PLACE ADDITIONAL BOOKS INSIDE FIRST BOOK

CLASSROOM BOOK BOOK No. _____

McGILL UNIVERSITY

DO YOU EXPECT TO GRADUATE AT THE NEXT CONVOCATION? ☐

Last Name only: MIDTERM EXAMINATION FALL 2008
(Please Print)

_____ _____
(Print other Names in full)

Student Number _____

Course and Year _____
(e.g. B.A. U 1)

Subject MECH572 INTRO. TO ROBOTICS
(Include course number)

Date of Examination THURSDAY OCT. 30, 2008

Row/Seat Number _____ **Class Section** _____

INSTRUCTIONS

Fill in the above carefully.

Write your answers on the RULED SIDE ONLY — Use the unruled side for rough work or calculations.

Do not write in the margin — If a page is accidentally left blank write "P.T.O." on it.

Do not tear pages from this book; all your writing must be handed in.

Put additional books inside first book when handing in.

This book must not be taken from the Examination Room.

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For Examiner's use only

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1. $\underset{\sim}{Q} : \mathcal{F} \to \mathcal{F}'$

$\vec{\imath} \longrightarrow \underbrace{0.7071(-\vec{\imath}+\vec{\jmath})}_{\vec{\imath}'}$; $\vec{k} \to \underbrace{0.5774(\vec{\imath}+\vec{\jmath}+\vec{k})}_{\vec{k}'}$

$$\vec{\jmath}' = \vec{k}' \times \vec{\imath}' = 0.7071 \times 0.5774 \begin{vmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ 1 & 1 & 1 \\ -1 & 1 & 0 \end{vmatrix}$$

$$= 0.4082(-\vec{\imath} - \vec{\jmath} + 2\vec{k})$$

$$\Rightarrow \underset{\sim}{Q} = \begin{bmatrix} -0.7071 & -0.4082 & 0.5774 \\ 0.7071 & -0.4082 & 0.5774 \\ 0 & 0.8165 & 0.5774 \end{bmatrix}$$

$$\Rightarrow \vec{e}\sin\phi = \text{vect}(\underset{\sim}{Q}) = \frac{1}{2}\begin{bmatrix} 0.8165 - 0.5774 \\ 0.5774 \\ 0.7071 + 0.4082 \end{bmatrix} = \begin{bmatrix} 0.1196 \\ 0.2887 \\ 0.5577 \end{bmatrix}$$

$$\Rightarrow \sin\phi = \| \text{vect}(\underset{\sim}{Q}) \| = 0.6392 \Rightarrow \phi = 39.73^\circ \text{ or } 140.27^\circ$$

$$1 + 2\cos\phi = \text{tr}(\underset{\sim}{Q}) = -0.5379 \Rightarrow \cos\phi = -\frac{1.5379}{2} = -0.7690$$

$$\Rightarrow \phi = 140.26^\circ \text{ or } 219.74^\circ$$

$$\Rightarrow \phi = 140.26^\circ \quad \text{Ans.}$$

$$\vec{e} = \frac{\text{vect}(\underset{\sim}{Q})}{\sin\phi} = \begin{bmatrix} 0.1871 \\ 0.4517 \\ 0.8725 \end{bmatrix} \quad \text{Ans.}$$

2. (a) $\vec{f}_1 = \begin{bmatrix} 0 \\ 10 \\ 0 \end{bmatrix}$, $\vec{f}_2 = \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix}$

$\vec{n}_M = (\vec{e} - \vec{m}) \times \vec{f}_1 + (\vec{c} - \vec{m}) \times \vec{f}_2$

$\vec{e} - \vec{m} = \begin{bmatrix} 0 \\ 0 \\ a \end{bmatrix} - \frac{1}{2}\begin{bmatrix} a \\ a \\ a \end{bmatrix} = \frac{a}{2}\begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$

$\vec{c} - \vec{m} = \begin{bmatrix} a \\ a \\ 0 \end{bmatrix} - \frac{1}{2}\begin{bmatrix} a \\ a \\ a \end{bmatrix} = \frac{a}{2}\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$

$(\vec{e} - \vec{m}) \times \vec{f}_1 = \frac{10a}{2}\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & -1 & 1 \\ 0 & 1 & 0 \end{vmatrix} = 5a(-\vec{i} + \vec{k})$

$(\vec{c} - \vec{m}) \times \vec{f}_2 = \frac{10a}{2}\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & -1 \\ 0 & 0 & 1 \end{vmatrix} = 5a(\vec{i} - \vec{j})$

$\Rightarrow \vec{n}_M = 5a(-\vec{i}' + \vec{k} + \vec{i}' - \vec{j}) \Rightarrow \vec{n}_M = 5a\begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$

$\vec{f} = \vec{f}_1 + \vec{f}_2 = \begin{bmatrix} 0 \\ 10 \\ 10 \end{bmatrix} \Rightarrow \vec{w} = 5[0 \; -a \; a \; 0 \; 2 \; 2]^T$

Ans.

(b) Recall eq. (3.111): $\vec{p}_0'' = \frac{1}{\|\vec{f}\|^2}\vec{f} \times (\vec{n}_M - \vec{f} \times \vec{m})$

$\vec{f} \times \vec{m} = 10 \times \frac{a}{2}\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} = 5a(\vec{j} - \vec{k}) = 5a\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$

$\vec{n}_M - \vec{f} \times \vec{m} = 5a\begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} - 5a\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} = 5a\begin{bmatrix} 0 \\ -2 \\ 2 \end{bmatrix} = 10a\begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$

$\vec{f} \times (\vec{n}_M - \vec{f} \times \vec{m}) = 10 \times 10a\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 1 & 1 \\ 0 & -1 & 1 \end{vmatrix} = 100a(2\vec{i}) = 200a\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$\|\vec{f}\|^2 = 100 + 100 = 200$

$\Rightarrow \vec{p}_0'' = \frac{200a}{200}\begin{bmatrix}1\\0\\0\end{bmatrix} = a\begin{bmatrix}1\\0\\0\end{bmatrix}$

Unit vector $\parallel$ wrench axis $\vec{e}'' = \frac{\vec{f}}{\|\vec{f}\|} = \frac{\sqrt{2}}{2}\begin{bmatrix}0\\1\\1\end{bmatrix}$

$\vec{n}'' = \vec{p}_0'' \times \vec{e} = a\frac{\sqrt{2}}{2}\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 1 & 0 & 0\\ 0 & 1 & 1\end{vmatrix} = \frac{\sqrt{2}}{2}a(-\vec{j}+\vec{k}) =$

$= \frac{\sqrt{2}}{2}a\begin{bmatrix}0\\-1\\1\end{bmatrix}$

$\Rightarrow \vec{k}'' = \frac{\sqrt{2}}{2}[0 \;\; 1 \;\; 1 \;\; 0 \;\; -a \;\; a]^T$: Plücker coords.

Ans.

3. (a) $\vec{w} = \begin{bmatrix} \vec{n} \\ \vec{f} \end{bmatrix}$

Conditions: 1) $\| [\vec{n}]_1 \| = \| [\vec{n}]_6 \|$

2) $\| [\vec{f}]_1 \| = \| [\vec{f}]_6 \|$ Ans.

3) $[\vec{n} \cdot \vec{f}]_1 = [\vec{n} \cdot \vec{f}]_6$

$\| [\vec{n}]_1 \| = 1$, $\| [\vec{n}]_6 \| = 1 \Rightarrow$ 1) OK

$\| [\vec{f}]_1 \| = 1$, $\| [\vec{f}]_6 \| = 1 \Rightarrow$ 2) OK

$[\vec{n} \cdot \vec{f}]_1 = 0$, $[\vec{n} \cdot \vec{f}]_6 = 0 \Rightarrow$ 3) OK

b) $[\vec{n} \times \vec{f}]_1 = [\vec{i} \times \vec{k}]_1 = [-\vec{j}]_1 = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$

$[\vec{n} \times \vec{f}]_6 = [\vec{k} \times \vec{i}]_6 = [\vec{j}]_6 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

$\underset{\sim}{Q} : \mathcal{F}_1 \to \mathcal{F}_6 \Rightarrow \underset{\sim}{Q} : [\vec{v}]_6 \to [\vec{v}]_1$ or $[\vec{v}]_1 = \underset{\sim}{Q} [\vec{v}]_6$

$\Rightarrow \underbrace{[\vec{n} \ \vec{f} \ \vec{n} \times \vec{f}]_1}_{\underset{\sim}{A}} = \underset{\sim}{Q} \underbrace{[\vec{n} \ \vec{f} \ \vec{n} \times \vec{f}]_6}_{\underset{\sim}{B}} \Rightarrow \underset{\sim}{Q} = \underset{\sim}{A} \underset{\sim}{B}^{-1}$

$\underset{\sim}{A} =$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

$\underset{\sim}{B} =$

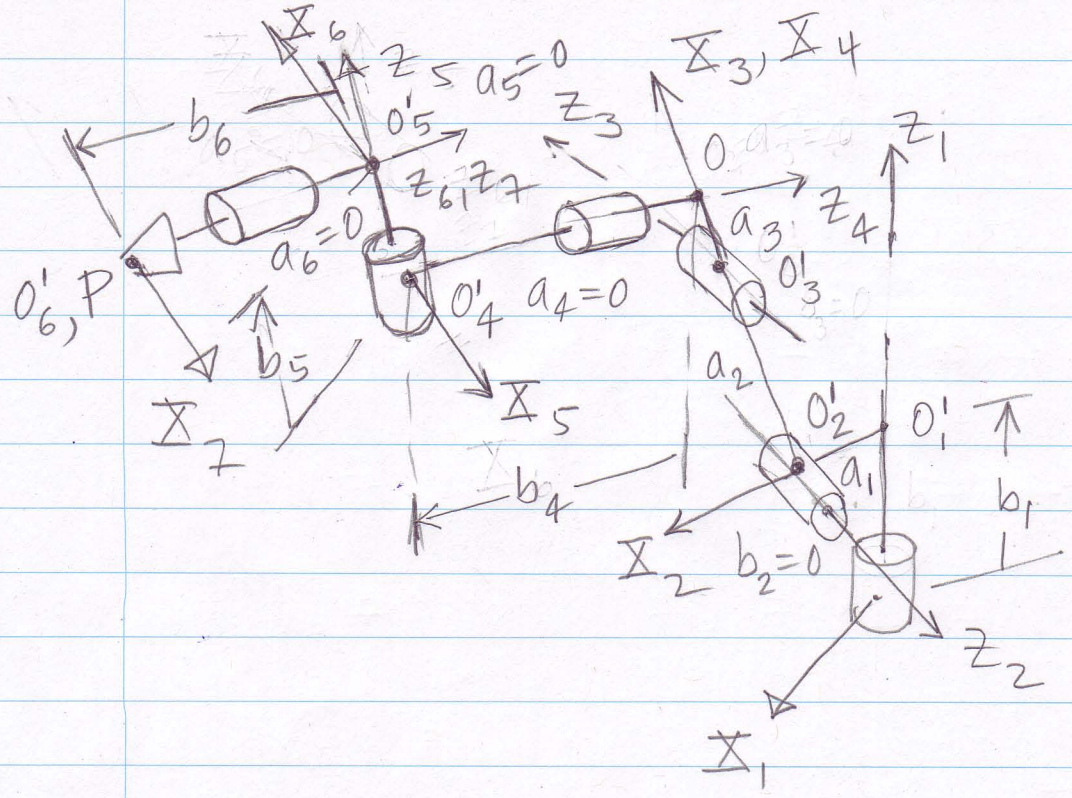
$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} = [\vec{k} \ \vec{i} \ \vec{j}]_6$$

$\Delta = \det(\underset{\sim}{B}) = \vec{k} \times \vec{i} \cdot \vec{j} = \vec{j} \cdot \vec{j} = 1$

$$\underset{\sim}{B}^{-1} = \frac{1}{1} \begin{bmatrix} (\vec{i} \times \vec{j})^T \\ (\vec{j} \times \vec{k})^T \\ (\vec{k} \times \vec{i})^T \end{bmatrix}_6 = \begin{bmatrix} \vec{k}^T \\ \vec{i}^T \\ \vec{j}^T \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\Rightarrow \underset{\sim}{Q} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$ Ans.

4. Kinematic Chain:



i	a_i (mm)	b_i (mm)	α_i (°)
1	100	900	-90°
2	700	0	180°
3	162	0	-90°
4	0	800	90°
5	0	140	90°
6	0	160	0°

DH parameters of the Fanuc S700

Ans