MECH 573 Mechanics of Robotic Systems

Class Test 1 **February 11, 2010**

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1. Shown in Fig. 1 is a planar, two-link serial manipulator with equal link-lengths.

(a) (20%) Find an expression for its condition number based on the Frobenius norm. *Hint: the product* $\mathbf{J}^T\mathbf{J}$ *leads to a simpler expression than* $\mathbf{J}\mathbf{J}^T$.

(b) (40%) Find the posture at which the above condition number attains its minimum value. What is this value?

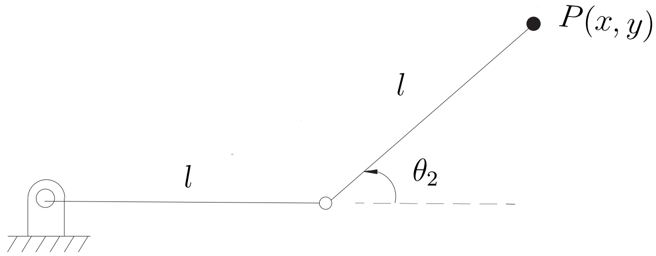

Figure 1: A planar two-link manipulator

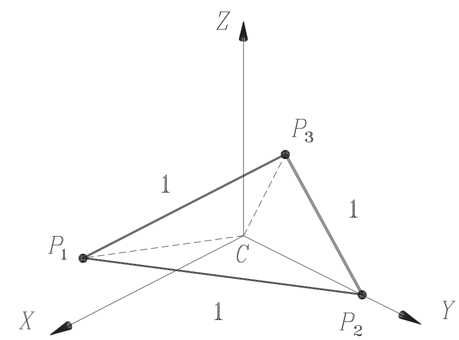

Figure 2: A triangular plate

2. (20%) The same manipulator of Fig. 1 is to execute a PPO that requires its joints to sweep angles of $\Delta\theta_1 = \pi/3$ and $\Delta\theta_2 = 4\pi/3$. If the maximum speeds that the motors can deliver are $(\dot{\theta}_1)_{\max} = 2\pi/3\ \text{s}^{-1}$ and $(\dot{\theta}_2)_{\max} = 4\pi/3\ \text{s}^{-1}$, compute the minimum time in which the robot can execute the operation with a 4-5-6-7 polynomial.

3. (20%) The rigid triangular plate of Fig. 8.2 is reproduced in Fig. 2 for quick reference. Matrix $\mathbf{D}$, defined as

$$\mathbf{D} = \frac{1}{2}[\text{tr}(\mathbf{P})\mathbf{1} - \mathbf{P}]$$

was found in Example 8.3.1 to be nonsingular when all vectors are represented in the given coordinate frame. Under a change of frame, obtained upon rotating $\{O,\ X,\ Y,\ Z\}$ through an angle θ about the Z-axis, it is possible to render $\mathbf{D}$ singular. The rotation is represented by matrix $\mathbf{Q}$, given by

$$\mathbf{Q} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Notice that, under this change of frame, $\mathbf{P}$, defined as

$$\mathbf{P} = [\,\mathbf{p}_1 - \mathbf{c} \quad \mathbf{p}_2 - \mathbf{c} \quad \mathbf{p}_3 - \mathbf{c}\,]$$

in which $\mathbf{c}$ is the position vector of the centroid of triangle $P_1P_2P_3$, changes to

$$\mathbf{P}' = \mathbf{Q}\mathbf{P}$$

Find the angle of rotation θ that renders $\mathbf{D}$ singular.