MECH577 Optimum Design

Project # 1: The Maximum Reach of a Three-joint Robot

Assigned: January 8, 2013 Due: January 29, 2010

N.B.: the figure and the exercise cited below are included in the course Lecture Notes.

In the design of the robot of Fig. 4.2 it is required to express its maximum reach in the form $r_M = Ra$, where a is the common distance between neighbouring axes. To this end, an expression for the distance $d = \sqrt{c_1^2 + c_2^2}$ of point $C(c_1, c_2, c_3)$ from the Z_1-axis was found in Exercise 4.5.1. An expression for the gradient ∇d of d with respect to $\mathbf{x} = [\,\theta_2,\, \theta_3\,]^T$—$\theta_1$ does not affect d—was also found in the same exercise. Moreover, angles θ_i are defined as those made by axes X_i and X_{i+1}, measured positive in the direction of Z_i.

All stationary points (SPs) of d can be found as the roots of $\nabla d = \mathbf{0}_2$, with $\mathbf{0}_2$ denoting the two-dimensional zero vector. Using the Newton-Raphson method, find all such roots that lie within the square $-\pi \le \theta_i \le \pi$, for $i = 2, 3$.

Furthermore, the gradient of ∇d, represented as $\nabla\nabla d$ and denoting the 2×2 matrix of second derivatives of d w.r.t. $\mathbf{x}$, can be used to decide on the nature of every SP of d: if positive-definite, then a minimum; if negative-definite, then a maximum; if sign-indefinite, then a saddle point. Produce a table with all the roots of the equation of interest, including values of $\nabla\nabla d$ at every root and the nature of each root. Using this table, decide what the value of R is.

Hint: shown in Fig. 4.3 are 10 intersections of the contours $\mathcal{C}_1$ and $\mathcal{C}_2$ defined by each of the two scalar components of the above vector equation. Use this plot to provide estimates of the SPs, and hence, initial guesses that target these points.