FUNDAMENTALS OF ROBOTIC MECHANICAL SYSTEMS
Third Edition

Theory, Methods, and Algorithms

Errata

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`Introduction: In order to ease the finding of items in this document, we have kept the page format and the original fonts of the book; we have also typeset with typewriter font---the one used in this Introduction---text that does not belong to the book.`

p. 62: `Text from eq.(2.98) and up to the paragraph below eq.(2.110) should read:`

$$\{\mathbf{T}\}_{\mathcal{A}} \equiv \begin{bmatrix} [\,\mathbf{Q}\,]_{\mathcal{A}} & [\,\mathbf{b}\,]_{\mathcal{A}} \\ \mathbf{0}^T & 1 \end{bmatrix} \tag{2.98}$$

where $\mathbf{0}$ is a three-dimensional array of zeros; neither the latter nor the real unity need be expressed in any particular frame. A similar expression follows for $\{\mathbf{T}\}_{\mathcal{B}}$, if with the two subscripts in the right-hand side of the above expression replaced with $\mathcal{B}$. Furthermore, as the reader can readily realize from Fig. 2.6,

$$[\,\mathbf{b}\,]_{\mathcal{B}} = -[\,\mathbf{Q}^T\,]_{\mathcal{B}}[\,\mathbf{b}\,]_{\mathcal{A}}$$

and hence,

Theorem 2.5.5 *The representations of $\{\mathbf{T}\}$ carrying coordinates in frame $\mathcal{B}$ into coordinates in frame $\mathcal{A}$, in these two frames, are related by:*

$$\{\mathbf{T}\}_{\mathcal{B}} = \begin{bmatrix} [\,\mathbf{Q}\,]_{\mathcal{A}} & -[\,\mathbf{Q}^T\,]_{\mathcal{A}}[\,\mathbf{b}\,]_{\mathcal{A}} \\ \mathbf{0}^T & 1 \end{bmatrix} \equiv \{\,\mathbf{R}^T\,\}_{\mathcal{A}}\{\mathbf{T}\}_{\mathcal{A}}\{\mathbf{R}\}_{\mathcal{A}} \tag{2.99}$$

where Theorem 2.5.2 has been invoked, while $\{\mathbf{R}\}_{\mathcal{A}}$ is defined as

$$\{\mathbf{R}\}_{\mathcal{A}} \equiv \begin{bmatrix} -[\,\mathbf{Q}\,]_{\mathcal{A}} & \mathbf{0} \\ \mathbf{0}^T & 1 \end{bmatrix} \tag{2.100}$$

It will become apparent in Section 2.6 that eq.(2.99) expresses a *similarity transformation* between $\{\mathbf{T}\}_{\mathcal{A}}$ and $\{\mathbf{T}\}_{\mathcal{B}}$.

Further, the inverse transformation of that defined in eq.(2.98), carrying $\mathcal{A}$-coordinates into $\mathcal{B}$-coordinates, can be derived from eq.(2.94b), thus obtaining

$$\{\,\mathbf{T}^{-1}\,\}_{\mathcal{B}} = \begin{bmatrix} [\,\mathbf{Q}^T\,]_{\mathcal{B}} & -[\,\mathbf{Q}^T\,]_{\mathcal{B}}[\,\mathbf{b}\,]_{\mathcal{A}} \\ \mathbf{0}^T & 1 \end{bmatrix} \equiv \begin{bmatrix} [\,\mathbf{Q}^T\,]_{\mathcal{B}} & -[\,\mathbf{b}\,]_{\mathcal{B}} \\ \mathbf{0}^T & 1 \end{bmatrix} \tag{2.101}$$

Notice that the foregoing inverse appears naturally in $\mathcal{B}$, rather than in $\mathcal{A}$.

Furthermore, homogeneous transformations can be concatenated. Indeed, let $\mathcal{F}_k$, for $k = i-1$, i, $i+1$, denote three coordinate frames, with origins at O_k. Moreover, let $\mathbf{Q}_{i-1}$ be the rotation carrying $\mathcal{F}_{i-1}$ into an orientation coinciding with that of $\mathcal{F}_i$. If a similar definition for $\mathbf{Q}_i$ is adopted, then $\mathbf{Q}_i$ denotes the rotation carrying $\mathcal{F}_i$ into an orientation coinciding with that of $\mathcal{F}_{i+1}$. First, the case in which all three origins coincide is considered. Clearly,

$$[\,\mathbf{p}\,]_i = [\,\mathbf{Q}_{i-1}^T\,]_{i-1}[\,\mathbf{p}\,]_{i-1} \tag{2.102}$$

$$[\,\mathbf{p}\,]_{i+1} = [\,\mathbf{Q}_i^T\,]_i[\,\mathbf{p}\,]_i = [\,\mathbf{Q}_i^T\,]_i[\,\mathbf{Q}_{i-1}^T\,]_{i-1}[\,\mathbf{p}\,]_{i-1} \tag{2.103}$$

the inverse relation of that appearing in eq.(2.103) being

$$[\,\mathbf{p}\,]_{i-1} = [\,\mathbf{Q}_{i-1}\,]_{i-1}[\,\mathbf{Q}_i\,]_i[\,\mathbf{p}\,]_{i+1} \tag{2.104}$$

If now the origins do not coincide, let $\mathbf{a}_{i-1}$ and $\mathbf{a}_i$ denote the vectors $\overrightarrow{O_{i-1}O_i}$ and $\overrightarrow{O_iO_{i+1}}$, respectively. The homogeneous-coordinate transformation matrices $\{\mathbf{T}_{i-1}\}_{i-1}$ and $\{\mathbf{T}_i\}_i$ thus arising are, apparently,

$$\{\mathbf{T}_{i-1}\}_{i-1} = \begin{bmatrix} [\,\mathbf{Q}_{i-1}\,]_{i-1} & [\,\mathbf{a}_{i-1}\,]_{i-1} \\ \mathbf{0}^T & 1 \end{bmatrix}, \quad \{\mathbf{T}_i\}_i = \begin{bmatrix} [\,\mathbf{Q}_i\,]_i & [\,\mathbf{a}_i\,]_i \\ \mathbf{0}^T & 1 \end{bmatrix} \tag{2.105}$$

Further, let $\mathbf{p}_i$ denote vector $\overrightarrow{O_iP}$ *in any frame.* The transformations of the components of this vector are, thus,

$$\{\mathbf{p}_{i-1}\}_{i-1} = \{\mathbf{T}_{i-1}\}_{i-1}\{\mathbf{p}_i\}_i \tag{2.106}$$

$$\{\mathbf{p}_{i-1}\}_{i-1} = \{\mathbf{T}_{i-1}\}_{i-1}\{\mathbf{T}_i\}_i\{\mathbf{p}_{i+1}\}_{i+1} \tag{2.107}$$

the corresponding inverse transformations being

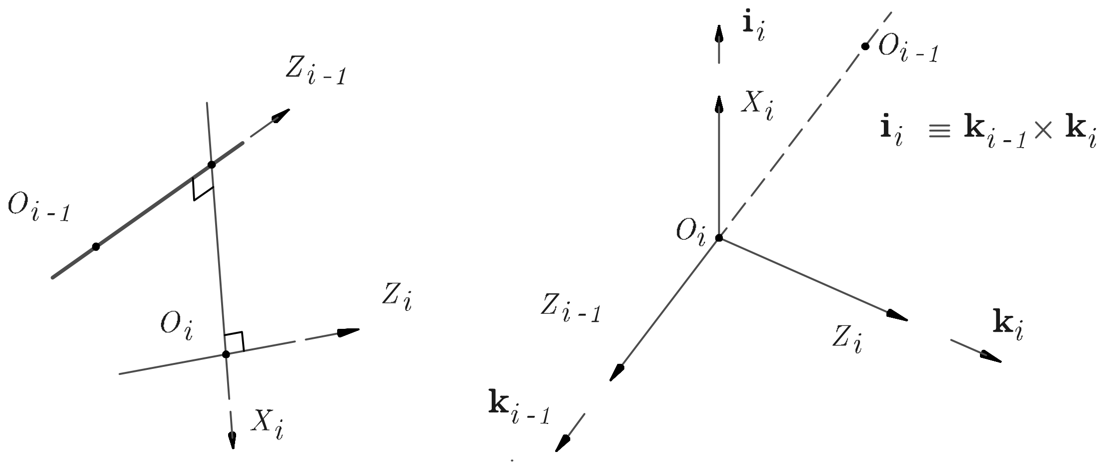
$$\{\,\mathbf{p}_i\,\}_i = \{\mathbf{T}_{i-1}\}_{i-1}^{-1}\{\,\mathbf{p}_{i-1}\,\}_{i-1} \tag{2.108}$$

$$\{\,\mathbf{p}_{i+1}\,\}_{i+1} = \{\mathbf{T}_i\}_i^{-1}\{\,\mathbf{p}_i\,\}_i = \{\mathbf{T}_i\}_i^{-1}\{\mathbf{T}_{i-1}\}_{i-1}^{-1}\{\,\mathbf{p}_{i-1}\,\}_{i-1} \tag{2.109}$$

Moreover, if $\mathcal{F}_i$ and $\mathcal{F}_{i+1}$ are regarded as $\mathcal{A}$ and $\mathcal{B}$, respectively, in eq.(2.101), then

$$\{\mathbf{T}_i\}_i^{-1} = \begin{bmatrix} [\,\mathbf{Q}_i^T\,]_{i+1} & -[\,\mathbf{Q}_i^T\,]_{i+1}[\,\mathbf{a}_i\,]_i \\ \mathbf{0}^T & 1 \end{bmatrix} \equiv \begin{bmatrix} [\,\mathbf{Q}_i^T\,]_i & -[\,\mathbf{Q}_i^T\,]_i[\,\mathbf{a}_i\,]_i \\ \mathbf{0}^T & 1 \end{bmatrix} \tag{2.110}$$

p. 132: `Figures 4.2(a) & (b) should be replaced by:`



(a) (b)

p. 151: `Equation displayed should read:`

$$a_1 = a_3 = 0, \quad b_1 = b_2 = b_3 = 0, \quad \alpha_1 = 90^\circ, \quad \alpha_2 = 0^\circ, \quad \alpha_3 = 90^\circ$$

p. 152: `First equation displayed should read:`

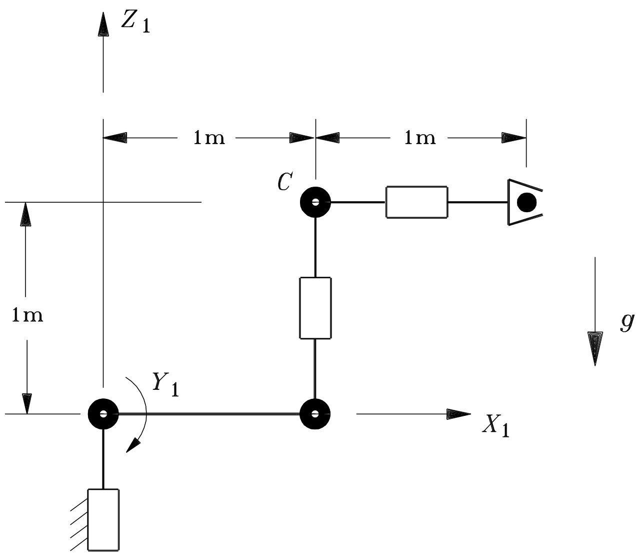
$$C = 0, \quad D = 2a_2b_4, \quad E = a_2^2 + b_4^2 - \|\mathbf{c}\|^2$$

p. 153: First equation displayed should read:

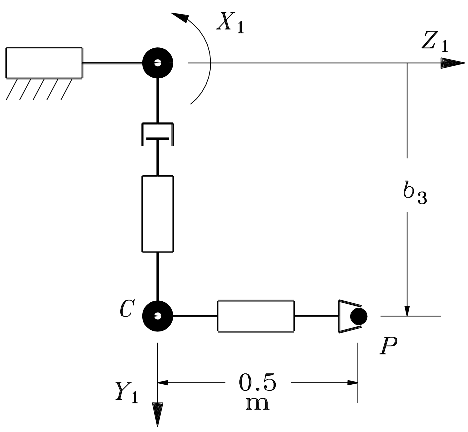
$$(\tau_3)_{1,2} = \frac{-2a_2b_4 \pm \sqrt{4a_2^2b_4^2 - (a_2^2 + b_4^2 - \|\mathbf{c}\|^2)^2}}{a_2^2 + b_4^2 - \|\mathbf{c}\|^2}$$

p. 162: In the problem statement for Exercice 4.2, "section 4.3" should read "section 4.4"

p. 227: Exercise 5.2 makes reference to a specific posture (configuration in the text) of the robot of Fig. 4.19, but that figure shows an arbitrary posture. The posture in question is that of the figure below:



p. 229: Exercise 5.8 makes reference to a specific posture (configuration in the text) of the robot of Fig. 4.20, but that figure shows an arbitrary posture. The posture in question is shown in the figure below:



p. 278: Algorithm 7.4.2 (Inward Recursions) should read:

$$
\begin{array}{lll}
\mathbf{f}_n^P & \leftarrow & m_n\ddot{\mathbf{c}}_n - \mathbf{f} \\
\mathbf{n}_n^P & \leftarrow & \mathbf{I}_n\dot{\boldsymbol{\omega}}_n + \boldsymbol{\omega}_n \times \mathbf{I}_n\boldsymbol{\omega}_n - \mathbf{n} + \boldsymbol{\rho}_n \times \mathbf{f}_n^P \\
\texttt{If R then} & & \\
\tau_n & \leftarrow & (\mathbf{Q}_n\mathbf{n}_n^P)_z \\
\texttt{else} & & \\
\tau_n & \leftarrow & (\mathbf{Q}_n\mathbf{f}_n^P)_z \\
\texttt{For i} = \texttt{n} - \texttt{1 to 1 step} -\texttt{1 do} & & \\
\quad \boldsymbol{\phi}_{i+1} & \leftarrow & \mathbf{Q}_{i+1}\mathbf{f}_{i+1}^P \\
\quad \mathbf{f}_i^P & \leftarrow & m_i\ddot{\mathbf{c}}_i + \boldsymbol{\phi}_{i+1} \\
\quad \mathbf{n}_i^P & \leftarrow & \mathbf{I}_i\dot{\boldsymbol{\omega}}_i + \boldsymbol{\omega}_i \times \mathbf{I}_i\boldsymbol{\omega}_i + \boldsymbol{\rho}_i \times \mathbf{f}_i^P + \mathbf{Q}_{i+1}\mathbf{n}_{i+1}^P + \boldsymbol{\delta}_i \times \boldsymbol{\phi}_{i+1} \\
\texttt{If R then} & & \\
\tau_i & \leftarrow & (\mathbf{Q}_i\mathbf{n}_i^P)_z \\
\texttt{else} & & \\
\tau_i & \leftarrow & (\mathbf{Q}_i\mathbf{f}_i^P)_z \quad \texttt{enddo}
\end{array}
$$

Note that, within the do-loop of the foregoing algorithm, the vectors to the left of the arrow are expressed in the $(i+1)$st frame, while $\mathbf{f}_{i+1}^P$ and $\mathbf{n}_{i+1}^P$, to the right of the arrow, are expressed in the $(i+2)$nd frame.

p. 284: Third line should read:

as explained in Chapter 4. Furthermore, ...

The third displayed equation should read:

$$\boldsymbol{\tau} = \mathbf{I}(\boldsymbol{\theta})\ddot{\boldsymbol{\theta}} + \mathbf{C}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}})\dot{\boldsymbol{\theta}} = \frac{1}{T^2}[s''(x)\mathbf{I}(x)\Delta\boldsymbol{\theta} + {s'}^2(x)\mathbf{C}(x, \Delta\boldsymbol{\theta})\Delta\boldsymbol{\theta}] \equiv \frac{1}{T^2}\mathbf{f}(x)$$

p. 298: The caption of Fig. 7.7 makes reference to Fig. 4.19, which should be Fig. 4.15:

p. 306: First and third components of the vector displayed are swapped. Correct display should be:

$$\mathbf{C}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}})\dot{\boldsymbol{\theta}} = \begin{bmatrix} -Ip^2 + (1/2)a^2mp^2 \\ Ip^2 - (1/4)a^2mp^2 \\ -Ip^2 + a^2mp^2 \end{bmatrix}$$

p. 316 Exercise 7.3 makes reference to a specific posture of the robot of Fig. 4.19, but that figure shows an arbitrary posture. The posture in question is that displayed in the erratum item of p. 227.

p. 321 Exercise 7.13 makes reference to algorithm 7.1, which should read: algorithm 7.6.1 found in p. 308

p. 358 `All` d `in Table. 9.1 should read` b`, i.e.,`

Item	Expression	Item	Expression
f_1	$c_4\iota_1 + s_4\iota_2 + a_3$	r_1	$c_4 m_1 + s_4 m_2$
f_2	$-\lambda_3(s_4\iota_1 - c_4\iota_2)$ $+\mu_3\iota_3$	r_2	$-\lambda_3(s_4 m_1 - c_4 m_2)$ $+\mu_3 m_3$
f_3	$\mu_3(s_4\iota_1 - c_4\iota_2)$ $+\lambda_3\iota_3 + b_3$	r_3	$\mu_3(s_4 m_1 - c_4 m_2)$ $+\lambda_3 m_3$
ι_1	$c_5 a_5 + a_4$	m_1	$s_5\mu_5$
ι_2	$-s_5\lambda_4 a_5 + \mu_4 b_5$	m_2	$c_5\lambda_4\mu_5 + \mu_4\lambda_5$
ι_3	$s_5\mu_4 a_5 + \lambda_4 b_5 + b_4$	m_3	$-c_5\mu_4\mu_5 + \lambda_4\lambda_5$
h_1	$c_1 p + s_1 q - a_1$	n_1	$c_1 u + s_1 v$
h_2	$-\lambda_1(s_1 p - c_1 q)$ $+\mu_1(r - b_1)$	n_2	$-\lambda_1(s_1 u - c_1 v)$ $+\mu_1 w$
h_3	$\mu_1(s_1 p - c_1 q)$ $+\lambda_1(r - b_1)$	n_3	$\mu_1(s_1 u - c_1 v)$ $+\lambda_1 w$
p	$-l_x a_6 - (m_x\mu_6 + n_x\lambda_6)b_6$ $+p_x$	u	$m_x\mu_6 + n_x\lambda_6$
q	$-l_y a_6 - (m_y\mu_6 + qn_y\lambda_6)b_6$ $+p_y$	v	$m_y\mu_6 + n_y\lambda_6$
r	$-l_z a_6 - (m_z\mu_6 + n_z\lambda_6)b_6$ $+p_z$	w	$m_z\mu_6 + n_z\lambda_6$

p. 361 `First two sentences of last paragraph should read:`

Additional equations free of θ_1 and θ_2 can be derived from any six of the eight equations in eq.(9.37b). Indeed, these six equations is all that is needed to solve for $\tilde{\mathbf{x}}_{12}$ in terms of θ_3, θ_4 and θ_5; the expressions thus resulting would be then substituted into the remaining two equations of the same set, to obtain two additional equations free of θ_1 and θ_2.