The Conceptual Design of Robotic Architectures by Means of a Complexity Measure

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Abstract

At the conceptual stage of the design process the designer has at her or his disposal a set of alternative design solutions, sometimes called "variants". However, as these are not as yet embodied, a mathematical model is not available, which means that the usual tools of optimum design, such as mathematical programming, calculus of variations, or even optimum control, are not applicable. We propose here a performance measure, based on the concept of complexity, that is intended to allow the designer to rank the various alternatives from simplest to most complex. The application of this measure to the design of the kinematic chain of the robotic structure of Schönflies-Motion Generators is outlined in the paper. This concept has led to a family of robots that we term McGill Schönflies-Motion Generators. One prototype of these robots is disclosed in the paper.

Keywords: Design theory, complexity, diversity, entropy

1 Introduction

The word "design" is derived from the Latin "designare," meaning "to mark out." This word bears many meanings in English, as it is used as a noun and as a verb. As a noun, moreover, the "D" word is used to *designate* the product, the process, the visual representation, etc. As a verb, to design is to conceptualize a product, tangible or intangible, intended to satisfy a human need.

Design is thus a broad activity, exhibiting several manifestations. In ascending order of technical content, we have: *art; graphic design; industrial design; architectural design*; and *engineering design.* Sometimes, the last two are referred to as *technical design.* The term "technical" derives from Greek $\tau\eta\chi\nu\eta$ (techneh), which means art or craft. "Technical" bears the connotation of material realization. Art itself has its technical content as the painter, the sculptor, the composer must obey certain rules that govern their medium of expression and that are independent of the purely artistic content of their work. However, the artistic, emotive content of art is much higher than that of industrial design, and is superseded, to a certain extent, by technical content in engineering design.

Engineering design is a pervasive activity that appears in every engineering project. Engineering design is also heavily dependent on subjective concepts, such as thinking processes. Although attempts have been made to formalize engineering design as a science, the appearance of various, quite disparate, schools of thought indicates that the theory of engineering design is an intellectual activity in evolution, whereby research is still needed to lay its foundations in a broadly acceptable framework.

This paper is an attempt to lay the foundations of a *design theory* within the engineering realm. Such a theory should be built on *design principles* that are common to all engineering activities, regardless of the underlying disciplinary details. A design theory should thus be equally applicable whether the objective of the task at hand is a coffee filter or a low-pass filter, a Wheatstone bridge or a stone bridge.

2 The Nature of Design

In a *design job* there is always a client, who submits the need to the *designer*, be this an individual or a *design team.* It is only seldom that one single individual is capable of undertaking and successfully completing a design job.

As outlined above, design is a complex activity that draws from creativity, scientific and engineering knowledge as well as hard and soft skills (team work, communication, interpersonal relations, and so on). Because of its complexity, design researchers have devised models of the design process. The three most frequently cited models are those due to French [1], Pahl et al. [2] and the

VDI[1] [3]. In the foregoing models we can identify four major stages (or steps) in the design process, namely,

1. **Analysis of the problem:** first, the need motivating the design job is analyzed by the designer in consultation with the client, and then formulated in terms of *functions* and *subfunctions*; more precise conditions are also included, as per the client's needs and resources, these conditions constituting the *design specifications*.

2. **Conceptual design:** Once the problem has been formulated and analyzed, a rich set of alternative solutions is generated by means of creativity-enhancing methods (brainstorming, synectics, IPS, a.k.a. TRIZ, etc.); finally, this set is thoroughly analyzed and scrutinized, until a reduced set of alternatives is identified as the candidate solutions that stand the highest chances of meeting all the functions and specifications, while observing all the constraints (timelines, budget, etc.)

3. **Embodiment design:** the alternatives identified in item 1 are given form by means of sketches and preliminary design drawings; several criteria are then put in place in order to decide which of these alternatives is the most likely to satisfy the client's need. In complex design jobs, like in aircraft design, a *parametric model* of the selected alternative(s) is(are) produced, with purposes of *analysis* and, eventually, *optimization* of the *designed object*.

4. **Detail design:** Once the design team has zeroed-in on one design solution, all parts and components are either designed or selected from catalogue. The final product is a design report, containing: a summary of the need that motivated the job; all design solutions considered; a thorough description of the design solution of choice; the manufacturing drawings; tables; plots; estimated budget to produce the object designed; and all kinds of information needed to understand the proposed solution.

It is then apparent that at the outset, at the conceptual stage, a mathematical model is not available. This will come once an embodiment of a solution has been produced. At the conceptual stage, only rules of the `if...then` type are possible. These rules are *heuristic*, in that they are based on common sense and experience, some based on solid science and engineering knowledge.

Some tools have been developed over the years to help the engineer at the early design stages. These have been proposed in the form of principles

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[1] This is the acronym of *Verein Deutscher Ingenieurer* or Association of German Engineers.

that are applicable to all engineering design jobs, regardless of the discipline. Three main schools are to be cited: The German School [2, 3]; Axiomatic Design [4, 5]; and Robust Design [6].

2.1 Entropy in the Mathematical Theory of Communication

The concept of information content, of the utmost importance in conceptual design, derives from the thermodynamic concept of *entropy*, as proposed by Rudolf Clausius (1822–1888) in 1865. Clausius introduced the concept in a seminal paper that shed light in the *Second Law of Thermodynamics.* According to Clausius, "if an *isolated* thermodynamic system is left alone, entropy can only increase."

The *mathematical theory of communication* (MTC), sometimes referred to as *information theory*, or even as *cybernetics*, is a far more recent discipline than statistical thermodynamics, or than classical thermodynamics for that matter. The credit of the creation of the theory goes to Claude E. Shannon, who published his milestone paper, "A mathematical theory of communication," in the *Bell System Journal* in July and October, 1948. In fact, the history of Shannon's theory has been traced back [7] to Boltzmann (ca. 1894), then to Szilard [8], von Neumann (ca. 1932) and Wiener [9], one of the creators of cybernetics.

The main item in MTC is *information*, and how to quantify the amount of information in a message, which is important in order to measure the *capacity*, in bits/s, of a communication channel. Within the MTC the concept of information is different from its connotation in everyday's language. As Weaver put it, "information in communication theory is associated with the amount of freedom of choice we have in constructing a message" [7]. In this context, *freedom* is to be understood as the opposite of *order*: A perfectly ordered society is one overwhelmed with laws and rules, to the point that it removes freedom from its individuals. A society without laws and rules, on the contrary, has complete freedom, but becomes totally disordered.

Shannon's notion of information, or its content in a message, in fact, can also be understood as the *level of ambiguity of a message.* Hence, contrary to the notion of information in everyday's language, information in the MTC is something we want to keep to a minimum, a perfect message carrying zero ambiguity and hence, zero information.

In formulating the Independence Axiom, Suh first defines the *functional requirements* and the *design parameters*, to be determined by the designer. In the next step, Suh presupposes *linear* relations between the two foregoing items, related by a *design matrix.* Within the paradigm of axiomatic design, the design matrix would better be square, although rectangular matrices can also be accommodated. Indeed, Suh calls “ideal” a design job in which the number of design parameters equals that of functional requirements, thus leading to a square design matrix; a “coupled design” is one design task in which the number of design parameters is smaller than that of functional requirements; the opposite of a coupled design, in turn, is called “redundant.” Apparently, in the last two instances the design matrix is rectangular. However, this framework faces some problems when one tries to apply it to a concrete design task. The first problem is that, at the conceptual stage, there is no mathematical model available, not to speak of a linear model, to begin with. The second problem arises when attempting to compute the information content of a design. Suh resorts to the concept of *entropy* in the mathematical theory of communication, as proposed by Shannon [7]:

$$H = -\sum_{1}^{n} p_i \log(p_i), \quad \sum_{1}^{n} p_i = 1 \tag{1}$$

where $\log(p_i)$ is the logarithm of p_i to a certain base. If we make abstraction of the Boltzmann constant in the definition of the thermodynamic entropy [10] and use natural logarithms in eq.(1), we find that the entropy expressions in the two cases are identical. While in thermodynamics the base e of natural logarithms is preferred, in information theory we are free to choose the logarithm base. The natural choice is *binary* logarithms, namely, those that use 2 as a base, in which case H is measured in *bits.* If the Neperian logarithms are used, those to a base e, then the unit of measure is the *nat*; if decimal or Briggs logarithms are used to measure H, then the units are *decibels.*

In information theory, the base 2 is preferred mainly because the theory is based on the concept of *choice*, which can always be reduced to a *binary* search. For example, to choose one letter of the 26 of the English alphabet, we can always start by deciding between two halves, the first comprising letters A to M, the second from N to Z. Then, we would continue with a splitting of each half into two “halves”, which needn’t be of the same size, and so on.

Suh proposes to interpret the p_i in eq.(1) as the *probability of meeting a tolerance.* However, at the conceptual stage, we do not have as yet a parametric model of the object under design, and hence, we cannot associate a tolerance to each parameter.

3 Design Diversity Based on Entropy

What we propose here is to regard the entropy of a message or of a design concept produced, e.g., during a brainstorming session, as the *complexity* of this concept. Here, we liken *complexity* to diversity: The more diverse the design concept at hand, the more complex it is. In other words, the higher the *diversity content* of an alternative solution to a design problem, the more complex the solution.

In order to assess the diversity of a design alternative, we will introduce first some definitions:

Function: A *generic* task imposed by the need to be satisfied by means of the object under design. Examples: fasten; separate; sort; support; transport; energize; entertain; actuate; sense; etc.

Function-carrier: A component or assembly intended to implement a function [2]. Examples: a bolt, a welding seam or a screw are components that serve the function *fasten*, and hence, they are *carriers* of this function.

Design specification: A *quantitative* condition to be met by the object under design. Example: A system to transport persons and materials through a span of 0.5 km should serve 1000 people/h and transport 1000 ton/h of merchandise.

Functions can be decomposed into *subfunctions*. Example: The function *separate* can have the subfunction *allow relative rotation of two neighbouring disks about one common axis.*

Let us assume that a given function F entails N subfunctions $f_1, f_2, \ldots f_N$, with f_i to be implemented with a number ν_i of *identical carriers* C_i, so that the total number N_c of function-carriers for F is

$$N_c = \nu_1 + \nu_2 + \ldots + \nu_N \tag{2}$$

Let us now denote with ϕ_i the *frequency* of occurrence of function-carrier C_i, namely,

$$\phi_i = \frac{\nu_i}{N_c}, \quad i = 1, \ldots, N \tag{3}$$

and hence,

$$\sum_{1}^{N} \phi_i = 1 \tag{4}$$

Apparently, the frequencies ϕ_i in eqs.(3) and (4) play the role of the probabilities p_i in eq.(1). We can thus define a *diversity* in the design-solution alternative at hand as the equivalent of the entropy in eq.(1). It would just be fair to refer to this *design entropy* as the *design diversity* H, namely,

$$H = -\sum_{1}^{N} \phi_i \log_2(\phi_i) \tag{5}$$

It is noteworthy that Shannon's formula, eq.(1), is an approximation to a combinatoric number, as appearing in statistical thermodynamics [10]. This approximation, based on the *Stirling formula*, is quite accurate for "large" values of N, as the numbers of particles occurring in thermodynamic systems are of the order of the number of Avogadro, 6.022×10^{23}. The numbers N_c of function carriers in a design are comparatively modest. Nevertheless, the Stirling formula gives a relatively low error, of about 14%, for numbers of the order of 10 [10], and hence, can be adopted within a design-theoretical framework.

Here we illustrate with a simple design example how to assess the *entropy content* of a design alternative in terms of the *diversity* of the function-carriers used to implement a function. We illustrate the concept by focusing on one specific feature, the diversity of the actuators in robot design. The design problem at hand reads:

> *Design a fast robot for pick-and-place operations requiring three independent translations and one rotation about a vertical axis.*

The desired motion is similar to that of the tray of a waiter (no tilt allowed). As this motion has four degrees of freedom (dof), i.e., three translations and one rotation, four functions carriers are needed. The function *move* can be divided into 1, 2, 3 or 4 subfunctions, depending on how the subfunctions are implemented. Let us compare four solutions. First, if we use four

Figure 1: The Adept Cobra s600

Figure 2: The ABB IRB660

distinct motors, thereby leading to a serial robot, we have

$$\nu_1 = \nu_2 = \nu_3 = \nu_4 = 1, \quad N_c = 4$$

From eq.(3),

$$\phi_1 = \phi_2 = \phi_3 = \phi_4 = \frac{1}{4} \tag{6}$$

Accordingly, the *actuation diversity* of the robot is evaluated with eq.(5), namely,

$$H_1 = -4\left[\frac{1}{4}\log_2\left(\frac{1}{4}\right)\right] = 4 \times \frac{1}{4} \times 2 = 2 \tag{7}$$

The embodiment and subsequent detailed design of this alternative would lead to a robot like the Adept Cobra s600, illustrated in Fig. 1.

If we use three distinct motors, we need to decompose the function *move* into three *subfunctions*. ABB Robotics has produced an ingenious function-decomposition: (*i*) *turn* the payload (PL) about one vertical axis fixed to the ground; (*ii*) *move* the PL under pure translation in a vertical plane; and (*iii*) *turn* the PL about a moving vertical axis . The ensuing embodiment and detailed design led to the ABB IRB660-1 depicted in Fig. 2. For this case, we have

$$\nu_1 = 1, \nu_2 = 2, \nu_3 = 1, \quad N_c = 4$$

From eq.(3),

$$\phi_1 = \frac{1}{4}, \quad \phi_2 = \frac{1}{2}, \quad \phi_3 = \frac{1}{4} \tag{8}$$

Accordingly, the actuation diversity of the robot is

$$H_2 = -2\left[\frac{1}{4}\log_2\left(\frac{1}{4}\right)\right] - \frac{1}{2}\log_2\left(\frac{1}{2}\right) = 2 \times \frac{1}{4} \times 2 + \frac{1}{2} = 1.5 \tag{9}$$

which is a lower value when compared with the first solution.

In a third solution, we use two distinct motors. We also end up with a serial-parallel robot. In this case, we need to decompose the function *move* into two subfunctions: (*i*) *translate* the PL into three independent directions; and (*ii*) *turn* the PL about a moving vertical axis. ABB Robotics implemented this solution by concatenating one parallel Delta Robot with a fourth vertical axis mounted on its moving plate. The motor moving this axis was fixed to the base; its torque and motion being transmitted by a serial combination of a prismatic joint and two universal joints at its ends. The result is the ABB IRB660-1 illustrated in Fig. 3. For this case, we have

$$\nu_1 = 3, \nu_2 = 1, \quad N_c = 4$$

Figure 3: The ABB FlexPicker

From eq.(3),

$$\phi_1 = \frac{3}{4}, \quad \phi_2 = \frac{1}{4} \tag{10}$$

Accordingly, the actuation diversity of the robot is

$$\begin{aligned} H_3 &= -\frac{3}{4}\log_2\left(\frac{3}{4}\right) - \frac{1}{4}\log_2\left(\frac{1}{4}\right) \\ &= \frac{3}{4}\left[2 - \log_2(3)\right] + \frac{1}{4} \times 2 = \frac{8}{4} - \frac{3}{4}\log_2(3) = 0.8112 \end{aligned} \tag{11}$$

which is an even lower actuation-diversity value than in the second solution.

Figure 4: The ABB Adept Quattro s650

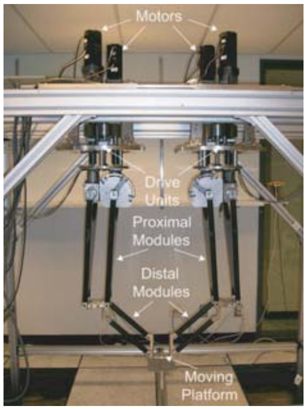

Figure 5: The McGill SMG

Finally, we can use four identical motors, thereby leading to a parallel robot. In this case, the function *move* has only one subfunction; in fact, the function is its own subfunction. Two alternative embodiments have been implemented with this solution, namely, the ABB Adept Quattro s650 and the McGill SMG, illustrated in Figs. 4 and 5, respectively. We thus have, in this case,

$$\nu_1 = 4, \quad N_c = 4$$

From eq.(3),

$$\phi_1 = 1 \tag{12}$$

The actuation diversity of the robot is

$$H_4 = -1 \times \log_2(1) = 0 \tag{13}$$

which is the lowest possible value.

4 Complexity-based Rules

At the conceptual stage, the designer of a robot has very limited information. The information typically includes the type, number and the relative arrangement of joints, along with the number of loops. Based on the functional requirements, the designer is usually able to decide on the type and the diversity of the actuators. Khan et al. [11] derived a set of design rules related to performance criteria, i.e., stiffness, life-cycle cost, workspace volume and agility, and the topology of a concept, i.e., number of joints, number of loops, type of joints, joint configuration, type of actuators and diversity of actuators. From the rules derived in [11], we define six aspects of robot complexity:

4.1 Joint-Number Complexity K_N

The joint-number complexity is defined as:

$$K_N = 1 - \exp(-q_N N) \tag{14}$$

where N is the number of joints used in the topology at hand and q_N is the resolution parameter, to be adjusted according to the resolution required. Note that $K_N \in [0, 1]$.

4.2 Loop Complexity K_L

$$K_L = 1 - \exp(-q_L L); \quad L = l - l_m \tag{15}$$

where l is the number of kinematic loops in the topology of the robot, l_m being the minimum number of loops required to produce the corresponding displacement set, group or subgroup, and q_L a resolution factor.

Table 1: Geometric complexity of the six lower kinematic pairs [11]

$K_{G\|R}$	$K_{G\|C}$	$K_{G\|P}$	$K_{G\|H}$	$K_{G\|F}$	$K_{G\|S}$
0.5234	0	1	0.8064	0.6954	0

4.3 Joint-Type Complexity K_J

Joint-Type Complexity K_J is that associated with the type of LKPs used in a kinematic chain. We define this complexity as

$$K_J = \frac{1}{n}\left(n_R K_{G|R} + n_P K_{G|P} + n_C K_{G|C} + n_F K_{G|F} + n_S K_{G|S} + n_H K_{G|H}\right) \tag{16}$$

where n_R, n_P, n_C, n_F, n_S and n_H are the numbers of revolute, prismatic, cylindrical, planar, spherical and helical joints, respectively, while n is the total number of pairs and $K_{G|\mathrm{x}}$ is the geometric complexity of the pair x, as recorded in Table 1.

4.4 Link Diversity K_B

At the conceptual design stage, partial information about the geometric relations between neighboring joints is available. However, this partial information suffices to allow us to distinguish five possible link topologies (Figure 6), as the relative layout between its two associated joint axes defines a binary link[2]. We use again the concept of entropy to evaluate the effect of geometric

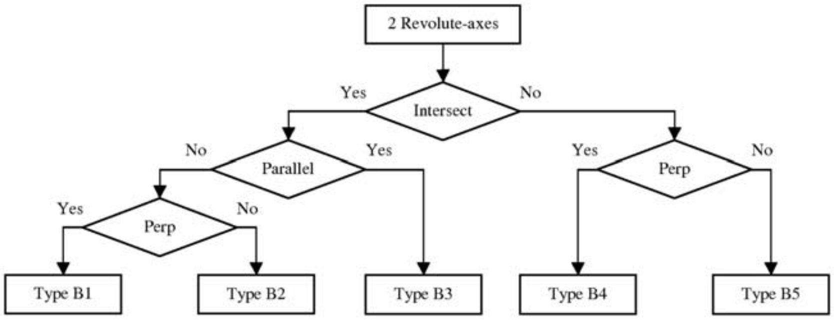

Figure 6: Binary tree displaying possible link topologies

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[2] Ternary and higher-order links can be accommodated, but we will leave the discussion of these aside in the interest of brevity. As well, we assume only revolute joints in this brief discussion.

diversity. In this vein we define the geometric diversity as:

$$K_B = \frac{B}{B_{max}} \tag{17a}$$

$$B = -\sum_{i=1}^{c} b_i \log_2(b_i)\,; \quad b_i = \frac{M_i}{\sum_{i=1}^{c} M_i} \tag{17b}$$

where B is the entropy of the link topologies and B_{max} is the maximum possible value of B—$B_{max} = \log_2(5) = 2.32$ bits—while c is the number of distinct joint-relation types—as displayed in Fig. 6—used in a concept, and M_i is the number of instances of each type of joint-relation.

4.5 Actuator-Type Complexity K_A

The actuator-type complexity is defined as

$$K_A = 1 - \exp(-q_A A); \quad A = a - a_m \tag{18}$$

where a is the number of electromagnetic actuators in the robot topology at hand, while a_m is the minimum number of electromagnetic actuators allowed.

4.6 Complexity due to Actuator Diversity K_H

The concept of entropy can be used again to evaluate the complexity due to actuator diversity. We define this complexity as:

$$K_H = \frac{H}{H_{max}} \tag{19}$$

for one design alternative, as defined in eqs.(2)–(5), where H is the entropy of the set of actuators and H_{max} is the maximum possible value of H, attained when no two actuators are identical. We thus have, for the worst-case scenario,

$$\phi_i = \frac{1}{N_c}, \quad i = 1, \ldots, \nu$$

whence

$$H_{max} = -\sum_{i=1}^{\nu} \frac{1}{N_c} \log_2\left(\frac{1}{N_c}\right) \tag{20}$$

in which ν is the number of distinct actuator types or sizes and N_i is the number of instances of each type or specification.

4.7 The Overall Complexity of a Robot Design

We define the complexity $K \in [0, 1]$ of a robot as a convex combination [12] of its various complexities:

$$K = w_N K_N + w_L K_L + w_J K_J + w_B K_B + w_A K_A + w_H K_H \tag{21}$$

where w_N, w_L, w_J, w_B, w_A and w_H denote their corresponding weights, such that

$$w_N + w_L + w_J + w_B + w_A + w_H = 1$$

These weights must be assigned by the designer based on the type of functions for which the robot is designed. Various methods have been proposed in the literature to select the relative weights [2].

5 Conclusions

We proposed *complexity* as a measure of the *diversity* content of a design alternative at the conceptual stage of the robot-design process. The measure proposed here is computable with the information content encountered in the mathematical theory of communication and entropy in statistical thermodynamics. Finally, the concept was illustrated by examples from the design of Schönflies-Motion Generators.

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