

MECH 573 Mechanics of Robotic Systems
Exercises in Linear Algebra
June, 2005

Note: The use of both components and computer algebra is strongly recommended against.

1. Let \mathbf{a} be an arbitrary three-dimensional vector and \mathbf{A} its cross-product matrix.

(a) Let $\mathbf{B} \equiv \mathbf{1} + \mathbf{A}$, with $\mathbf{1}$ defined as the 3×3 identity matrix. Then, prove that

$$\det(\mathbf{B}) = 1 + \|\mathbf{a}\|^2 > 1$$

and hence, \mathbf{B} is nonsingular.

(b) Find \mathbf{B}^{-1} in terms of \mathbf{A} or, equivalently, of \mathbf{a} .

2. For given three-dimensional, non-zero \mathbf{a} and \mathbf{b} , find \mathbf{v} that verifies

$$\mathbf{v} + \mathbf{a} \times \mathbf{v} = \mathbf{b}$$

When finding an expression for \mathbf{v} in terms of \mathbf{a} and \mathbf{b} , answer the questions below:

(a) Is it possible to find \mathbf{v} for arbitrary \mathbf{a} and \mathbf{b} ?

(b) Can \mathbf{v} be orthogonal to \mathbf{a} ? If so, under which conditions?

(c) Can \mathbf{v} be orthogonal to \mathbf{b} ? If so, under which conditions?