion, as a function of the number of spaces between dots (in units of dot diameter).

3.4 Colors Effect

In this section, we base our idea on the factor of similarity of Gestalt psychology. Two dots are perceived together if they have a cue in common, color, for example. In one session of our experiments, we vary the colors of the dots randomly, and we ask the subjects whether they see an illusion or not. Four subjects performed the experiment in this case and the results were recorded.

3.4.1 The Sun Illusion

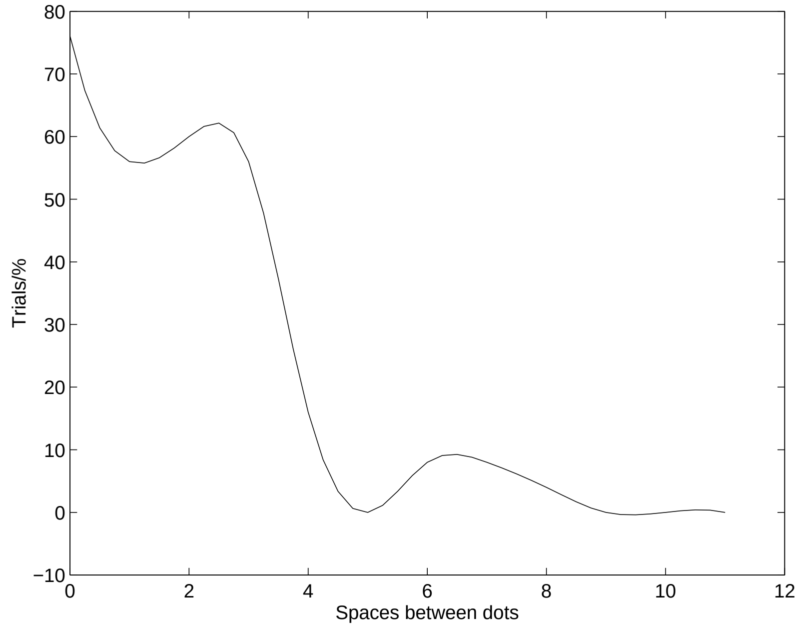
The fact that the dots had random colors made it more difficult for the subject to perceive the illusion. In Figure 19, we show the average correct percentage over all subjects of perceiving the illusion in both the black/white and colors/white scenarios. It can be seen from Figure 19 that the percentage of correct trials is 80% for the black/white case when it should be at its maximum, and 45% for the colored/white case, this is when the distance between the dots is 0 min. We can also see that the curve drops off at a smaller dot spacing in the colored dots case, which means that the subjects stop perceiving the illusion at a smaller dot distance on an average. These findings show that the colored dots case was harder for the subject to perceive the illusion. We shall also add that the subjects themselves noted that this case was the most difficult case for them to perceive the illusion, in general, and to detect the contour of the sun, in particular.

3.4.2 The Kanizsa Edge

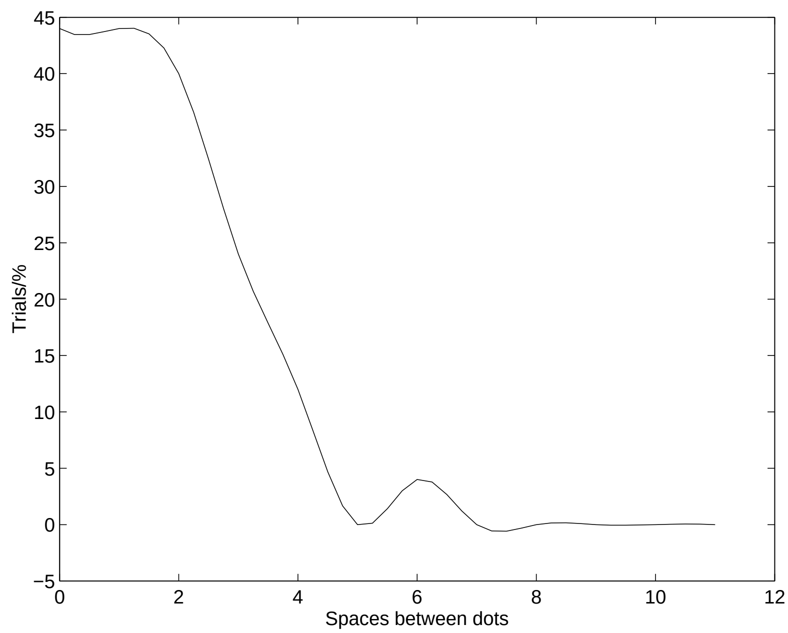
The same subjects who did the experimental session in Section 3.4.1 were asked to indicate whether they perceived an illusion in the Kanizsa edge at different dot spacings. Figure 20 depicts the average correct percentage over all subjects of perceiving the ilusion (the subjective edge and the occlusion). It can be seen from Figure 20 that the percentage of correct trials is 95% for the black/white case when it should be at its maximum, and 70% for the colored/white case, this is when the distance between the dots is 0 min. We can also see that the curve drops off with a larger slope in the colored dots case, which means that the subjects stop perceiving the illusion at a smaller dot distance on an average. These findings show that the colored dots case was harder for the subject to perceive the illusion. We shall also note that the subjects themselves noted that this case was the most difficult case for them to perceive the illusion, in general, and to detect the occlusion, in particular, since they couldn't make out the lines from the image easily.

4 Conclusions and Future Work

The *size/spacing constraint* and its influence on visual illusions has been the topic of research in this paper. Fixing the size of a dot on a dot display image, we show how the spacing between dots affects the perception of visual illusions. These visual illusions can be subjective edges or apparent occlusion in the Kanizsa Edge case, and apparent brightness in the center of the sun or a contour defining the sun in the sun illusion case.

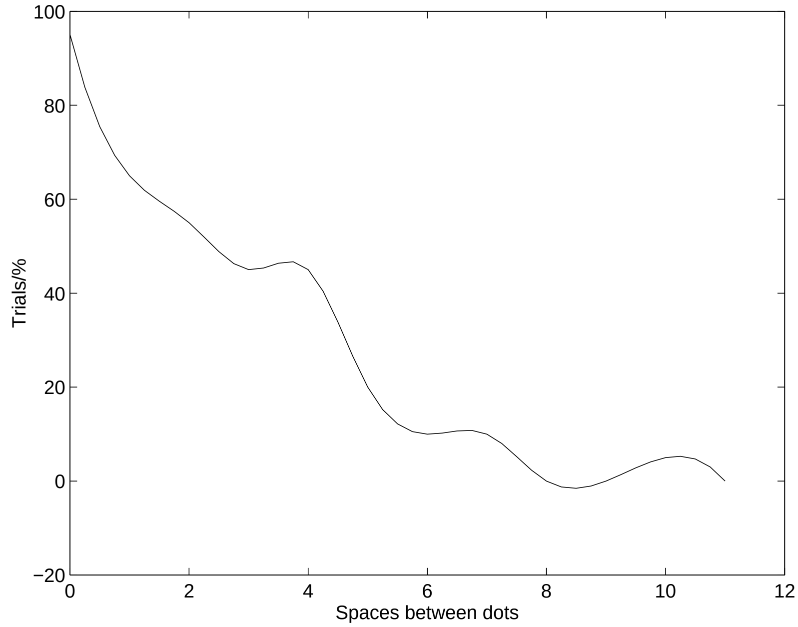


(a)

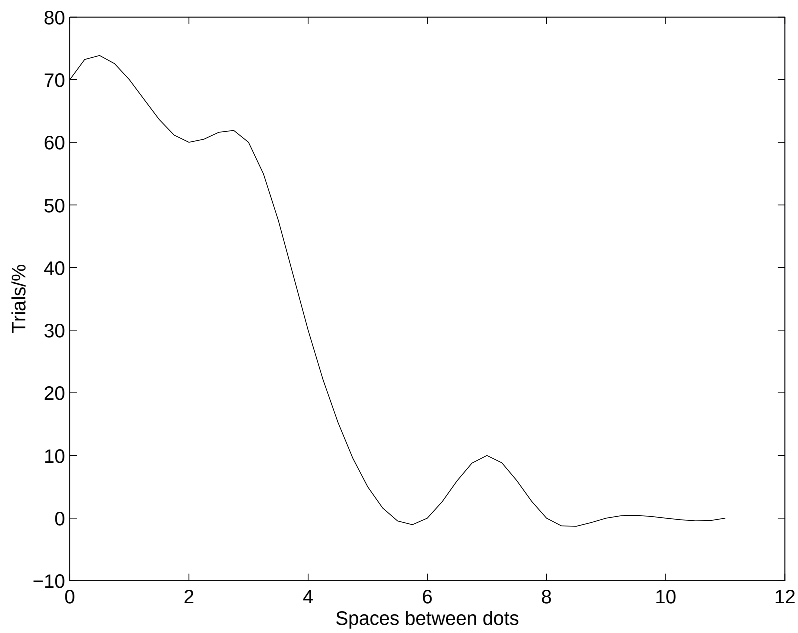


(b)

Figure 19: The average percentage of trials during which all subjects saw the sun illusion, as a function of the number of spaces between dots (in units of dot diameter): (a) black dots were rendered on a white background, (b) randomly colored dots were rendered on a white background



(a)



(b)

Figure 20: The average percentage of trials during which all subjects saw the Kanizsa edge illusion, as a function of the number of spaces between dots (in units of dot diameter): (a) black dots were rendered on a white background, (b) randomly colored dots were rendered on a white background

The displays or models tested on subjects in this paper extend Zucker and Davis's [11] models obtaining insight from the characteristics of human vision. The human visual system tends to perceive two colors of close lightness contrast near an edge or where they meet as having a larger contrast [1]. This is done in order to make out the details of what is being perceived. Furthermore, humans tend to group objects according to the laws of Gestalt psychology. Objects are grouped perceptually according to similar cues, one of which is color. We show that when the color of the dots is random, the subjects can no longer group the dots as well as before, and therefore the perception of illusions is impeded.

The experimental results performed on subjects are analyzed. In Section 3.2, the thresholds obtained for the sun illusion and the Kanizsa edge illusion are between 4-6, which is similar to Zucker and Davis's findings [11]. However, they do not show any results for the Kanizsa edge case; they just mention that they get informal results. Moreover, we should mention that we do not exclude any outliers from our results. Had we done so, we would have gotten much smoother curves. We show one subject who was an outlier in Section 3.2, in perceiving the sun illusion in the black/white case. Subject six was also an outlier as can be seen from Table 1. The trials percentage would have had smaller values for larger dot spaces, had we excluded the outliers who were claiming to having seen an illusion in all cases. Finally, a point worth mentioning here is that Zucker and Davis [11] seem a bit unrealisitic as their results seem too perfect. They start with 100% correct trials, for the sun illusion example, and then decrease gradually after the threshold down to 0. Their subjects all seemed to be perfect.

Zucker and Davis [11] do not explain in great detail how the illusions are perceived. The Kanizsa edge is a sinusoidal contour, so it is an example of an object that needs complex processing by the brain (see Section 1.2). The edge can be extracted by the trace inference process [5]. The lines can be perceived by early orientation selection (Section 1) neurophysiologically [4] and by the factors of similarity and proximity of Gestalt psychology, by grouping the dots into lines, and then grouping each set of lines into a surface. Since the surfaces are not at the same level, thus inducing the sinusoid, one tends to be perceived infront of the other. The sun illusion is made up of a sun and rays. The sun has a contour which is perceived by the factor of similarity of Gestalt psychology. As for the brightness, it arises from the fact that there are two different objects in the scene, the sun and the background. Since people tend to segregate the object from the background [6], they perceive one as having a different color or lightness from the other. An interesting point is that if one were to compare the brightness of the center of the sun with the immediate surrounds, no illusion would be perceived in an image that is supposed to induce an illusion. Therefore, the brightness of the center should be compared with that of the background which is a bit further away. This might be due to the fact that the rays which have the same color of the dots in the center should not interfere with the other brightness color so that the contrast would be more evident.

It is believed that the segmentation of an image into figures and background is a first and critical step for the perception of an illusion [6]. In other words, it is basically part of the organizational process performed by the visual system. When a figure is isolated from the background, the dots can be organized or grouped by the visual system into a form and consequently the visual illusion can be perceived. When the colors are made random, the factor of similarity which the visual system is inherently using to group the dots in the black dots on white background and blue dots on blue backgound is violated. Therefore, the grouping process is impeded, and so the visual system cannot group the dots and make out the shape as easily as before. Therefore, the illusion cannot be perceived as clearly. After the experiments, the subjects claimed that they could not perceive the contour as clearly as the previous cases, in the sun illusion case. This is because the grouping process of the circle and the rays is impeded. This implied that they could not see the apparent brightness in the center as clearly too. Also, they could not perceive the apparent edge as clearly as in the cases before, in the Kanizsa edge illusion. This is because the grouping process of the lines is also hindered. This implied that they could not see the occlusion because the surfaces formed by the two sets of lines cannot be perceived if the dots appeared scattered all over.

It was nice to see how the blue display results agreed with the black and white display results. It was very counter-intuitive at first; however the idea of lightness perception across junctions explained it [1].

Zucker and Davis [11] use black and white in order to get maximum contrast between the two colors, thus inducing an illusion. However, had they used two different colors they would not have obtained such good results. Furthermore, had they used different colors of dots, they might not have guaranteed such good results either as we have seen in this paper.

Some improvements to the results would be training the subject for a longer time, and that would involve displaying a larger number of images at the start. Moreover, Zucker and Davis [11] perform the experiment on paper. There is no use for such an experiment. Also, they provide no statistical measurement of how good or bad each of their subjects performed in the experiment. It would be good to give the reader an idea of the performance of the subjects. It would be also good to have more subjects, as they only have three subjects. It seems that they performed perfectly. Finally, the dot size used in this paper was 1 min diameter, which was that used by Zucker and Davis [11], and the subject was seated at 50 cm from the monitor. However, not all the subjects performed well, that is some had a low hit rate. This might be due to the fact that the dot size used was not optimal for all subjects' eyes. Changing the dot diameter might help improving the performance of some, but, on the other hand, it might degrade the performace of others. Using the dot size that is best for each subject and then combining all the results from different subjects with the different dot sizes is an interesting problem.

Acknowledgements I would like to express my gratitude to the instructor Kaleem Siddiqi. Next, thanks to all the people who performed the experiments, as without their assistance this work would not have been possible. I would also like to thank my colleagues Ziad Hafed and Gurman Gill for their insightful discussions. Finally, the students who took the course COMP-766 are acknowledged for having provided a pleasant environment throughout the term.

References

[1] E. H. Adelson. *The New Cognitive Neurosciences*, chapter Lightness Perception and Lightness Illusion, pages 339–351. Cambridge, MA: MIT Press, 2nd ed. edition, 2000.

[2] C. H. Coombs, R. M. Dawes, and A. Tversky. *Mathematical Psychology*, chapter 6, pages 165–201. Englewood Cliffs, NJ: Prentice-Hall, 1970.

[3] E. B. Goldstein. *Sensation and Perception.* Brooks/Cole Publishing Company, Pacific Grove, California, 5th ed. edition, 1999.

[4] D. H. Hubel and T. N. Wiesel. Brain mechanisms of vision. *Scientific American*, 247:150–162, 1979.

[5] P. Parent and S. W. Zucker. Trace inference, curvature consistency, and curve detection. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 11(8):823–839, 1989.

[6] N. Rubin, K. Nakayama, and R. Shapley. Enhanced perception of illusory contours in the lower versus upper visual hemifiels. *Science*, 271:651–653, February 1996.

[7] P. Sinha and E. H. Adelson. Recovering reflectance in a world of painted polyhedra. In *Proceedings of Fourth International Conference of Computer Vision*, pages 156–163, Berlin, Germany, May 1993. Los Alamitos, California: IEEE Computer Society Press.

[8] S. Skaff. http://www.cim.mcgill.ca/~sandra/shape_project/images.

[9] http://www.opengl.org.

[10] M. Wertheimer. *A source book of Gestalt psychology*, chapter Laws of Organization in Perceptual Forms, pages 71–88. London: Routledge & Kegan Paul, 1938.

[11] S. W. Zucker and S. Davis. Points and endpoints: A size/spacing constraint for dot grouping. *Perception*, 17:229–247, 1988.

A The Visual Angle

A subject is at a and is subtending an angle t at two points b and c. In other words, he/she is looking at the two points through angle t. The angle t is the visual angle, which is to be computed given the distance between b and c, x, and the distance of the subject from the monitor, d. The triangle amb is right; therefore, $tan(\frac{\theta}{2} = \frac{x/2}{d})$. From this latter equation, $x = tan(\frac{\theta}{2}) * 2d$.

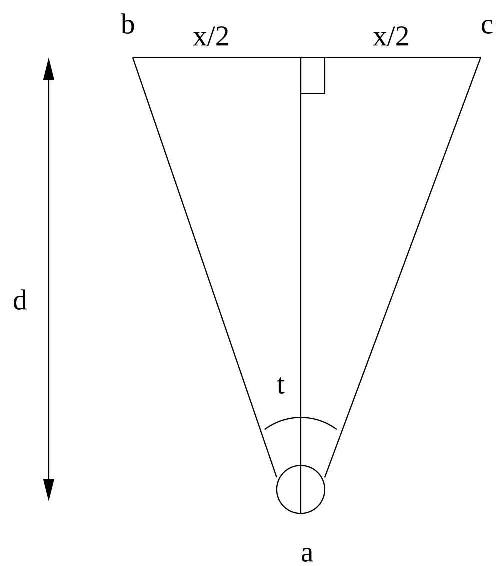


Figure 21: A subject at a subtending a visual angle t.