The Kinematics and Dynamics of Parallel Schönflies-Motion Generators

Jorge Angeles

Department of Mechanical Engineering
Centre for Intelligent Machines
McGill University
Montreal, QC, Canada
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3 Manipulator Architecture

4 Kinetostatic Design

5 Dynamics

6 Conclusions
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What is a Schönflies Motion?
Objectives

- Design a parallel manipulator, which outperforms current SMG (serial and parallel SCARA)
- Pick and place operations and machining
Objectives

- Design a parallel manipulator, which **outperforms** current SMG (serial and parallel SCARA)
- **Pick and place** operations and **machining**
Specifications

- The robot is expected to beat the record-setting 500-ms cycle time.

- Kinetostatic robustness
- Identical motors fixed to the base
- Large workspace
- High stiffness
Manipulator Architecture

2RΠΠR

Drive Unit
Animation
Kinetostatics

- Mechanical analysis of rigid-body mechanical systems moving under static, conservative conditions
- Relations between the feasible twists—point-velocity and angular velocity—and the constraint wrenches—force and moment—pertaining to the various links of a kinematic chain
- Kinetostatic design $\equiv$ dimensioning of the links under kinetostatic conditions
Kinetostatic Design

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Jacobian matrices

\[ At = B \dot{\theta} \]  

- \( \mathbf{t} \) is the twist
- \( \dot{\theta} \) is the vector of active joint rates
- \( \mathbf{A} \) is the forward Jacobian
- \( \mathbf{B} \) is the inverse Jacobian

\[ \mathbf{A} \equiv \begin{bmatrix} \mathbf{A}_{/} \\ \mathbf{A}_{//} \end{bmatrix} \in \mathbb{R}^{6 \times 4}, \quad \mathbf{B} \equiv \begin{bmatrix} \mathbf{B}_{/} & \mathbf{O}_{32} \\ \mathbf{O}_{32} & \mathbf{B}_{//} \end{bmatrix} \in \mathbb{R}^{6 \times 4} \]  

- Redundancy is introduced to add robustness to the model and to avoid formulation singularities
Jacobian matrices

\[ At = B \dot{\theta} \quad (1) \]

- \( t \) is the twist
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\[ A \equiv \begin{bmatrix} A_I \\ A_{II} \end{bmatrix} \in \mathbb{R}^{6\times4}, \quad B \equiv \begin{bmatrix} B_I \\ O_{32} \\ B_{II} \end{bmatrix} \in \mathbb{R}^{6\times4} \quad (2) \]

- Redundancy is introduced to add robustness to the model and to avoid formulation singularities
Singularity Analysis

Two types of singularities

- \( \text{Rank}(A) < 4 \Rightarrow \text{parallel singularity} \)
- \( \text{Rank}(B) < 4 \Rightarrow \text{serial singularity} \)
Singularity Analysis

Two types of singularities

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Singularity Analysis

Serial singularities

(a) (b) (c)
Singularity Analysis

Parallel singularities

(a) (b)
Dimensioning

A 300 mm
B
C 25 mm
D

Trajectory Planning

SMG Dynamics

Drive Dynamics

Motor and Gear-Head Selection

OK?

Yes
Optimum Dimensions

No
Trajectory Optimization

Use a combination of 4-5-6-7 polynomials
Displacement along X and Y

\[ z \text{ [mm]} \]

\[ x \text{ or } y \text{ (pan or normal plane) [mm]} \]

\[ t \text{ [s]} \]

\[ \dot{x} \text{ or } \dot{y} \text{ [mm/s]} \]

\[ \ddot{x} \text{ or } \ddot{y} \text{ [mm/s}^2\text{]} \]
Displacement along Z

\[ \begin{align*}
  x \text{ or } y \text{ (pan or normal plane)} & \text{ [mm]} \\
  z & \text{ [mm]} \\
  t & \text{ [s]} \\
  \dot{z} & \text{ [mm/s]} \\
  \ddot{z} & \text{ [mm/s}^2]\end{align*} \]
Orientation of the end-effector

\[ \phi \text{ [rad]} \]

\[ \dot{\phi} \text{ [rad/s]} \]

\[ \ddot{\phi} \text{ [rad/s}^2] \]

\[ x \text{ or } y \text{ (pan or normal plane) [mm]} \]

\[ 5\pi/12 \text{ to } -5\pi/12 \]

\[ 0 \text{ to } 2 \]

\[ -200 \text{ to } 200 \]

\[ 0 \text{ to } 0.4 \]

\[ -2 \text{ to } 2 \]

\[ -1000 \text{ to } 1000 \]

\[ -500 \text{ to } 500 \]
SMG Model

SMG

\[ \omega_2 = \dot{\theta}_{11} \mathbf{k} \]
\[ \omega_4 = \dot{\theta}_{111} \mathbf{k} \]

system of five rigid bodies coupled by massless joints undergoing Schönflies displacements
Equations of motion

Natural Orthogonal Complement:

\[ I(\theta)\ddot{\theta} + C(\theta, \dot{\theta})\dot{\theta} = \tau + \gamma - \delta \]  

- \( I : 4 \times 4 \) inertia matrix
- \( C : 4 \times 4 \) matrix of Coriolis and centrifugal forces
- \( \theta : \) the four-dimensional vector of actuated joint variables
- \( \tau : \) the four-dimensional vector of actuated joint torques
- \( \gamma : \) the four-dimensional vector of gravity forces
- \( \delta : \) the four-dimensional vector of dissipative forces, which are neglected
Equations of motion

- **Inertia matrix:**
  \[ I = I_A + I_B + I_C \]  
  \[ I_i = T_i^T M_i T_i, \quad i = A, B, C \]  
  \( A \) and \( B \) being the two elbows and \( C \) the end-plate of the manipulator.

- **Matrix of Coriolis and centrifugal forces:**
  \[ C(\theta, \dot{\theta}) \dot{\theta} = T_A^T M_A T_A \dot{\theta} + T_B^T M_B T_B \dot{\theta} + T_C^T M_C T_C \dot{\theta} \]  

- **Gravity forces:**
  \[ \gamma = \sum_{i=A, B, C} T_i^T w_i^G \]  
  \[ w_i^G = \begin{bmatrix} 0_3 \\ m_i g \end{bmatrix}, \quad i = A, B, C \]
Pan and Tilt: Velocities and Torques

(a) PAN velocities

(b) TILT velocities

(c) PAN torques

(d) TILT torques
**Drive Units**

**Kinematics**

\[ \omega_{in} = J_D^{-1} \omega_{out} \]  

\[ \begin{align*}  
\omega_{in} &= \begin{bmatrix} \omega_A \\ \omega_B \end{bmatrix}, \\
\omega_{out} &= \begin{bmatrix} \omega_p \\ \omega_t \end{bmatrix} 
\end{align*} \]

\[ J_D^{-1} = -r \begin{bmatrix} 1 & -r_{6,5} \\ 1 & r_{6,5} \\ 1 & 1 + 2r_{6,5} \end{bmatrix} \]
Dynamics

\[ T = \frac{1}{2} l_S \omega_S^2 + \frac{1}{2} l_R \omega_R^2 + \frac{1}{2} l_C \omega_C^2 + 3 \frac{1}{2} l_P \omega_P^2 + \frac{3}{2} m_P \left( \frac{d_C}{2} \omega_C \right)^2 \]  

\[ \text{Lagrange equations:} \]

\[ \frac{d}{dt} \frac{\partial T}{\partial \dot{q}_i} - \frac{\partial T}{\partial q_i} = \tau_{SR} - J_D^T \tau_{CP}, \quad i = 1, 2 \]  

\[ \tau_{SR} = I_D \ddot{q} + J_D^T \tau_{CP} \]
Sun and Ring: Velocities and Torques

(a) SUN Velocities

(b) RING Velocities

(c) SUN Torques

(d) RING Torques
## Motor and Gearhead specifications

### Table: Peak requirements

<table>
<thead>
<tr>
<th>$\bar{\omega}_S$ [rpm]</th>
<th>$\bar{\tau}_S$ [Nm]</th>
<th>$\bar{\omega}_R$ [rpm]</th>
<th>$\bar{\tau}_R$ [Nm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>192</td>
<td>72</td>
<td>179</td>
<td>84</td>
</tr>
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</table>

### Table: Motor and Gearhead specifications

<table>
<thead>
<tr>
<th>Gear head ratio</th>
<th>$\bar{\omega}_M$ [rpm]</th>
<th>$\bar{\tau}_M$ [Nm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>5000</td>
<td>7.1</td>
</tr>
</tbody>
</table>
Conclusions

- A novel parallel robot generator of Schönflies motions was introduced
- Link-dimensioning was based on kinetostatic conditions
- Kinematics modelled using $6 \times 4$ Jacobian matrices for robustness against formulation singularities
- Motor and gearhead selection were based on dynamics model
- Dynamics model was based on a MBS of five rigid links coupled by massless joints
- Dynamics model is a 4-dimensional systems of second-order ODEs
- Currently undergoing tests and adjustments to reach a 500 ms (or shorter) cycle time
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