Towards a Theory of Engineering Design

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Outline

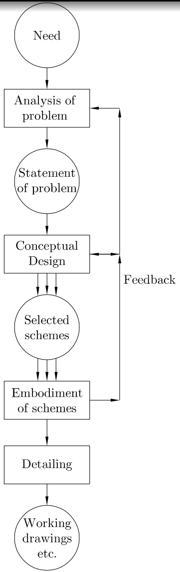
Theory of Engineering Design

- The *theory* supporting the engineering design activity ✓
- The *methodology* applied to accomplish the activity

The concept of *theory*:

- "Theory" is a fairly general concept, to the point that the *Oxford Dictionary of Philosophy* does not include it as such, but always preceded by a qualifier or followed by a complement as in "theory of knowledge"
- Bertrand Russell (1972) offers an extensive account of the evolution of the word "theory," starting with its origins as "an Orphic word, which Cornford interprets as 'passionate sympathetic contemplation,' " but stays short of giving a modern interpretation of the word
- Wikipedia: "**Theory** is a contemplative and rational type of abstract or generalizing thinking, or the results of such thinking"

Three main design schools can be cited:
Systematic Design; *Axiomatic Design*; and *Robust Engineering*



- The first stage:
  to produce a) design requirements, and
  b) *design specifications* ⇒ *Disciplinary knowledge*
- The second stage:
  to produce a set of *design variants*
- The third stage:
  to produce a preliminary model of the design solution, and a *parametric model* of each of the short-listed candidate variants (*multiobjective optimization*)

} *Synthesis*

- The final stage:
  materials selection, manufacturing issues, and production-cost analysis
  ⇒ Discipline-dependent

• : Design process model after French

The “German School”

- Finds its roots in the work of Rodenacker (1968–69), who proposed a general framework under the title “Methodisches Konstruiren,” translated as “Systematic Design”
- Rodenacker was preceded by Releaux (Moon, 2003), who first proposed a *language* to describe symbolically, in a compact form, the layout of a mechanism, and set up rules to generate mechanisms based on “elementary operations”
- A major contribution of this school is the formalization of the design process, which has had a major influence worldwide
- VDI, the Association of German Engineers, has issued a *guideline* for the design and development of engineering systems and products, that is based on the principles of systematic design

At the conceptual stage of the design process:
• a key step is the definition of *functions* and their decomposition into *subfunctions*, to be met by *function-carriers*, or *subfunction-carriers*—the concept of function decomposition and the terminology function-carrier are taken from Systematic Design

Suh's methodology relies largely on two *axioms*:

The Independence Axiom

The best design is one in which all functions are independent

- Questionable: there are instances in both engineering and nature where the same function-carrier satisfies more than one function

The Minimum-Information Axiom

The best design is the one containing the minimum amount of information

- Suh borrows from Shannon's *mathematical theory of communication* the concept of *information content*
- What Shannon's information content actually measures is the *ambiguity* in a message, which should be, ideally, eliminated, or at least, minimized
- The information content of a message is defined by Shannon as

$$I = -\sum_{1}^{n} p_i \log(p_i), \quad \sum_{1}^{n} p_i = 1$$

p_i: the probability of choosing one communication item out of n possible choices

Example: spelling a name by telephone: *Anne*

A can be understood as A (rightly), but also as $H \Rightarrow$ a probability of 1/2 for being correctly understood

N can be understood as N (rightly), but also as $M \Rightarrow$ a probability of 1/2 for being correctly understood

E can be understood as E (rightly), but also as B, C, D, G, P, T, $V \Rightarrow$ a probability of 1/8 for being correctly understood

Thus, the information content of each of the four sub-messages are

$$I_A = -\sum_{1}^{2} \frac{1}{2} \log_2(\frac{1}{2}) = 2 \times \frac{1}{2} \times 1 = 1.0,$$

$$I_N = -\sum_{1}^{2} \frac{1}{2} \log_2(\frac{1}{2}) = 2 \times \frac{1}{2} \times 1 = 1.0,$$

$$I_E = -\sum_{1}^{8} \frac{1}{8} \log_2(\frac{1}{8}) = 8 \times \frac{1}{8} \times 3 = 3.0$$

Example: spelling a name by telephone: *Anne* (Cont'd)

Hence, the total information content of the four sub-messages becomes

$$I = I_A + 2 \times I_N + I_E = 1.0 + 2 \times 1.0 + 3.0 = 6 \text{ bits}$$

If, however, the same name is spelled using the International Alphabet, displayed in the table, then, the sequence is *Alpha*, *November*, *November*, *Echo*, which removes every possible ambiguity; hence, $p_i = 1$, for $i = 1, 2, 3$, or $I_A = I_N = I_E = 0$, thereby leading to $I = 0$, i.e., a complete elimination of ambiguity

alpha	hotel	Oscar	Victor
bravo	India	papa	whiskey
Charlie	Juliett	Quebec	xray
delta	kilo	Romeo	yankee
echo	Lima	sierra	zulu
foxtrot	Mike	tango	
golf	November	uniform	

• : The International Alphabet

- Applying the above concept to design is not obvious
- Suh proposes to regard p_i in Shannon's formula as the probability of meeting a tolerance specification
- Suh defines the *functional requirements* (FR) of the design task and the *design parameters* (DP) to be determined by the designer

Problems with Suh's paradigm

- At this stage there is no mathematical model $\Rightarrow$ no quantitative relations between FR and DP are available
- According to Suh's paradigm, a design satisfies the Independence Axiom when the design matrix is both square and *diagonal.*

  P: a diagonal design matrix with diagonal entries of quite disparate orders of magnitude implies a high sensitivity of the design functions to changes in the design-parameter values $\Rightarrow$ *matrix condition number*

Problems with Suh's paradigm (Cont'd)

- in Axiomatic Design, two scalar metrics, *reangularity* (R) and *semangularity* (S) are introduced, as means to quantify the *degree of coupling* between FR and DP
  - R is 0, the optimum case, when the square matrix at stake is orthogonal
  - Assuming that the design matrix is of $n \times n$, S can attain a minimum of n when the matrix is diagonal
  - technical problems: *diagonality* is not an intrinsic property of a square matrix, as any matrix with a *complete set* of eigenvectors can be diagonalized by means of a change of vector basis—a one-to-one change of variable—and *orthogonality* is not essential for a linear transformation to be "optimal" in some way

In fact, the optimality condition at stake can only be defined as the *matrix condition number*. Optimally-conditioned matrices, whether square or rectangular, are isotropic, i.e., their condition number is unity, the worst being *singular* or, correspondingly, *rank-deficient*, with an unbounded ($\rightarrow \infty$) condition number

Robust Engineering

- Taguchi (1993) developed his design framework, labeled *Robust Engineering*, in the fifties
- Motivation: to explore novel means of solving engineering problems. One of his first jobs as an engineer: to devise the right proportions of the ingredients of caramels—the problem faced by the caramel manufacturer was the transportation of the goods in the presence of a full spectrum of weather conditions in Japan, from the temperate south to the extreme north in the island of Hokkaido
- Taguchi realized that the key to the production of engineered goods was to design them with the least *sensitivity* to environmental conditions, which can be summarized as

> *Variations in the objective functions of products (or technologies) are primarily due to three sources: environmental effects, deteriorative effects, and manufacturing imperfections. The purpose of robust design is to make the products and the processes less [sic] sensitive to these effects.*

Robust Engineering

Two main concepts constitute the core of robust engineering:

1. *Signal-to-noise Ratio*: in every engineering task signal and noise invariably appear. Signal is the intended operation, the *design objective* in our context. Noise is any disturbance that cannot be predicted accurately, but whose statistics are known *to some extent.* Noise is closely related to *uncertainty*, which comes from the unpredictability of the specific conditions under which an engineered good, be it tangible or intangible, will operate, including the user herself. Therefore, a robust design exhibits a maximum signal-to-noise ratio
2. *Loss Function*: this is the loss to society caused by a faulty engineering work, a faulty design in our context. The loss function thus quantifies a negative aspect of the designed object; this quantity should be reduced to a minimum

Conceptual Design

- "Statement of Work" ⇒ to produce a rich set of variants of the object of design ⇒ *brainstorming*
- to base any evaluation of design alternatives in terms of the *complexity* of each *design variant*
- to regard the entropy of a message as the *complexity* of one design variant at the conceptual stage
- Here, we liken complexity to *diversity*: The more diverse the variant, the more complex it is ⇒ the higher the *diversity content* of an alternative solution to a design problem, the more *complex* the alternative

Conceptual Design

Some definitions drawn from the German School:

Function: A *generic* task imposed by the need to be satisfied by means of the object under design. Examples: fasten; separate; sort; support; transport; energize; entertain; actuate; sense; etc.

Function-carrier: A component or assembly intended to implement a function. Examples: a bolt, a welding seam or a screw are components that *carry* the function *fasten*

Design specification: A *quantitative* condition to be met by the object under design. Example: A system to transport persons and materials through a span of 0.5 km should serve 1000 people/h and transport 1000 ton/h of merchandise

Functions can have *subfunctions*

Example: The function *move* in the design of a wheeled mobile robot (WMR) can have the subfunctions *drive* and *steer*

Assume that a given function F entails N subfunctions $f_1, f_2, \ldots f_N$, with f_i to be implemented with a number ν_i of identical carriers $C_i \Rightarrow$ the total number N_c of function-carriers for F is $N_c = \nu_1 + \nu_2 + \ldots + \nu_N$

Further, let us denote with ϕ_i de *frequency* of occurrence of function-carrier C_i, namely,

$$\phi_i = \frac{\nu_i}{N_c}, \quad i = 1, \ldots, N, \quad 0 \le \phi_i \le 1 \Rightarrow \sum_1^N \phi_i = 1 \tag{1}$$

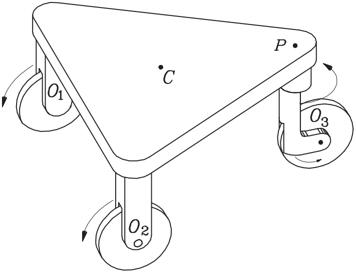
The frequency ϕ_i can play the role of the probability p_i in Shannon's formula $\Rightarrow$ We can define a *disorder* in the design-solution alternative at hand as the equivalent of the entropy $\Rightarrow$ refer to this *design entropy* as the *design complexity* K, namely,

$$K = -\sum_1^N \phi_i \log_2(\phi_i) \tag{2}$$

Example: The Conceptual Design of a Mobile Platform

The design task

Design a mobile platform capable of transporting pallets of finished products in a production plant.



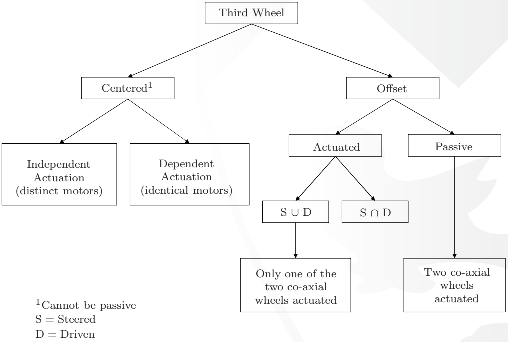
• : The basic structure of a tricycle under design

- with three *conventional* wheels of identical size and shape (minimum diversity of wheel models!)
- the third wheel is shown as offset, but this need not be the case: an alternative layout to implement roll & steering of the wheel will be proposed here
- the vehicle has a mobility of 2, as each wheel can only move under *rolling* (driving) and *turning* (steering)

Example: The Conceptual Design of a Mobile Platform (Cont'd)

Question: where to place the two motors?

- Three wheels and four components turn w.r.t. the platform—the three wheels plus the bracket
- Wheels 1 and 2: can only be actuated under driving (D);
- Wheel 3: can be both driven and steered; either centered or offset



• : Actuation alternatives for an autonomous tricycle

Example: The Conceptual Design of a Mobile Platform (Cont'd)

As per the decision tree, five variants are identified:

A Independent actuation: the third wheel is actuated by two motors to provide, independently, S and D, the motors thus being distinct. We have thus two functions, (1) S and (2) D $\Rightarrow \nu_1 = \nu_2 = 1$ and $N_C = 2$ $\Rightarrow \phi_1 = \phi_2 = 1/2 \Rightarrow$ the complexity K_A of this alternative is

$$K_A = -\sum_1^2 \frac{1}{2} \log_2 \left(\frac{1}{2} \right) = 1.0$$

B Dependent actuation: the two motors act in a coordinated fashion so as to produce both S and D concurrently. The two motors can thus be assumed to be identical $\Rightarrow$ We thus have two function-carriers for one single function $\Rightarrow \nu_1 = 2$ and $N_c = 2 \Rightarrow \phi_1 = 2/2 = 1 \Rightarrow$ The complexity K_B of this variant is

$$K_B = -\sum_1^1 1.0 \log_2(1.0) = 0$$

Example: The Conceptual Design of a Mobile Platform (Cont'd)

C S ∪ D ⇒ two sub-variants, each with $N_C = 2$ and two sub-functions, each with its own function-carrier ⇒ f_1 is "steer (S) the third wheel," and f_2 "drive (D) one of the two coaxial wheels." ⇒ $\nu_1 = \nu_2 = 1/2$ ⇒ The two sub-variants are

C.1 S the third (offset) wheel, while leaving free its D axis, and D one co-axial wheel. Hence,

$$K_{C.1} = -\sum_{1}^{2} \frac{1}{2} \log_2 \left(\frac{1}{2} \right) = 1.0$$

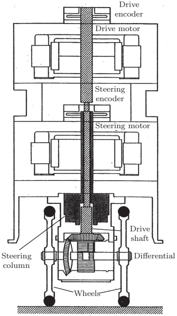
C.2 D the third (offset) wheel, while leaving free its S axis, and D one co-axial wheel—we have two distinct functions, even if the two actuated wheels are under D because the third wheel has its S axis idle, while the actuated co-axial wheel is blocked under S ⇒

$$K_{C.2} = -\sum_{1}^{2} \frac{1}{2} \log_2 \left(\frac{1}{2} \right) = 1.0$$

Example: The Conceptual Design of a Mobile Platform (Cont'd)

D S $\cap$ D, which leads to two functions, S and D $\Rightarrow N_C = 2$, each function with its own carrier $\Rightarrow f_1$ is S and f_2 is D the third wheel $\Rightarrow \nu_1 = \nu_2 = 1/2 \Rightarrow K_D = K_A = K_C = 1.0$

E Two co-axial wheels are actuated. In this case we have one single function, f_1, which is D the ith co-axial wheel, for $i = 1, 2 \Rightarrow N_C = 2$ and $\nu_1 = 2 \Rightarrow \phi_1 = 1$ and $K_E = K_B = 0$

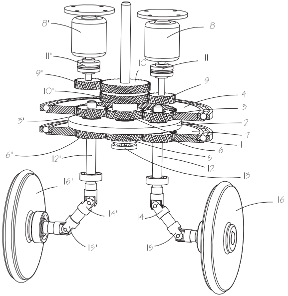


- : Moravec's implementation of **A**

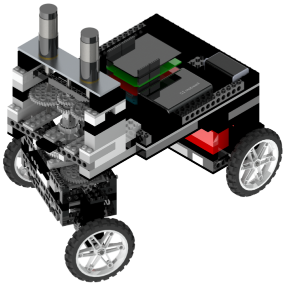
- Moravec (1983) implemented **A**, using three times this variant to produce an omnidirectional tricycle $\Rightarrow$ over-redundant actuation: six motors to actuate three degrees of freedom
- Angeles (2005) implemented **B** to produce a dual-wheel transmission

Example: The Conceptual Design of a Mobile Platform (Cont'd)

Variant **B**, embodied in what is dubbed the *Dual-wheel Transmission* (DWT):



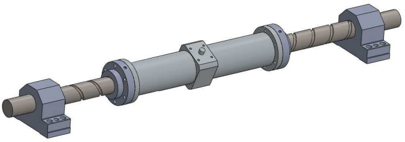
- : The dual-wheel transmission, a two-dof mechanism for independent driving and steering



- : A Lego Mindstorms prototype of the dual-wheel transmission

Example: The Conceptual Design of a Mobile Platform (Cont'd)

The DWT is a carrier of the *drive and steer* double function. Other instances of a double function occur in practice, e.g., *pitch and roll* (PR), *tilt and yaw* (TY, or "point," as in a *pointing operation*) and *slide and turn* (ST)



• : The C-drive, a two-dof mechanism that produces independent *slide* and *turn* functions under *coupling*

- One example of ST is the C-drive
- This drive is intended to produce sliding and turning of a collar, to drive a robotic link (Harada et al., 2014)

Example: The Conceptual Design of a Mobile Platform (Cont'd)

C-drive

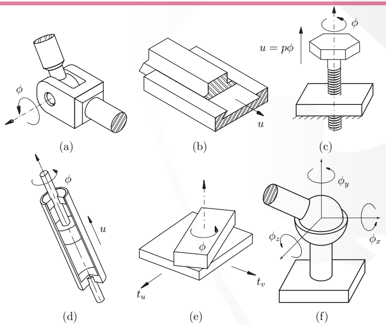
Pure rotation

Pure translation

Helix

Example: The Conceptual Design of Kinematic Chains

- *Kinematic chain*: a mechanical system composed of a set of rigid bodies, termed *links*, coupled by *lower kinematic pairs* (LKP)
- LKP: the coupling of two rigid links by means of a *wrapping action*—the two links share one common surface with certain symmetries that allow for relative motion in one, two or three independent directions



- : The six lower kinematic pairs: (a) revolute; (b) prismatic; (c) screw; (d) cylindrical; (e) planar; and (f) spherical

Example: The Conceptual Design of Kinematic Chains (Cont'd)

- A key decision to be made by the machine designer at the conceptual stage is the type of joints to be used to produce a certain motion
- The designer needs a hierarchical ordering of the LKP from "best to worst", based on an agreement of what a measure of "goodness" is in this context ⇒ *complexity*
- The key issue: to assign a complexity value to a *paradigm surface* characterizing each LKP that is based on the *curvature distribution* along the surface

• : An illustration of *surface complexity*

Example: The Conceptual Design of Kinematic Chains (Cont'd)

- A measure of complexity must be dimensionless ⇒ the ratio of the rms value of the derivative of the curvature, $\kappa'_{\rm rms}$, w.r.t. a *dimensionless variable* that grows monotonically with an arc length along the surface, to the rms value $\kappa_{\rm rms}$ of the curvature itself
- Drawing from Taguchi's concept of *loss function*, we define the *loss of regularity* (LOR) of a curve as the foregoing ratio, namely,

$$\mathrm{LOR} = \frac{\kappa'_{\rm rms}}{\kappa_{\rm rms}}$$

The paradigm surface associated with each of the first three LKP

- A revolute joint allowing for pure rotation about one axis, its wrapping surface must have one symmetry of rotation, but not of extrusion, which disqualifies a circular cylinder. The paradigm surface is generated upon rotating about the x-axis the polynomial $P_R(x) = -x^6 + 3x^4 - 3x^2 + 1.1132$, the value of the constant term being chosen so as to make the LOR a minimum, namely 10.3

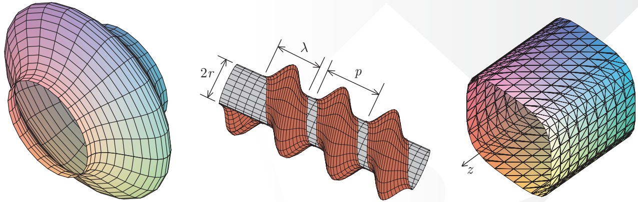
Example: The Conceptual Design of Kinematic Chains (Cont'd)

The paradigm surface associated with each of the first three LKP (Cont'd)

- A prismatic joint allowing for translation in one direction, its wrapping surface must have a symmetry of extrusion, but not of rotation. The paradigm surface is generated upon extruding a Lamé curve of 4th degree: $L_4(x, y) = x^4 + y^4 - 1 = 0$ in a direction normal to the x-y plane $\Rightarrow$ The Lamé curve was smooth everywhere, with G^2 continuity. The LOR for this surface was found to be 19.6802
- The screw joint allowing for both a rotation about an axis and a sliding along a direction parallel to the axis, with rotation and sliding related by the pitch p of the screw, the surface associated with this joint is obtained based on the polynomial $P_H(x) = -(x-p)^6 + 3(x-p)^4 - 3(x-p)^2 + r \Rightarrow$ a modified $P_R(x)$. Upon giving this polynomial a helical motion of pitch $p = 4.87r$ about the x-axis, the helicoidal surface is obtained $\Rightarrow$ the optimum value of the ratio, $p/r = 1$, was chosen so as to minimize the LOR of the resulting surface, the corresponding LOR value being 15.8702

Example: The Conceptual Design of Kinematic Chains (Cont'd)

Conclusion: The revolute joint is preferred over the screw joint, and the latter over the prismatic joint



(a) (b) (c)

• : The surfaces characterizing the basic LKP: (a) the revolute pair; (b) the screw pair; and (c) the prismatic pair

- The foregoing LOR values, or any monotonic function thereof, can be used as a measure of the *type complexity* of each of the three LKP under discussion

Embodiment Design

In an attempt to formulate a theoretical framework that would allow the implementation of Taguchi's robust design, we start by distinguishing between the design variables (DV) and the design-environment parameters (DEP) (Al-Widyan and Angeles, 2005):

- *Design variables*: those variables appearing in the mathematical model of an object under design that the designer has to assign in order to meet the design objectives. As such, these variables are *deterministic*, and grouped in the n-dimensional vector array $\mathbf{x}$:

$$\mathbf{x} = \left[x_1 \; x_2 \; \ldots \; x_n\right]^T$$

Embodiment Design

- *Design environment parameters*: random variables describing the environment on which the designed object will operate, over which the designer has no control. It is assumed, however, that the designer knows the statistical properties of these variables *to some extent*. These properties include the type of probability distribution of these variables and the parameters associated with them. DEP are grouped in the ν-dimensional array $\mathbf{p}$:

$$\mathbf{p} = \left[p_1 \; p_2 \; \ldots \; p_\nu\right]^T$$

- *Performance functions*: relations that represent the performance of the design in terms of design variables and design-environment parameters. The performance functions are grouped in a m-dimensional array:

$$\mathbf{f} = \mathbf{f}(\mathbf{x}; \mathbf{p})$$

which encapsulates the mathematical model at hand

Probability Distributions of the DEP

- Given the randomness of the DEP, we need models for their probability distributions
- For the sake of conciseness, we assume that the variations of the DEP obey Gaussian distributions with nonzero mean and nonidentical standard deviations

Further, if $\mathbf{p}_0$ indicates the nominal operating conditions, then the expected value of the variation in DEP, denoted by $\boldsymbol{\mu}_p$, can be represented as

$$\boldsymbol{\mu}_p \equiv E[\,\Delta\mathbf{p}\,] \equiv E[\,\mathbf{p} - \mathbf{p}_0\,]$$

$E[\,\cdot\,]$: the *expected-value* operator

Moreover, the covariance matrix $\mathbf{P}$ of $\Delta\mathbf{p}$ is evaluated as

$$\mathbf{P} \equiv V[\,\Delta\mathbf{p}\,] \equiv E[\,(\Delta\mathbf{p} - \boldsymbol{\mu}_p)(\Delta\mathbf{p} - \boldsymbol{\mu}_p)^T\,] = E[\,\Delta\mathbf{p}\Delta\mathbf{p}^T\,] - \boldsymbol{\mu}_p\boldsymbol{\mu}_p^T \in \mathbb{R}^{\nu\times\nu}$$

$V[\,\cdot\,]$: the *covariance* operator

Probability Distributions of the DEP (Cont'd)

Robust design aims to render the performance vector $\mathbf{f}$ of a design as insensitive to variations $\Delta\mathbf{p}$ as possible. In this vein, we assume that $\mathbf{f} = \mathbf{f}(\mathbf{x}; \mathbf{p})$ is differentiable w.r.t. the DEP $\Rightarrow$ upon expansion around the nominal point $(\mathbf{x}, \mathbf{p}_0)$:

$$\mathbf{f}(\mathbf{x}; \mathbf{p}_0 + \Delta\mathbf{p}) = \mathbf{f}(\mathbf{x}; \mathbf{p}_0) + \left(\frac{\partial \mathbf{f}}{\partial \mathbf{p}}\right)\Bigg|_{\mathbf{p}=\mathbf{p}_0} \Delta\mathbf{p} + \mathrm{HOT}$$

HOT: higher-order terms; $\partial\mathbf{f}/\partial\mathbf{p}$: evaluated at $\mathbf{p} = \mathbf{p}_0$

Upon deleting the HOT:

$$\Delta\mathbf{f} = \mathbf{F}\Delta\mathbf{p}, \quad \mathbf{F}((\mathbf{x}; \mathbf{p}_0) = \frac{\partial \mathbf{f}}{\partial \mathbf{p}}\bigg|_{\mathbf{p}=\mathbf{p}_0} \in \mathbb{R}^{m\times\nu}$$

Moreover, $\mathbf{F}$ is the $m \times \nu$ Jacobian matrix of $\mathbf{f}$ w.r.t. $\mathbf{p}$
$\Rightarrow$ $\mathbf{F}$ measures the sensitivity of the design performance to variations in the design-environment parameters
$\Rightarrow$ $\mathbf{F}$ is called the *performance matrix* of the design at hand

Probability Distributions of the DEP (Cont'd)

Now, the expected value $\boldsymbol{\mu}_f$ of $\Delta\mathbf{f}$ is

$$\boldsymbol{\mu}_f = E[\,\Delta\mathbf{f}\,] = \mathbf{F}E[\,\Delta\mathbf{p}\,] = \mathbf{F}\boldsymbol{\mu}_p$$

Furthermore, the corresponding covariance matrix $\boldsymbol{\Phi}$ of $\Delta\mathbf{f}$ is evaluated as

$$\boldsymbol{\Phi} = E[\,(\Delta\mathbf{f} - \boldsymbol{\mu}_f)(\Delta\mathbf{f} - \boldsymbol{\mu}_f)^T\,]$$

$$\Rightarrow \boldsymbol{\Phi} = E[\,\mathbf{F}(\Delta\mathbf{p} - \boldsymbol{\mu}_p)(\Delta\mathbf{p} - \boldsymbol{\mu}_p)^T\mathbf{F}^T\,] = \mathbf{F}E[\,(\Delta\mathbf{p} - \boldsymbol{\mu}_p)(\Delta\mathbf{p} - \boldsymbol{\mu}_p)^T\,]\mathbf{F}^T$$

which simplifies to

$$\boldsymbol{\Phi} = \mathbf{F}(E[\,\Delta\mathbf{p}\Delta\mathbf{p}^T\,] - \boldsymbol{\mu}_p\boldsymbol{\mu}_p^T)\mathbf{F}^T$$

Recalling the definition of $\mathbf{P}$, the above expression simplifies to

$$\boldsymbol{\Phi} = \mathbf{F}\mathbf{P}\mathbf{F}^T$$

Probability Distributions of the DEP (Cont'd)

As the objective of robust design is to minimize the sensitivity of a design to variations in the DEP, and this sensitivity is encapsulated in the $m \times m$ covariance matrix $\mathbf{\Phi}$, the obvious next step is to define a (scalar) *measure* of the "magnitude" of $\mathbf{\Phi}$. This measure is chosen here as the *matrix Frobenius norm*, which will be represented here as the *objective function* σ_f, namely,

$$\sigma_f \equiv \|\mathbf{\Phi}\| = \sqrt{\text{trace}(\mathbf{\Phi}\mathbf{\Phi}^T)}$$

Now the *robust-design problem* can be formulated within the framework of *mathematical programming* (Hillier and Liebermann, 1995) as

$$z \equiv \frac{1}{2}\sigma_f^2(\mathbf{x}) \rightarrow \min_{\mathbf{x}}$$

subject to

$$\mathbf{f}(\mathbf{x};\, \mathbf{p}_0) = \mathbf{f}_0$$

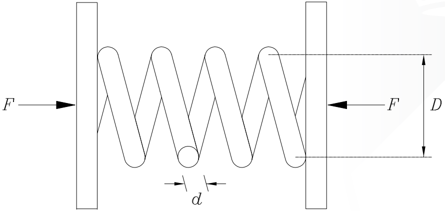
Probability Distributions of the DEP (Cont'd)

From the definition of $\mathbf{\Phi}$, however, the designer must know the two matrices $\mathbf{F}$ and $\mathbf{P}$. Frequently, however, the designer has no access to data on the statistical properties of the DEP, and hence, $\mathbf{P}$ is not known. This shortcoming shouldn't stop us, for we can resort to an inequality of norms, as proposed earlier (Al-Widyan and Angeles, 2005):

$$\|\mathbf{\Phi}\| \leq \|\mathbf{F}\|$$

- which means that minimizing $\|\mathbf{F}\|$ we are implicitly minimizing $\|\mathbf{\Phi}\|$
- The downside here is that the design may be suboptimum in that it is too restricting
- Nevertheless, in some instances it is preferably to overdesign than the other way around

Example: the Robust Design of a Helical Spring



D: the mean spring diameter
d: the wire diameter
N: the number of turns

- : An axially loaded helical spring

The stiffness k, the shear stress τ, and the natural frequency ω_n of the spring are given by

$$k = \frac{d^4 G}{8D^3 N}, \ \tau = K_s \frac{8FD}{\pi d^3}, \ \omega_n = \frac{1}{2}\sqrt{\frac{k}{M}}$$

where G: the shear modulus; M: the mass of the spring;
K_s: the shear-stress correction factor

Example: the Robust Design of a Helical Spring (Cont'd)

Based on the frequency response of a harmonic oscillator we have

$$\frac{X_o}{F_o} = \frac{k}{1-\gamma^2}$$

X_o and F_o: the amplitude of the translation and the force
$\gamma = \omega_o/\omega_n$: the frequency ratio, with ω_o denoting the frequency of the harmonic excitation $F(t) = F_o \cos \omega_o t$

- The excitation amplitude F_o and its frequency ω_o are unpredictable $\Rightarrow$ they lie beyond the control of the designer
- The values of the parameters defining the spring, d, D and N, are up to the designer to assign, while the shear stress τ and the translation amplitude X_o represent the response of the spring, and hence, the *performance functions* $\Rightarrow$

$$\mathbf{x} = [\,d,\ D,\ N\,]^T,\ \mathbf{p} = [\,F_o,\ \omega_o\,]^T,\ \mathbf{f} = [\,\tau,\ X_o\,]^T$$

Example: the Robust Design of a Helical Spring (Cont'd)

X_o is a function of both F_o and ω_o, of the latter via $\gamma \Rightarrow$ we can thus write, to a first-order approximation,

$$\frac{\Delta X_o}{X_o} = \frac{\Delta F_o}{F_o} + \frac{2\gamma^2}{1-\gamma^2}\frac{\Delta\omega_o}{\omega_o}$$

while the variation in the shear stress is given by

$$\frac{\Delta\tau}{\tau} = \frac{\Delta F_o}{F_o}$$

The variation in the DEP vector, whose statistical properties are unpredictable, induces a variation in the performance vector given, to a first-order approximation, by

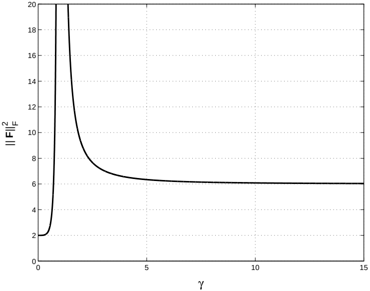
$$\underbrace{\begin{bmatrix}\Delta\tau/\tau \\ \Delta X_o/X_o\end{bmatrix}}_{\Delta\mathbf{f}} = \underbrace{\begin{bmatrix}1 & 0 \\ 1 & 2\gamma^2/(1-\gamma^2)\end{bmatrix}}_{\mathbf{F}} \underbrace{\begin{bmatrix}\Delta F_o/F_o \\ \Delta\omega_o/\omega_o\end{bmatrix}}_{\Delta\mathbf{p}}$$

As we don't have knowledge of $\mathbf{P}$, rather than minimizing σ_f, we aim to minimize $\|\mathbf{F}\|_F$

Example: the Robust Design of a Helical Spring (Cont'd)

A straightforward calculation leads to

$$\|\mathbf{F}\|_F^2 = 1 + \frac{2\gamma^4}{(1-\gamma^2)^2}$$



- : Plot of $\|\mathbf{F}\|_F^2$ for the helical spring

- $\|\mathbf{F}\|_F^2$ attains its minimum at very low frequencies, which is not practical
- The foregoing quantity becomes unbounded when $\gamma = 1$, which corresponds to resonance
- However, the plot flattens out quickly past a value of $\gamma = 5$
- As a matter of fact experienced designers usually design springs for a value of $\gamma = 10$

Conclusions

- An eclectic approach that *may* lead to a *theory of engineering design* was proposed
- The three best known schools were scrutinized, while taking the best of each, which led to a novel formulation of the two critical stages of the design process, *conceptual design* and *embodiment design*
- At the conceptual design stage complexity was proposed as the main issue to minimize—complexity can be attributed to virtually anything
- At the embodiment design stage, robustness was proposed as the main objective, to be maximized under the constraints imposed by a particular design task
- The proposed framework is discipline-independent. It can be applied to any engineering-design task

Thank you for your attention!