Diploma Thesis

The Development and Implementation of Kinematics Algorithms on RVS (Robot Visualization System)

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Erklaerung

“Hiermit erklaere ich, die Diplomarbeit selbstständig verfasst und keine anderen als die angegebenen Hilfsmittel verwendet zu haben.”.

Brilon, August 25, 2006

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Abstract

This report describes the development of a computer program for the simulation of robots. The program is called *Robot Visualization System (RVS) 2006* and is based on a software package, also called RVS, that was developed over 10 years ago. Because the original RVS was adapted to a Silicon Graphics Workstation, the software is not compatible to current computer architectures. Therefore, the source code of RVS needs to be modified in order to use the tool on an IBM-compatible PC. The aim of this report is to document all steps that are required for this task.

Zusammenfassung

in deutsch

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1 Introduction

1.1 Introduction

Like in any other field of industrial development, simulation programs have become a standard tool in robotics. These tools can provide invaluable help during the design stage of a new robot or a whole work environment for a specific robot. A lot of 3D-visualization software packages are commercially available. During the 1990s the McGill University developed its own noncommercial software tool for the simulation of robots called *Robot Visualization System (RVS)*. This tool was developed for educational use and never claimed to be as professional as its commercial counterparts. Because the tool was adapted to a specific computer architecture and operating system, it cannot be used on an IBM-compatible PC using Windows or LINUX. The aim of this project is to develop a new software which provides the same functionality as the original RVS. In order to minimize the required work, parts of the old software should be used in the new one if possible. Therefore the existing software needs to be analyzed.

In Chapter 2 the existing program structure is analyzed, to get an overview which could be used in the new software. Based on the results of the analysis, three main steps are necessary to develop the new software. First of all a new user interface is required. Chapter 3 presents three different packages which can create a Graphical User Interface and describes the one which was used for this project more detailed. The original RVS offers a Primitives Library which was used to create 3D-objects. Every robot model was built from several of these objects. This Primitives Library is based on an old graphics engine and cannot be used in the new programm. Chapter 4 describes the modification with a new graphics engine, in order to make the library compatible. Finally, the required Kinematics Engine is mentioned in Chapter 5.

Once these parts were programmed / ported, the functionality of the program could be extended. Chapter 6 presents four new features which were implemented in the new tool called RVS2006. Chapter 6 also includes an overview of the structure of the program, in order to support other programmers in the future.

Chapter 7 gives an introduction in the Geometry of Serial Robots, which is required for the development of new robots in RVS2006. The last part of this project was to develop basics for an optimizations algorithm which should be implemented in RVS in the future. These tools are provided in Chapter 8.

1.2 Robot Visualization System

The Robot Visualization System (RVS) is a small and easy to use tool for simulation of robots. It was developed at the McGill University's Centre for Intelligent Machines on a *Silicon Graphics (SG) Workstation*. The aim was to support the user during the design process of a new robot. With just a few input variables (4 for each link) RVS is able to produce a 3D rendering of the robot's skeleton. RVS provides the following additional functionalities:

- Each joint variable can be controlled individually;
- joint limits can be set;
- the robot can be moved to pre-stored postures;
- the robot can be made to follow pre-stored trajectories;
- frames for each link can be displayed; and
- obstacles can be placed in the workspace.

Finally, it is possible to program a fully rendered model for a robot.

1.3 Using RVS

To start RVS the user has to type `./sim1` in the UNIX prompt. The main RVS window appears and shows the RVS Workspace. Using the pop-up menu, different functions can be chosen. The whole package is a window-driven environment, meaning that for each function a new window is shown. Inside the program, the user can navigate using the mouse [1].

1.4 Objectives

Since the development of RVS, computers have improved significantly both in their quantity and in their computation power. Today, nearly everyone has access to a desktop PC or a laptop. The computation power and the graphical support of current systems is enough to support the functionality of RVS. Hence, a rather expensive, SG machine is no more required.

To make RVS available to PC users, the program must be ported to a current operating system, mainly Microsoft Windows$^{\mathrm{TM}}$ and LINUX. This leads to the following goals of this project:

- The development of a new software package which offers the same functionality as the original RVS but available on a usual computer;
- to debug existing features;
- to provide new features;

  Because the whole expenditure of time is not foreseeable at the beginning, it is not possible to define specific aims for new features. Therefore, only a outline is determined, which should be processed as time permits:

  - *programming of new features which weren't included or didn't work probably in the original software,*
  - *implementation of an optimization algorithm.*
- and to provide a complete documentation for programmers to follow.

The package should offer two important attributes in the end:

- **open source** - To publish the package as free software, therefore no commercial packages should be used.
- **platform independent** - The new RVS should be available for use on the most common computers. If possible, it should be programmed completely hardware and operating system independent.

2 Overview of RVS

2.1 Introduction

An essential initial task in this project is to analyze the original software. Unfortunately, the existing documentation is not detailed. The RVS User Manual[1] includes some information about software libraries used and a small, but incomplete, overview of the file and folder structure of RVS. Hence, it is necessary to look for further details elsewhere. For the support on libraries used the internet is the first choice. Unfortunately, the only way to understand how RVS works is to analyze the source code. Below, a short overview of the structure of RVS is provided.

Notation

Typewriter `Typeface` is used for any kind of source code in this document.

2.2 The Main Structure of RVS

The original RVS is written in C. Also every library that is used is written in this programming language. The program needs to interact with the X Server[1]. This is done using Xlib, an X Window System protocol client library. Using the functions of Xlib the programmer is able to create a program without knowing the details of the protocol. However, Xlib is a low-level library and it does not provide objects such as buttons, menus, etc., for a *Graphical User Interface (GUI)*. For this reason, another library is needed. This second library called Xt Intrinsics and is based on Xlib. It provides support for creating and using widgets[2]. The programmer uses widgets to build the GUI, so that the user can easily interact with the program. However, Xt does not offer any widgets, so again one more library has to be used.

XForms The Forms Library for X or short XForms, is a widget tookit[3] based on Xlib. Therefore, XForms should run on all systems where an X Window System is installed. The main idea, as for every toolkit, is to create a form, a window,

[1] The X Server is the main component of an X Window System, which is used on UNIX/LINUX computers to generate graphical interface.
[2] A widget is an interface component, like a button, a small browser, a output element, etc.
[3] Widget toolkits (or GUI toolkits) are sets of basic building units for graphical user interfaces.

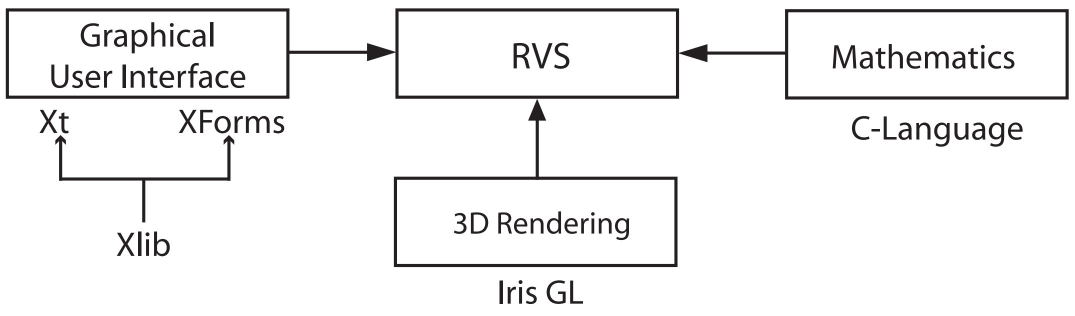


Figure 2.1: Basis Structure of RVS

and place different widgets inside the form. Once the form is displayed the user is able to interact with the program. The library offers many widgets, such as buttons, sliders, input fields, value outputs and so on. Also different styles of the widgets are available. Here are two examples of how to create a form and a button with XForms:

```
// CREATE A FORM/WINDOW
FL_FORM *My_form;
My_form = fl_bgn_form(FL_UP_BOX,320,120);

// CREATE A OBJECT -> BUTTON
FL_OBJECT *My_object;
My_object = fl_add_button(FL_NORMAL_BUTTON,40,70,80,30,"Yes");
```

The second requirement for the RVS software package is the graphical engine to generate the 3D-drawings.

IrisGL By using a SG Workstation for RVS, the choice of the graphical engine was quite simple. In the early 1990's, SGI[4] was the world's leader in 3D graphics and the company provided a graphical library for their own workstation. In combination with the best hardware, the Iris Graphics Language (IrisGL) became the standard API[5] of that time. For this reason, all the 3D renderings in RVS are created using IrisGL.

[4] Silicon Graphics Incorporated (until 1999)
[5] API stands for Application programming interface

3 User Interface

3.1 Separation of the Project

The whole project may be divided into the following three parts:

1. **New Platform-Independent User Interface**
2. **Graphics Library**
3. **Kinematics Engine**

The first task is to create the new user interface with the menu and all required windows and widgets. OpenGL should then be tested to ensure that it is working properly within the new GUI. Once this is done, the existing primitives to draw the robot can be modified with the new graphical engine. The last step is to include all the functions like the kinematics engine step by step into the new interface.

3.2 Graphics Library

Following the aims of this project, namely to be platform independent, the old graphical engine IrisGL must be replaced. Because IrisGL works only on Silicon Graphics workstations with the Iris operating system, it can not be used in the new software.

OpenGL The new engine, called OpenGL[1] works nearly the same way as IrisGL. The Open Graphics Library is a cross-platform API for 2D/3D computer graphics by SGI. About 120 commands can be used to create various graphical object, but these commands are low-level functions. It is not possible to build complex 3D objects with a single command. All objects have to be built from a small set of geometric primitives, like points, lines or polygons. In OpenGL, every command starts with the letters `gl`. A constant begins with `GL_` and is written in capital. For example, to define a rectangle you have to define the polygon primitive and its four vertices:

[1] During the early 1990's, SGI decided to provide an open standard graphics API for the developing portable 2D and 3D applications. The result was OpenGL, which was based on IrisGL.

```
glBegin(GL_POLYGON);
    glVertex2f(-0.5, -0.5);
    glVertex2f(-0.5, 0.5);
    glVertex2f(0.5, 0.5);
    glVertex2f(0.5, -0.5);
glEnd();
```

This means that every 3D object is based on a fairly large number of primitives. Some intermediate-level libraries built on OpenGL are available. The OpenGL Utility Library (GLU) is one of them and provides routines for 3D objects, such as a sphere, a cylinder, etc.
Like IrisGL, OpenGL is a *state* machine. A state will remain in a particular mode until it is changed. For example, if the color is set to red in the beginning of the program every functions call would continue to use the red color until the color is changed.
To keep OpenGL hardware independent, no GUI and data handling are included. It is up to the programmer to use a separate toolkit to handle such tasks.
Drawing objects using OpenGL is not enough. The programmer must also create the scene, e.g. virtual lights, camera etc. OpenGL offers functions to create a virtual 3D scene. A 3D scene with objects drawn within must be rendered on a 2D screen for display. OpenGL provides these projections. Different types of projections can be used for the visualization of the objects. As in reality only the objects in focus can be seen and will be displayed on the screen. Depending on the position of the light(s), the objects are fairly bright and colored.

Direct3D An alternative graphics library could be Direct3D, which is part of Microsoft's DirectX API. Like OpenGL this library is used to render 3D graphics. Therefore, Direct3D provides many commands to generate 3D objects. For this project Direct3D is not a possible choice, because it is only available for Microsoft's Windows operating systems.

3.3 Graphical User Interface

As explained above a new toolkit is required. The old Forms Library for X could not be used in the new software, because it is based on Xlib. It is connected to the X Window System and therefore it is not a cross-platform[2] toolkit. The new toolkit should provide functions for the window management, the input/output routines and it should offer all the needed widgets.

[2] Cross-platform (or platform independent) software means that the software works on different system platforms (e.g. Linux/Unix, Microsoft Windows, and Mac OS X)

Choosing the best toolkit is more difficult than it seems. Several packages of interest are available over the internet. With respect to the project aims, the new toolkit should achieve the following:

1. Cross-platform;
2. OpenGL support;
3. free; and
4. support C/C++.

In the following we discuss three different toolkits.

GLUT/GLUI In almost every OpenGL documentation [9] the OpenGL Utility Toolkit (GLUT) is mentioned. The GLUT library [6] is based on OpenGL, GLU and depending on the operating system functions to use OpenGL. Bindings are available for C and FORTRAN. GLUT provides window definition, window control, keyboard and mouse I/O, small popup-menus and routines to draw geometric objects. All of these functions allow the programmer to build a window that shows OpenGL graphics with a minimum amount of effort. Although GLUT is a cross-platform library, the main disadvantage of GLUT is that it does not provide enough functionality to build an extensive user interface.

A possible solution for this problem is GLUI; a GLUT-based User Interface library [7]. This C++ package starts where GLUT ends. GLUI builds windows and widgets to create a GUI. Because it is based on GLUT, it is also operating-system independent, however the types of widgets available are limited.

FOX FOX stands for Free Objects for X and is written in C++. In 1997 it was developed for LINUX applications, but the aim became to make it completely cross-platform.“Every line of code not written is a correct one.”[**?**], is one of ideas behind FOX. To minimize the number of lines of code, nearly every widget can be initialized in one single line. The types of FOX-widgets are much more than that of GLUI. To show OpenGL rendering a special window can be created. FOX would provide everything that the RVS-project would need.

FLTK The third library is the Fast and Light Toolkit (FLTK)[8]. Like FOX, it is a C++ GUI toolkit and supports Microsoft Windows, LINUX/UNIX and MacOS. The history of FLTK shows a direct relation to XForms. It was developed to fix the problems that appeared when graphical engine switched to OpenGL. This required a rewrite of XForms and as a result FLTK was developed. To a certain extent, the two toolkits are similar.

FLTK offers window definition, window control and input/output functions. Furthermore, it is designed to be statically linked. As a result, it is divided into small pieces and only the parts being used need to be linked. The result is that FLTK programs are very small and start quickly. There are 64 basic widgets in FLTK, which are extendible to 92 with minor modifications.

FLTK is GLUT compatible. With modified header files, an existing GLUT program can be implemented in FLTK. FLTK also offers the possibility to create a special widget for OpenGL applications (for more information see Section 3.5).

Along with FLTK comes FLUID; the Fast Light User Interface Designer. This tool is very handy to design an interface. Using "drag and drop", all widgets can be put in the desired place and relevant parameters can be set. When everything is correctly placed FLUID can then generate the C-code. This tool is very helpful when designing the layout of a new window.

Initially the new interface was created using GLUT and GLUI, for the reason explained above. An OpenGL window can quickly be developed using GLUT and GLUI. Menu and interface creation is also intuitive. By defining all the wanted widget, GLUI places them automatically in the best position inside the window. However, at one point it was recognized that it is not possible to build a new interface with these two packages. GLUI has limited widgets to offer. For example, browsers and sliders are not avaiable. Hence, simple tasks, like choosing the robot architecture from the list, would become complicated to manage.

Several other options were then considered for different toolkits that are not mentions here. The two packages that came to focused were FOX and FLTK. Finally FLTK was chosen, because of the following reasons:

- FLTK is closer to GLUT
- The FLTK syntax is easier
- FLTK comes with FLUID
- Sufficient to support RVS functionality.

3.4 How FLTK works

This section provide a quick overview of how a FLTK program works. As mentioned before, the package is separated into different parts. First, the static library **libfltk.a** must be added to the project options in the IDE. For every window, button, etc., there exists a header file which is identical in name to the widget type and have the ".H" extension. The following example shows how to create a simple window with a button, a value output and a callback function;

```
#include <stdlib.h>
#include <FL/Fl.H>
#include <FL/Fl_Window.H>
#include <FL/Fl_Button.H>
#include <FL/Fl_Value_Output.H>

// Callback function
void button_cb() {
    exit(0);
}

int main()
{
    Fl_Window *My_window;
    Fl_Button *My_button;
    Fl_Value_Output* My_output;

    My_window = new Fl_Window(100, 100,
                              180, 100, "How FLTK works!");

    My_button = new Fl_Button(50, 10, 80, 30, "exit");
        My_button->labelsize(12);
        My_button->callback((Fl_Callback*)button_cb);

    My_output = new Fl_Value_Output(75, 50, 40, 30, "Output   ");
        My_output->value(42);
        My_output->labelfont(FL_BOLD+FL_ITALIC);
        My_output->box(FL_PLASTIC_UP_BOX);

    My_window->end();
    My_window->show();

    return(Fl::run());
}
```

First, all required header files need to be included. Before the main routine, the function `void button_cb()` is defined. This is the *callback* function for the button widget. Every time the button is pressed the code in `button_cb()` is executed. Callbacks can be assigned to each widget which can change its value, like buttons (1 or 0), value inputs or sliders. In this example, the function just closes the program by executing the `exit(0)` command. However, callback functions usually include much more code and execute much more complex operations.

In the main routine, the FLTK-window is created. It starts with

```
My_window = new Fl_Window(100, 100, 180, 100, "How FLTK works!");
```

and ends with

```
My_window->end();
```

All widgets that are created between these two lines of code are placed inside the window. The following table shows the structure for most widgets:

	Command	Example
1.Type	Fl_Widget	Fl_Value_Output
2.Variable	name	My_output
3.Constructor	Fl_Widget(x, y, width, height, label)	Fl_Value_Output(75, 50, 40, 30, "Output ")
4.Method *(if necessary)*	name->method(parameter)	output->value(42);

The position parameters (x,y) for a widget define the distance from the upper left corner of the window. For a window the reference point is the upper left corner of the screen. These parameters are optional specifications. When no position is provided the program uses the default value of 0. Also, the label statement is optional. The method functions can be used to set values, change the design parameters, deactivate a widget, set a callback function, etc. For example, the `labelfont` for the `Fl_Button` is set to bold and italic in the source code above. Finally, the FLTK program ends with

```
My_window->show();
return(Fl::run());
```

The former line (a method) displays the defined window, with all widgets, on the screen. With the latter command the program enters the FLTK event loop. This loop runs continuously so that the program can react to any events such as button are pressed, mouse movement, etc. Fig. 3.1 shows the output of the program.

3.5 OpenGL in FLTK

There are a few methods to include OpenGL code in a FLTK interface. One is to set up an OpenGL context in a `Fl_Window` widget. This is managed by using the commands `gl_start()` and `gl_finish()` and writing the OpenGL code in between. A second possibility is to emulate a GLUT window for drawings. The simplest way, which is used in this project, is to generate a subclass for a widget called:

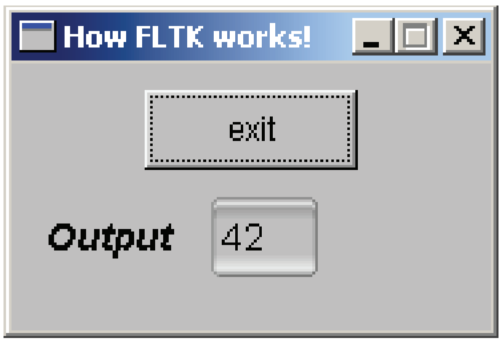


Figure 3.1: Small FLTK window

`Fl_Gl_Window()`.

This widget is then implemented into an `Fl_Window()` widget. For RVS the `Fl_Gl_Window()` is placed in the main FLTK window. At least three things must be defined in order to create an OpenGL subclass:

1. The class definition itself;
2. a `draw()` method, to display the drawing; and
3. a `handle()` method, for all the I/O action.

The following example explains how to display the rectangle mentioned in subsection 3.2. The libraries **libopengl32.a**, **libglu32.a** and **libfltk_gl.a** for correspondingly, OpenGL, GLU and FLTK-OpenGL support must be added to the project at hand. After all the header files for the widgets are included the subclass is then defined.

```
class MyGlWindow : public Fl_Gl_Window
{
    int handle(int event)
    {
        switch (event)
        {
            case FL_PUSH:
                int m_key = Fl::event_button();
                switch (m_key)
                {
                    case (FL_LEFT_MOUSE):
                         exit(0);
                }
        }
    }

    void draw()
    {
        // set the viewport
        glViewport(0,0,w(),h());

        // Indicates the buffers currently enabled for color writing.
        glClear(GL_COLOR_BUFFER_BIT);

        // set color (RGB mode)
        glColor3f(1,0,0);

        // draw Polygon
        glBegin(GL_POLYGON);
            glVertex2f(-0.5, -0.5);
            glVertex2f(-0.5, 0.5);
            glVertex2f(0.5, 0.5);
            glVertex2f(0.5, -0.5);
        glEnd();
    }

    public:
       MyGlWindow(int x,int y,int w,int h) : Fl_Gl_Window(x,y,w,h) {}
};
```

The event handler `handle()` checks if any event has been activated. Otherwise it would do nothing. For the case when an event is present `handle()` verifies the type of event. In the example above, `FL_PUSH` represents a mouse button being pushed while `Fl::event_button()` returns which button was pushed. Finally, the `exit(0)` command is executed when the left mouse button (`FL_LEFT_MOUSE`) is pressed.

The `draw()` method is called every time the OpenGL window is drawn (program starts) or needs to be redrawn (e.g. changing polygon parameters with a widget). The first two OpenGL commands set up the scene and the third command changes the color to red so that the rectangle will be red.

The last line is the constructor for the `Fl_Gl_Window` widget, which is used in the main routine.

```
int main() {
    Fl_Window* My_window;
       My_window = new Fl_Window(100, 100, 200, 200, "OpenGL in FLTK");
    MyGlWindow gl_window(10,10,window->w()-20,window->h()-20);
    My_window->end();
```

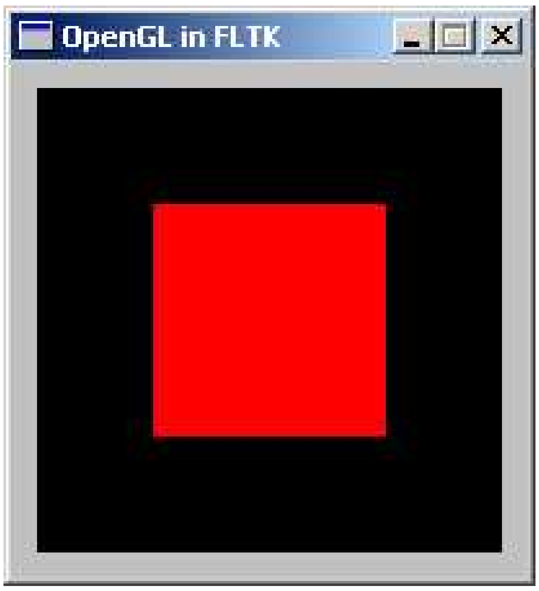


Figure 3.2: OpenGL in FLTK

```
    My_window->show();
    return(Fl::run());
}
```

This example shows that the size of a widget can be set with respect to another widget. The size of the `Fl_Gl_Window` changes when the main window gets bigger or smaller. The result of this example is shown in Fig. 3.2

4 RVS Primitives

As outlined in Chapter 2, OpenGL builds all geometric objects from simple primitives. Of course, it is possible to build a fully rendered robot by defining primitive after primitive and create all the required surfaces. However, it would be a lot of work to do this, especially when many robotic architectures must be created. For this reason, *intermediate-level* primitives were provided by RVS. The final robot design is built using a combination of these intermediate-level primitives. These primitives are referred as *RVS primitives* in the following.

4.1 The Structure of the RVS Primitive Library

4.1.1 Declaration of the Primitives

Nearly 40 different 3D objects can be created with the existing primitives in RVS. The basic idea behind every primitive is the same: The programmer must define the geometric parameters and rendering options for the primitive and a set of functions handle all necessary steps to generate the OpenGL rendering. The following five steps are required for this purpose.

Structure Definition First, the structure of the primitive is specified in the "primitives.h" header file. These are used to save all the necessary geometric parameters like height and radius as well as the rendering options like material and color.

Public Functions for Primitives In the above mentioned header file, a public function for every object is declared. The main definition is in the primitives source file "primitives.c". Only these functions and the final drawing function can be called from outside this file.

```
JwPrimitive JwDefCylinder(int n, float r, float h, int mat,
JwColor c)
```

n	= number of side
r	= radius
h	= height
mat	= material
c	= color

Private definition functions for Units For each object, a private function is defined which is called by the public function defined earlier. The task of this function is to calculate all the required vertices and normals to draw the object.

Private drawing functions for Units This function is needed to set up all OpenGL commands in order to generate the 3D rendering. It uses the calculations of the previous function to set all vertices in the OpenGL scene.

Draw the Primitive When everything has been defined and calculated, the object can be rendered. This is managed by calling the

```
JwDrawPrimitive(JwPrimitive P)
```

function.

4.1.2 Creating and Handling RVS Primitives

When an object is drawn in the OpenGL scene it is displayed in the base frame by default. To build a robot system, it is necessary to edit the position and put the object in the right place. For this reason, some editing primitives, translation and rotation are available:

```
JwEditTranslate(float x, float y, float z)
JwEditRotate(Angle angle, char axis)
```

These functions include the OpenGL commands to translate and rotate a created object. In order to edit only the desired primitive, the `JwEdit` commands have to be surrounded by

```
JwBgnEditPrimitive(JwPrimitive P)
```

and

```
JwEndEditPrimitive()
```

where "`JwPrimitive P`" represents the variable for the primitive. The following example shows how one can create and translate a cylinder using the RVS primitives library.

```
JwPrimitive My_Cylinder;
My_Cylinder = JwDefCylinder(24, 3.0, 3.0, JwShinyMetalMat, SeaGreen);

JwBgnEditPrimitive(My_Cylinder);
    JwEditTranslate(2.5, 1.0, 3.75);
JwEndEditPrimitive();

JwDrawPrimitive(My_Cylinder);
```

4.1.3 Creating and Handling Robot Parts

More complex objects are built from of a combination of simpler primitives. We provide here an example that defines the motor object comprising two cylinders, a ring and a cylinder fillet.

```
// Header File
typedef struct _JwMotor_ {
  float r;
  float len;
  float h;
} *JwMotor, JwMotorRec;


// Source File
JwPrimitive JwDefMotor(float r, float len, float h, int mat, JwColor c)
{
    JwPrimitive P;
    JwMotor B;
    float r1,r2,r3,la;

    P = (JwPrimitive)JwMalloc(sizeof(JwPrimitiveRec));

    P->type = JwMotorT;
    P->spec = B = (JwMotor)JwMalloc(sizeof(JwMotorRec));
    B->r = r;
    B->len = len;
    B->h = h;

    r1 = 0.7*r;
    r2 = 0.6*r;
    r3 = 0.2*r;
    la = 1.0*r;

    P->nu = 4;

    P->U = (JwUnit *)JwMalloc(4*sizeof(JwUnit));
    P->U[0] = def_cyl_fillet(24,r,len,(len/10),(len/10),mat,c);
    JwBgnEditUnit(P->U[0]);
    JwEditTranslate(0.0, 0.0,-(la+(len/2)));
    JwEndEditUnit();
    P->U[1] = def_cylinder(24,r2,(r3/6), mat,Grey|0xcc000000);
    JwBgnEditUnit(P->U[1]);
    JwEditTranslate(0.0, 0.0,-la+(r3/12));
    JwEndEditUnit();

    P->U[2] = def_ring(24,r1,r2,r3, mat,c);
    JwBgnEditUnit(P->U[2]);
    JwEditTranslate(0.0, 0.0,-la+(r3/2));
    JwEndEditUnit();

    P->U[3] = def_cylinder(24, r3,la,  mat,Grey|0xcc000000);
    JwBgnEditUnit(P->U[3]);
    JwEditTranslate(0.0, 0.0,-(la/2));
    JwEndEditUnit();

    return P;
}
```

In the first half of the function, the memory for the primitive is reserved (`JwMalloc`) and the type is specified (`P->type = JwMotorT;`). Furthermore all the necessary geometric parameters for the simpler primitives are calculated. The second half contains the definition of these primitives, which are defined as four units (`P->U[]`) of the motor primitive. All the required vertices for the OpenGL commands as well as the OpenGL commands themselves will be specified within the functions for the simpler primitive. When the `JwDraw` function is called for the `JwMotor` primitive the four objects will be rendered on screen.

4.1.4 Deleting an Object

During the definition process, memory space for the different variables and structures is reserved. To free this memory space again, for example when a new robot architecture is selected, the

```
JwFreePrimitive(JwPrimitive P)
```

function is called.

4.2 Porting the RVS Primitives Library

Porting the primitives from IrisGL to OpenGL was one of the main changes required during the programming. However, the question can be asked “Why write or port RVS primitives when toolkits like GLUT or GLU offer routines to create 3D-objects?”. The answer to this simple. All the existing robot models use RVS primitives. As a result, all these files have to be rewritten when other routines are used.

The porting task itself can be described as a simple change of commands. Although this task may seem to be straight forward, it is a bit tricky in the end. Often the OpenGL Porting Guide [2] can be used to find the new command for an old one. This is sometimes not so easy. Following three types of porting scenarios were faced.

Exact Equivalent

For some statements, the only difference is the notation. For example,

```
linewidth()
```

is the IrisGL call to change the width of a line. In the OpenGL it is:

```
glLineWidth()
```

In this case no further modifications are required.

Exact Equivalent but with Different Argument List

We provide here an example of the porting scenario at hand and a description of how it was handled. The OpenGL command `glMultMatrix()` is the equivalent to the IrisGL command `multmatrix()`. Both commands multiply the projection matrix by an arbitrary matrix. The difference is the way this is managed. In IrisGL the matrix values are saved in a 4x4 array and in OpenGL it is saved in a 1x16 array.

IrisGL

$$\begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} \end{bmatrix}$$

OpenGL

$$\begin{bmatrix} a_1 & a_5 & a_9 & a_{13} & a_2 & a_6 & a_{10} & a_{14} & a_3 & a_7 & a_{11} & a_{14} & a_4 & a_8 & a_{12} & a_{16} \end{bmatrix}$$

In general this would not be a significant problem because OpenGL reads the projection matrix in the correct order. However, one of the RVS primitives library's handling function `JwEditMultMatrix(Matrix M)` receives a matrix M of type 4x4 array. This poses compatibility problems. To solve this problem, the matrix entries must be placed in the correct format in a 1x16 array, before `glMultMatrix()` is called. This is managed by two for-loops as follows:

```
    GL_Matrix Mgl;
    for(i=0;i<4;i++)
    {
      for(j=0;j<4;j++)
      {
        Mgl [i*4+j] = M[j][i];
      }
    }
```

With this solution, all the pre-defined matrices can remain unchanged in the program. If the entries are in the wrong format the robot model is not rendered correctly; the result being misplaced parts in the OpenGL Scene.

No Exact Equivalent

This paragraph provides an example and the solution for the type of porting scenario without an exact equivalent. The IrisGL command `cpack` is used to change the active RGBA (red, green, blue, alpha) values. It expects a single integer value in hexadecimal notation, where the four components range from 0 to 255. For example,

```
    cpack(0xFF004080);
```

sets alpha to 0xFF (255), blue to 0x00 (0), green to 0x40 (64) and red to 0x80 (128). In RVS primitives library all the available color values are defined in "colors.h" and use definition names like `Black`, `SeaGreen` or `WarmGrey`. Unfortunately, OpenGL offers no command to set the color with a single value. As shown in the example in subsection 3.5, the color is set by three single values for the RGB mode. When the RGBA mode is used, the command changes to `glColor4f()`. There are two possible ways to make the pre-defined color values compatible with the new OpenGL command. The first possibility would be to rewrite the header file and separate all values into four new one. This involves rewriting all the primitives. The second option was to add two small functions to the program which take the hexadecimal value, save the RGBA entries in four new variables and then change the color with `glColor4f()`. One of these functions is as followed:

```
void JwSetColor(long JwColor)
{
    float nJwR, nJwG, nJwB, nJwA;
    nJwR=((JwColor&0x000000ff)>>2)/255.0;
    nJwG=((JwColor&0x0000ff00)>>10)/255.0;
    nJwB=((JwColor&0x00ff0000)>>18)/255.0;
    nJwA=((JwColor&0xff000000)>>24)/255.0;
    glColor4f(nJwR,nJwG,nJwB,nJwA);
}
```

The bitwise and-operator is used to separate color value. All values are divided by 255.0 because OpenGL expects values between 0.0 and 1.0 for each settings.

5 Kinematics Engine

5.1 Forward and Inverse Kinematics

With the new interface and the working primitives, RVS is ready to create robot architectures and render them in 3D. However, the program also offers animation of robots. To accomplish this, a *kinematics engine* is required to solve the inverse and direct displacement problem.

Direct Displacement Problem It may be defined as: Given the joint displacements of a serial robot, compute the position and orientation of each link of the robot.

Inverse Displacement Problem It may be defined as: Given the EE pose (position and orientation), find all the joint displacements of the serial robot. This problem accepts either unique, multiple or no solution.

For both problems, a algorithm is available in the original RVS software. The engine in hand uses only C language calls and has no elements of either OpenGL or XForms. Hence, this source code could be implemented in the new software without any change.

5.2 Animations in FLTK/OpenGL

Calculating the new link position in Cartesian space is only half the task. The remaining half is to display the animation in the OpenGL scene. It is quite easy to calculate the new position for each link and redraw the completed model. The effect is that the robot "jumps" from the old posture to the new one. In fact, this is the basic idea of motion in RVS. When the robot follows a predefined trajectory, RVS simply renders the intermediate postures sequentially. As long as these intermediate postures are close together, this appears to be a smooth movement. Three steps are required to generate an animation:

1. Load the new posture from a saved file;
2. calculate the new position for each link; and
3. update the robot model.

How these steps are managed is now explained for the "Joint Trajectory" feature. The saved trajectory file contains several lines with joint angles in the following format:

$$\begin{matrix} \theta_1(1) & \theta_2(1) & \theta_3(1) & \dots \\ \theta_1(2) & \theta_2(2) & \theta_3(2) & \dots \\ \theta_1(3) & \theta_2(3) & \theta_3(3) & \dots \\ \vdots & \vdots & \vdots & \ddots \end{matrix}$$

For example, for the 7-link REDIESTRO robot, the file would look like

$$\begin{matrix} -101.7014 & -17.6766 & 98.9590 & 88.8606 & -25.0257 & 119.4901 & -113.3444 \\ -101.6753 & -17.6985 & 98.8462 & 88.9656 & -25.2299 & 119.6729 & -113.5423 \\ -101.6438 & -17.7185 & 98.7321 & 89.0719 & -25.4328 & 119.8611 & -113.7414 \\ -101.6070 & -17.7365 & 98.6165 & 89.1794 & -25.6346 & 120.0545 & -113.9415 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{matrix}$$

and has, depending on the length of the trajectory, hundreds of these lines. In the end, it is up to the user to define how precise the whole trajectory will be and how many postures are saved in the file. When the saved file is chosen from the "JTraj" directory, RVS reads all the lines, saves the posture in an array and calculates link positions in the Cartesian space for each posture.

When the user presses the play button for the forward mode, the animation starts. At a first glance, this seems to be a simple task. Just one for-loop to set the new postures and redraw the model. However, this is not possible because of the FLTK routine to redraw the OpenGL scene. When the `redraw()` method for a widget, in this case the `Fl_Gl_Window`, is called, the widget will not be updated immediately. The widget is updated, when programm enters the FLTK main loop. The result for the aforementioned for-loop is that the program will set the `redraw()` method for every posture but only the last posture will be rendered. Therefore, the simulation would show the robot "jump" from the initial to the final posture.

The solution to this problem is the **idle** callback. This function is called every time the FLTK program waits for a new event. When the idle callback is inside the main FLTK loop, FLTK calls the idle callback continuously. This feature is used to get background processing done. With the `Fl::add_idle()` command, an idle callback can be added to the program. For the RVS project, it is initialized when the OpenGL window is created. Inside the callback an if-statement for each type of animation is defined and hence the source code will be executed when the condition is true. Fig. 5.1 shows the process inside the idle callback.

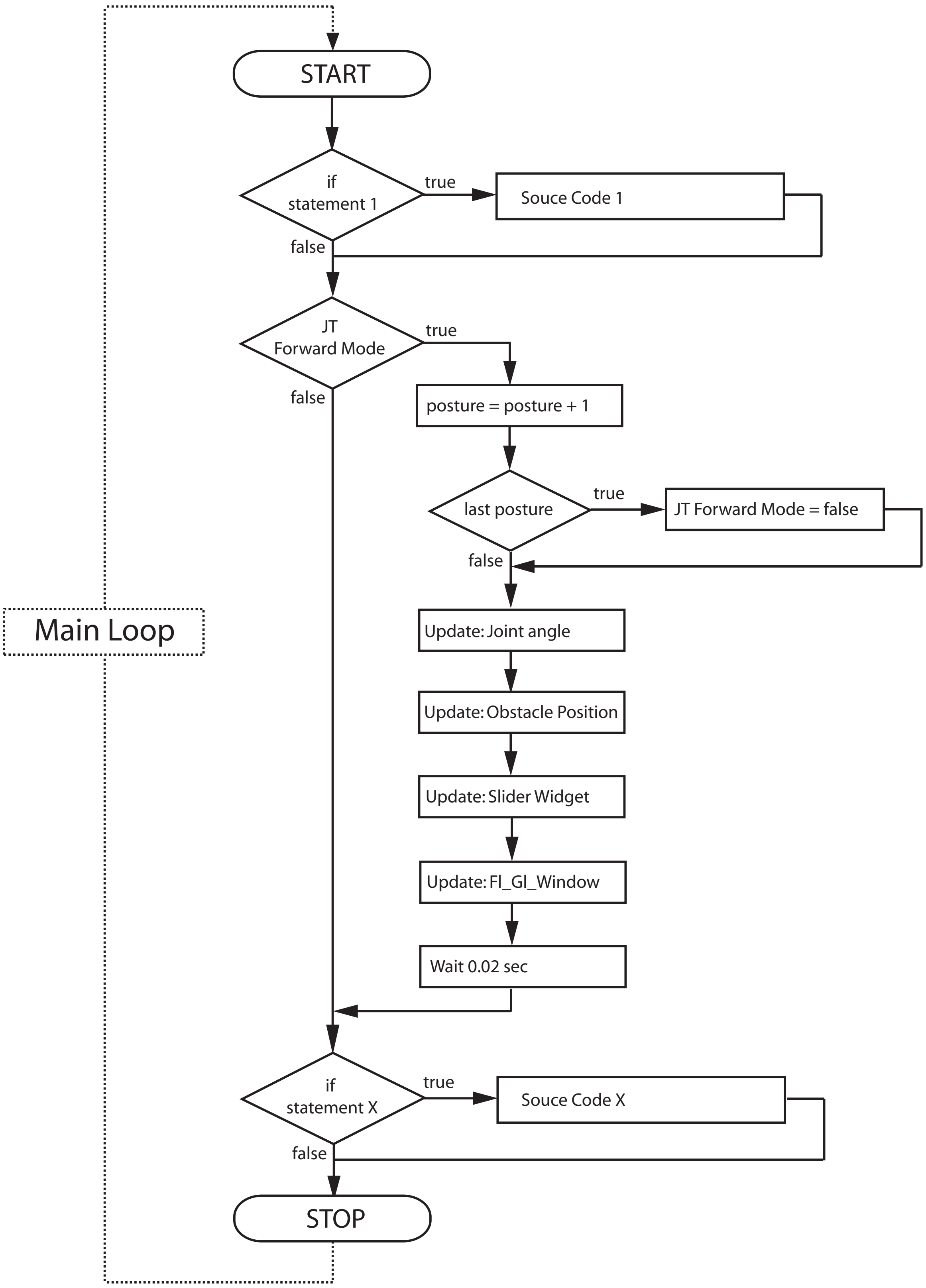


Figure 5.1: Flowchart : IdleCallback

6 RVS 2006

6.1 Integrated Development Environment

An integrated development environment (IDE) is a program that assists computer programmers to develop software. Usually, a source code editor, a compiler and/or interpreter and a debugger are integrated in those packages. All these features are very useful to programm big projects. The programmer includes several files to a project, sets all the linker and compiler options and the IDE builds automatically an executable file.

Initially, Microsoft Visual Studio .NET 2003 was selected. However, this choice was changed latter to Dev-C++ from Bloodshed Software. Of course the company's name sounds a bit curious, but Dev-C++ has some very useful features to offer. First of all it is quite small (around 10MB) and published under the GNU GPL[1], making it a free software that can be downloaded under:

`www.bloodshed.net/devcpp.html`

As a result, every user who wants to edit the source code later can easily install the IDE and open the RVS project on their computer.

Another feature of this programm are small packages called DevPaks. These packages are plug-ins (e.g. OpenGL, FLTK, GLUT) for the IDE and are also available online. On users demand Dev-C++ can download and install all the required libraries on the computer. The process is transparent to the IDE user/programmer.

The current version of Dev-C++ is only available for the Microsoft Windows operating system. An alternative could be Code::Blocks. This IDE runs also under LINUX, can open Dev-C++ projects as well as use DevPaks. Furthermore, any other IDE could be used.

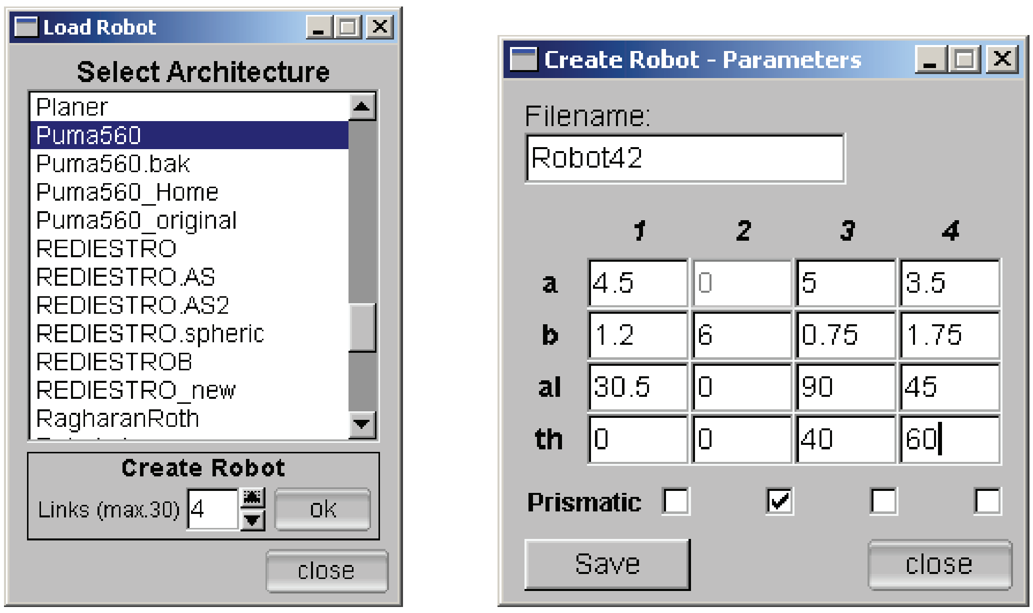
6.2 New Features in RVS

6.2.1 Feature: New Robot

In the original RVS software was no function to create a new architecture within the program. The user had to open an ASCII[2] text editor and write a file for the new

[1] GNU General Public License

[2] ASCII American Standard Code for Information Interchange



Load Robot

Select Architecture

- Planer
- Puma560
- Puma560.bak
- Puma560_Home
- Puma560_original
- REDIESTRO
- REDIESTRO.AS
- REDIESTRO.AS2
- REDIESTRO.spheric
- REDIESTROB
- REDIESTRO_new
- RagharanRoth

Create Robot

Links (max.30) 4 ok

close

Create Robot - Parameters

Filename: Robot42

	1	2	3	4
a	4.5	0	5	3.5
b	1.2	6	0.75	1.75
al	30.5	0	90	45
th	0	0	40	60
Prismatic	☐	✓	☐	☐

Save close

(a) (b)

Figure 6.1: Interface windows for "New Robot": (a) Menu: Load Robot; (b) Menu: Create Robot - Paramertes

architecture. In order to make RVS more user friendly, the program can now take care of this task within the user interface. In this vein the user must define the number of links (Fig. 6.1(a)), the file name, the DH parameters and the joint type (Fig. 6.1(b)). By clicking the "save" button the program creates a new architecture file in the "Arch" directory. The program verifies whether the selected filename already exist. In that case the user has the choice to overwrite the existing filename or to provide another filename. Once the new file is created it can be selected from the file browser.

6.2.2 Feature: Edit Robot

Another feature added is the ability to edit the saved parameters within the interface. When the user changes a DH parameter, the robot is redrawn *immediately* in the OpenGL scene. After all the modifications have been made, the new architecture can be saved. Furthermore, the save option is deactivated when a fully rendered model for the selected robot is available. For a fully rendered model, a mere change of the DH parameters would make the full rendering incompatible. Full-renderings should be edited as discussed in Section 6.3.

6.2.3 Feature: Work Environment

Generally, every robot is designed for a special family of task, e.g. pick-and-place operations. Furthermore, the robot should work in a special work environment. To simulate this, RVS offers a variety of objects to create a work environment. The available list of objects is as follows:

- Table,
- conveyor,
- pallet,
- battery,
- peg,
- surface and
- fuselage.

This list could be extended in the future. Figure 6.2 shows a pick and place work environment built around a PUMA robot. In the following two ways are discussed to create a work environment for RVS.

Creating a Work Environment at Source Code Level

The programming of these object is similar to that of a link of a fully rendered robot model. Every object is built from a number of primitives and defined as a new drawing object of the type `Thing`. In the source code, a workspace object is than created like a `JwPrimitive` object. A new variable of type `Thing` is declared, the geometric parameters and the color are defined, and finally the object is rendered. The following example shows how to create a conveyor.

```
Thing conveyor;
conveyor = DefConveyor(4, 2, 3, SeaGreen);
DrawThing(conveyor);
```

Two editing primitives are available to translate or to rotate the created object. After each object has been defined in the source code, the whole project needs to be recompiled. This approach becomes tedious for the case when only one position parameter for an object has to be changed. Finally, it is not possible to exchange a created work area easily between two computers, as it is possible for architecture files.

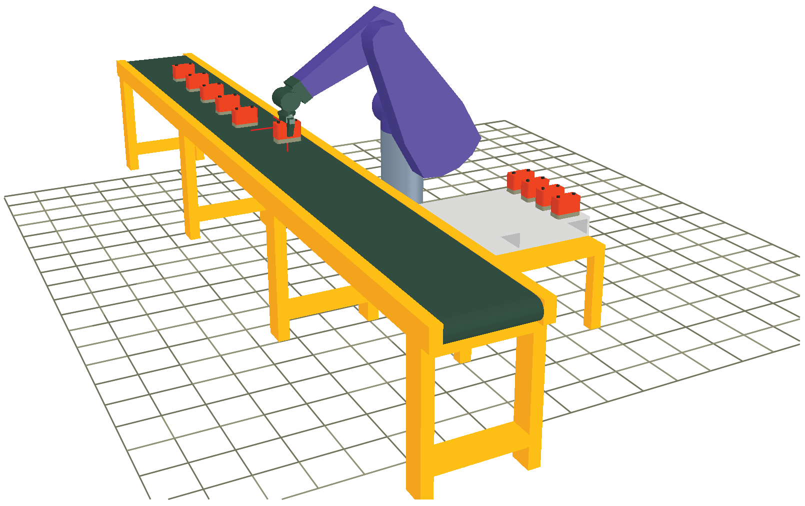
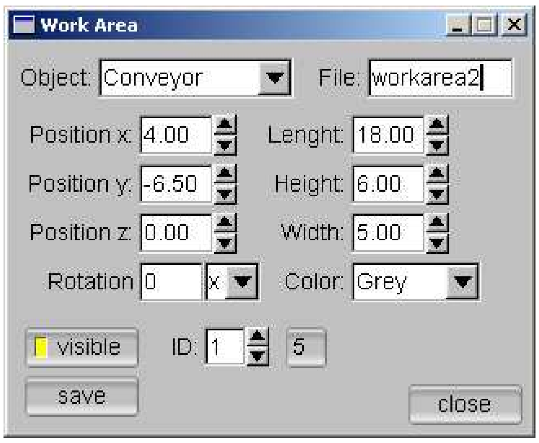


Figure 6.2: Puma robot: Pick and place operation



Object: Conveyor
File: workarea2
Position x: 4.00
Position y: -6.50
Position z: 0.00
Rotation 0 x
Lenght: 18.00
Height: 6.00
Width: 5.00
Color: Grey
ID: 1
5

Figure 6.3: Interface window for "Work Environment"

Creating a Work Environment within the Interface

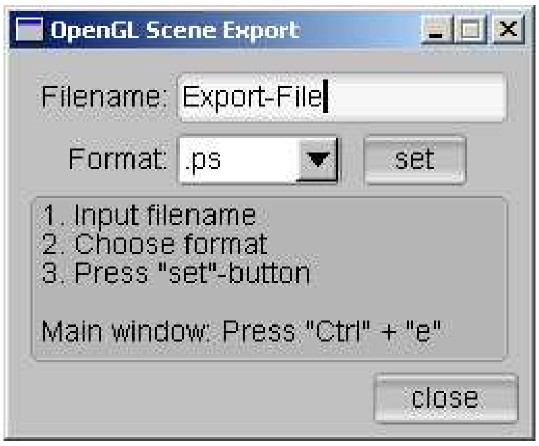
A new "Work Environment" entry has been added to the menu. A window (Fig. 6.3) displays all the necessary options to select an object, set parameters and place it around the robot. To create a new object, the user must increase the ID-number. Each object and its parameters can be identified with this ID-number. The limit for the maximum number of objects is set inside the source code and is currently set to 20 objects. Finally, the created scene can be save to a simple text file. Hence, it is now possible to create, save and reload a work environment.

6.2.4 Feature: OpenGL Scene Export

Once an OpenGL scene is created and displayed on screen, it is quite useful generate an image file for a presentation or a report. This could be managed by taking a screenshot[3], which is a raster/bitmap-based file or by creating a vector-based file. A screenshot is easy to generate, but the quality is not good enough for detailed printouts. The advantage of a vector-based file is that the exported file will always be a high quality image.

Therefore, a library called **gl2ps**, is implemented in RVS. The library can create a vector-based file (ps,eps,pdf,svg) of a OpenGL scene. The user has to set the filename and the desired format in the interface window and finally start the export with the shortcut[4] "`crtl + e`".

[3] Outputting the entire screen in a common format such as BMP, PNG, or JPEG
[4] A keyboard shortcut is a key or set of keys that performs a predefined function


OpenGL Scene Export

Filename: Export-File

Format: .ps

1. Input filename
2. Choose format
3. Press "set"-button

Main window: Press "Ctrl" + "e"

Figure 6.4: OpenGL Scene Export

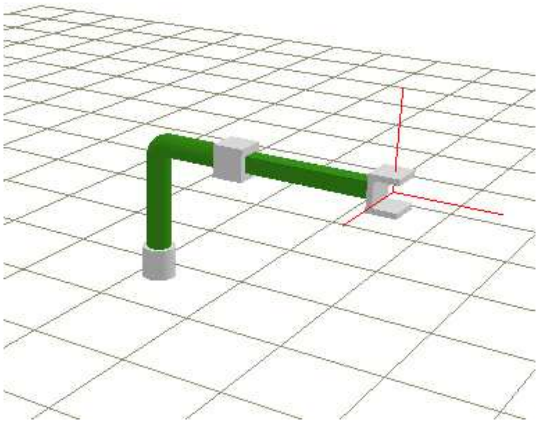
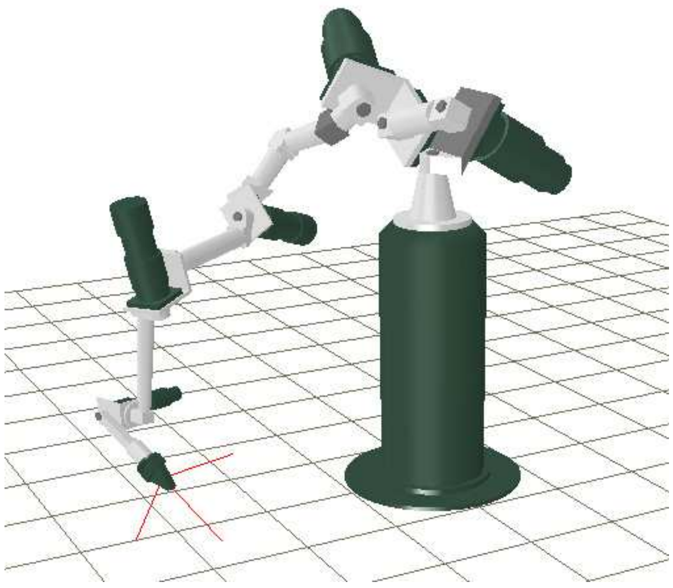


Figure 6.5: Skeleton drawing of a 2-link robot

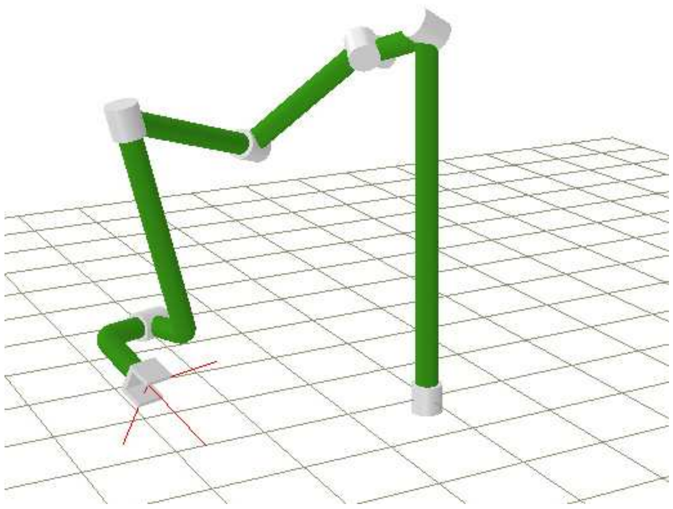
6.3 How to Create a Fully Rendered Robot

This section describes all the necessary steps to create a full rendered robot. However first, it is important to understand how a robot skeleton is displayed on the screen. The skeleton mode is the default mode when a fully rendered robot model is not available. The program reads the DH parameters for the selected robot from the architecture file and creates the links accordingly. A skeleton revolute link is L-shaped and is built with two cylinders and one elbow primitives. For a straight prismatic link, a prism primitive is used. Finally, the base and the end-effector drawings are always the same for a skeleton model (Fig. 6.5).

A little more effort is required to generate the fully rendered model. Four steps are required, to achieve the task at hand:



(a)



(b)

Figure 6.6: Model views of REDIESTRO: (a) Fully rendered model; (b) skeleton model

1. **Architecture File** – Every model starts with its architecture file, which could be generated using an ASCII text editor or the newly implemented function in the program. Appendix A shows the complete architecture file for “PUMA560”.

2. **Robot File** – Creating this C-language file, is the main task to obtain for a fully rendered model. It is a file with the same name as its corresponding architecture file. Each link is built using several RVS primitives. Further, the base and the end-effector have to be designed with the RVS primitives. The result is a file with several combinations of drawing and editing functions. For example, the first link of the “REDIESTRO” model (Fig. 6.6(a)) uses 10 RVS primitives and 42 editing functions. Figure 6.6(b) shows the skeleton model for the same robot with the standard link design.

3. **Modifications** – Once the Robot file is complete, the files “Links.h” and “Links.c” have to be modified. In these files, the new robot file must be implemented, so that the program can render the full geometry when the architecture is selected. The “Links.*” files include functions to draw the defined model.

4. **Recompiling** – The last step is to compile the new robot file and the modified files to create a new executable file.

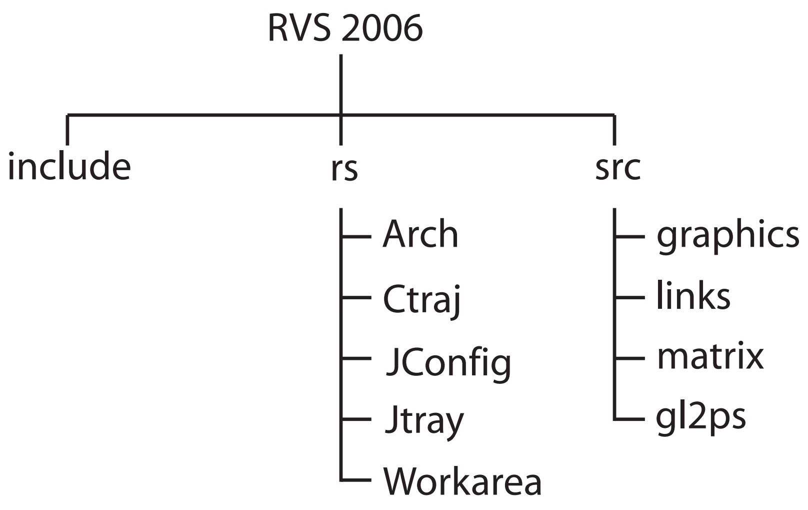


Figure 6.7: RVS2006 - Directory tree

6.4 File and Folder Overview

The major problem during the analysis of the original RVS was the lack of documentation. As mentioned in Chapter 2, the RVS user manual offers little in the overview of files and folders. This becomes particulary evident when some of the important files are not even mentioned in the available overview. Hence, the only way to find a functions was to go through the source code which was tedious and time consuming. To facilitate future programmers we aim to include a more detailed overview of the file structure. In the interest of space a general outline is provided. Figure 6.7 shows the directory tree of the RVS project and the following explains the folders and special files.

Directory	**Filename**	**Explanation**
\include		This directory contains the header files for some source files.
	colors.h	Available color definitions
	primitives.h	Type definition for all RVS primitives.
	robot.h	Structure definition for a link, the end effector and the robot.
\rs		This directory contains the main source files.
\rs\Arch		Robot architectures are saved in this directory.
\rs\Ctraj		Files with the cartesian trajectories are saved in this folder.
\rs\Jconfig		Posture files for the different robots.

Directory	**Filename**	**Explanation**
\rs\Jtraj		Saved files for joint trajectories. An example of such a file is available in Section 5.2.
\rs\Work_Env		The created work area scenes mentioned in subsection 6.2.3 are saved here
	main.c	Source code for the main program main-function declaration of global variables FLTK main window function `gl_win_redraw()` for the `redraw()`-method
	menu.*	Source code for the menu The menu.c file includes one block of functions for each menu entry. The “main” function (to create the window) for each block should give the name for each function and widget of the block. For example the “main” function to load a new architecture is `void arch()`, so all other names should start with `arch_`. Callbacks are named like the corresponding widget and have the extension `_cb`. Because the file got very large some functions are saved in new files which are named `menu_`*`filename`*`.c` The Function `Robot NewRobot(char *name)` is called when an new robot is selected from the list.
	opengl_win.c opengl_win.h	Source file for the `Fl_Gl_Window` Constructor and destructor for the widget. Settings for the OpenGL window, like background color, colormode, etc. The `draw()` which calls the functions to draw the robot, the floor, the effector and the trajectory course. `void IdleCallback()` for all animations. `void handle()` for all I/O actions. Header file for the declaration.
	reader.*	Header and source file for functions to load new robot data from the architecture file and the data for the trajectories.
	ikp.c	Source file with the functions for the kinematic engine.
	setting.c	Source file for all the workspace objects.
	RVS2006.exe	Executable file for WIN32 applications.
\src		This directory contains the source files for different functions
	axes.c	Provides two functions to draw frames at the base,

Directory	Filename	Explanation
		each joint and the end effector. `void JwAxes_xyz(float len, JwColor color)` `void JwAxes_XYZ(float len, JwColor color)`
	light.c	Source code for all the light settings in the OpenGL scene. `void JwInitLight()` Function `JwSetColor()` to change the defined color value in single RGBA values.
	primitives.c	Source code for all the basic primitives. (Chapter 4)
\links		Includes files for the fully rendered models.
	links.c	Functions to check for a fully rendered model.
	robotname.c	These are the files for the defined fully rendered models.
\matrix		This folder contains several files for mathematical operations, such as cross product, matrix vector multiplications or Cholesky decomposition.
\gl2ps		This folder contains all files for the gl2ps library

6.5 Required Library files

To compile the source code some libraries have to be added to the linker options. The following table includes all required libraries for a WIN32-application.

FLTK library	libfltk.a
Libraries used by FLTK	libole32.a libuuid.a libcomct32.a libwsock32.a libm.a
OpenGL library for FLTK	libfltk_gl.a
Standard OpenGL library	libopengl32.a
GLU library	libglu32.a
Windows GDI	libgdi32.a

7 Geometry of Serial Robots

7.1 Revolute and Prismatic Links

The architecture of a serial robot can be described as an assembly of several rigid bodies, or *links*, thereby forming what is known as a *kinematic chain*. Two links are coupled by a kinematic pair, also termed a *joint*. Depending on the type of contact between the two links, the kinematic pair can be either lower or higher. Only the two basic lower types are considered here, as their higher counterparts appear in robots only exceptionally.

Turning Pair This type is also called a *revolute* joint. The contact surface between the two links is a surface of revolution not allowing sliding along its axis of symmetry. As a result, the two links can only rotate about the foregoing axis with respect to each other.
Example: Journal bearing

Sliding Pair This pair, also called a *prismatic* joint, allows for a relative pure-translation motion. This pair, contrary to the revolute, has no axis only a direction of motion.
Example: Dovetail coupling

The kinematic chain can be also of one of two types. In a *closed chain*, each link is coupled to two other links. This chain is also called a *linkage*. The chain is an open chain, when it contains exactly two links that are connected to only one other link. Hence, a serial robot is an open kinematic chain, because the first and the last link are only connected to one other link. In robotics the first link is called the *base*, the last link is the *end-effector*.

7.2 Denavit-Hartenberg Notation

For a robotic architecture each link, except the base and the end effector, lies between two kinematic pairs. In order to describe the robot geometry precisely, the *Denavit-Hartenberg notation* (Denavit and Hartenberg, 1955) is introduced. All links are numbered from 0 (base) to n (end-effector) and each pair is defined as the coupling between the (i–1)st and ith link. Next, a coordinate frame $\mathcal{F}_i$ (O_i, X_i,Y_i, Z_i) is defined, which is connected to the (i–1)st link. Therefore, the frames are numbered from $i = 1, 2, \ldots, n+1$. Following the rules for the Denavit-Hartenberg notation, illustrated in Fig. 7.1, and assuming only revolute joints,

1. Z_i is the axis of the i*th* kinematic pair
2. X_i is the common perpendicular from Z_{i-1} to Z_i (If these two axes are parallel, then the location of X_i is undefined.) Y_i is defined by the right-hand rule.
3. a_i is the distance between Z_i and Z_{i+1}, this is the *link lengths*
4. b_i is the Z_i-coordinate, of the intersection of Z_i with X_{i+1}. Parameters b_i is also called the *link offset* and can be positive or negative.
5. α_i is the angle between Z_i and Z_{i+1} measured positive in the direction of X_{i+1}. This parameter is called *twist angle.*
6. θ_i is the angle between X_i and X_{i+1} measured positive in the direction of Z_{i+1}. This angle is called the *joint angle.*

Because a robot does not have a $(n+1)$st link, these rules do not apply to the last frame. Therefore, the frame can be defined arbitrarily, but its origin is placed at a specific point, the *operation point P*, of the end-effector. In summary, the kinematic chain contains $n+1$ links, $n+1$ frames and n kinematic pairs.

The three variables a_i, b_i and α_i are called the *joint parameters* and represent the fundamental geometry of the robot. The *architecture* of each link is defined by the set of these parameters. They are constant because the architecture of a robot does not change when the robot moves. What changes is the *joint variable* θ_i.

In order to describe the complete architecture and posture of a n-link robot $3n$ joint parameters and n joint variables are needed. This leads to $4n$ *Denavit-Hartenberg parameters* for the robot.

Position vectors $\mathbf{e}_i$

The vector e_i is defined along the Z-axis of the frame $\mathcal{F}_i$. In frame $\mathcal{F}_i$, this vector components are

$$[\mathbf{e}_i]_i \equiv \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \tag{7.1}$$

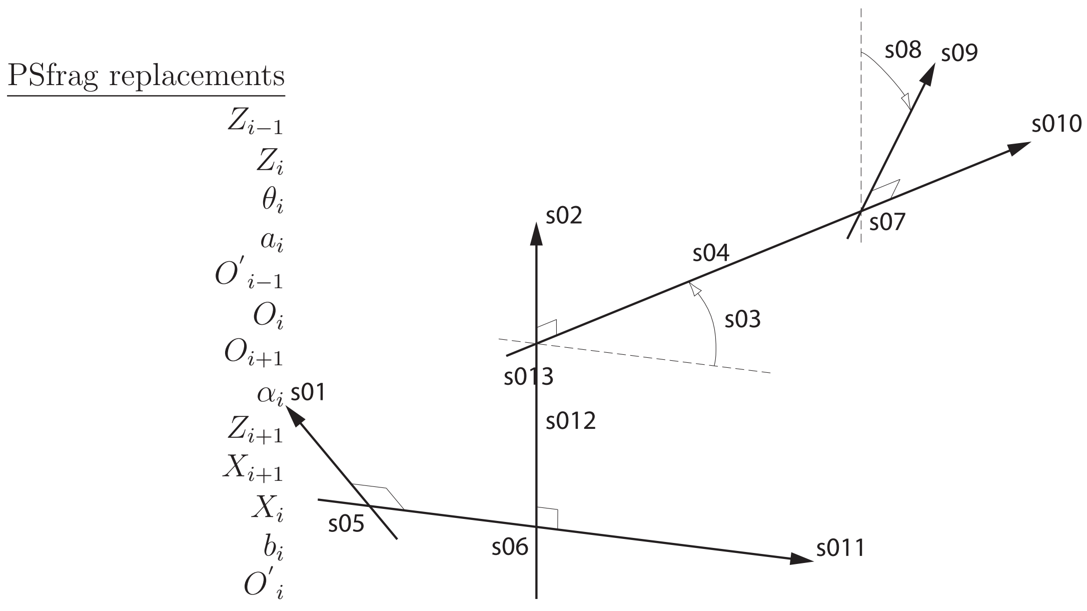


Figure 7.1: Denavit-Hartenberg Notation

Position vectors $\mathbf{a}_i$

The directed from the origin of the ith frame to the origin of the $(i+1)$st frame is defined by the vector $\mathbf{a}_i$, whose components in $\mathcal{F}_i$ are

$$[\mathbf{a}_i]_i = \begin{bmatrix} a_i \cos\theta_i \\ a_i \sin\theta_i \\ b_i \end{bmatrix} \tag{7.2}$$

Position vectors $\mathbf{r}_i$

The vector $\mathbf{r}_i$ is defined directed from the origin O_i of the ith frame to the *operation point* P of the end-effector, namely,

$$\mathbf{r}_i = \mathbf{a}_i + \mathbf{a}_{i+1} + \mathbf{a}_{i+2} + \ldots + \mathbf{a}_n \tag{7.3}$$

To calculate the above vector, all vectors on the right–hand side of the foregoing expression have to be expressed in the ith frame.

7.3 Rotation Matrix $\mathbf{Q}_i$

The matrix $\mathbf{Q}_i$ is used to transform a vector or a matrix from the $i + 1$st frame into the ith frame. This matrix is given by [10]

$$\mathbf{Q}_i = \begin{bmatrix} \cos\theta_i & -\cos\alpha_i \sin\theta_i & \sin\alpha_i \sin\theta_i \\ \sin\theta_i & \cos\alpha_i \cos\theta_i & \sin\alpha_i \cos\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i \end{bmatrix} \tag{7.4}$$

For example, vector $\mathbf{r}_i$ expressed in the ith frame is:

$$\mathbf{r}_i = \mathbf{a}_i + \mathbf{Q}_i\mathbf{a}_{i+1} + \mathbf{Q}_i\mathbf{Q}_{i+1}\mathbf{a}_{i+2} + \ldots + \mathbf{Q}_i \cdots \mathbf{Q}_{n-1}\mathbf{a}_n \tag{7.5}$$

8 Tools for the Optimum Design of Robots using Gradient Methods

8.1 Introduction

The design of a new robot starts with the determination of performance specifications that should be achieved by the robot. On the basis of these specifications the various links will be designed. There are different methods to define the fundamental geometry for a robot, like Burmester Theory or the minimization of the condition number of the Jacobian matrix at one robot posture. The latter approach was used of Khan and Angeles[3] and is also used in this project. The following sections give a brief introduction.

Remark

For brevity, the details of derivations below are not included in this report, but the pertinent references are included.

8.2 Mathematical Background

8.2.1 The Jacobian Matrix

In robotics the *Jacobian Matrix* ($\mathbf{J}$) is defined as a $6 \times n$ matrix mapping the set of joint rates $\dot{\boldsymbol{\theta}}$) into the twist ($\mathbf{t}$) of the end-effector, namely,

$$\mathbf{J}\dot{\boldsymbol{\theta}} = \mathbf{t}$$

where

$$\mathbf{J} = \begin{bmatrix} \mathbf{e}_1 & \mathbf{e}_2 & \cdots & \mathbf{e}_n \\ \mathbf{e}_1 \times \mathbf{r}_i & \mathbf{e}_2 \times \mathbf{r}_2 & \cdots & \mathbf{e}_n \times \mathbf{r}_n \end{bmatrix}$$

8.2.2 Condition Number

The condition number is a measure of the roundoff error amplification of the computed results with respect to the roundoff error of the input data. The general definition of the condition number of a nonsingular matrix $\mathbf{A}$ is possible when all matrix entries have the same physical units. In that case the condition number is defined as

$$\kappa(\mathbf{A}) = \|\mathbf{A}\|\|\mathbf{A}^{-1}\| \tag{8.1}$$

where $\|\cdot\|$ stands for a arbitrary matrix norm. In this report the Frobenius norm $\|\cdot\|_F$ is used throughout. The Frobenius norm of the $n \times n$ nonsingular matrix $\mathbf{A}$ s defined as

$$\|\mathbf{A}\|_F = \sqrt{\frac{1}{n}\mathrm{tr}(\mathbf{A}\mathbf{A}^T)} = \sqrt{\frac{1}{n}\mathrm{tr}(\mathbf{A}^T\mathbf{A})} \tag{8.2}$$

The Frobenius norm of $\mathbf{A}^{-1}$ is defined as

$$\|\mathbf{A}^{-1}\|_F = \sqrt{\frac{1}{n}\mathrm{tr}(\mathbf{A}^{-1}\mathbf{A}^{-T})} = \sqrt{\frac{1}{n}\mathrm{tr}[(\mathbf{A}^T\mathbf{A})^{-1}]} = \sqrt{\frac{1}{n}\mathrm{tr}[(\mathbf{A}\mathbf{A}^T)^{-1}]} \tag{8.3}$$

Finally, the Frobenius–norm condition number of $\mathbf{A}$ is given by

$$\kappa_F(\mathbf{A}) = \frac{1}{n}\sqrt{\mathrm{tr}(\mathbf{A}\mathbf{A}^T)\mathrm{tr}[(\mathbf{A}\mathbf{A}^T)^{-1}]} = \frac{1}{n}\sqrt{\mathrm{tr}(\mathbf{A}^T\mathbf{A})\mathrm{tr}[\mathbf{A}^T(\mathbf{A})^{-1}]} \tag{8.4}$$

For brevity, we shall refer to $\kappa_F(\mathbf{A})$ simply as the *condition number* of $\mathbf{A}$.

8.2.3 Concept of Homogenous Space

As long as, the Jacobian matrix contains entries with nonhomogeneous dimensions, the condition number cannot be calculated. To solve this problem, the concepts of *homogenous space* and *characteristic length* are introduced. The goal of this concept is to make all the entries dimensionless. The homogenous space is defined as a dimensionless Euclidean space.

Similar to the Denavit-Hartenberg notation frames, vectors and distances can be defined in homogenous space. With the same rules the homogenous counterparts $\overline{a}_i$, $\overline{b}_i$ of the DH parameters $\mathbf{a}_i$ and $\mathbf{b}_i$ are defined. The two angles α_i and θ_i bear no physical units while the two variables $\overline{a}_i$ and $\overline{b}_i$ are nothing but length ratios, i.e.,

$$\begin{aligned}\overline{a}_i &= \frac{a_i}{L}\\ \overline{b}_i &= \frac{b_i}{L}\end{aligned}$$

where L is the characteristic length, as yet to be defined. With the foregoing homogenous variables, the dimensionless counterparts $\overline{\mathbf{a}}_i$ and $\overline{\mathbf{r}}_i$ of the vectors $\mathbf{a}_i$ and $\mathbf{r}_i$ are then defined, the homogeneous Jacobian Matrix **H** taking the form

$$\mathbf{H} = \begin{bmatrix} \mathbf{e}_1 & \mathbf{e}_2 & \dots & \mathbf{e}_n \\ \mathbf{e}_1 \times \bar{\mathbf{r}}_1 & \mathbf{e}_2 \times \bar{\mathbf{r}}_2 & \dots & \mathbf{e}_n \times \bar{\mathbf{r}}_n \end{bmatrix} \tag{8.5}$$

8.3 Optimization Problem

The approach, mentioned in the Introduction, optimizing the robot dimensions over its architecture parameters and joint variables, is adopted here. For a n-axis robot, $4n$ design parameters are available over which the designer can minimize the condition number of the Jacobian matrix. Three of these parameters do not influence the condition number. The remaining $4n-3$ design variables are group in a *design vector* $\mathbf{x}$ in the form

$$\mathbf{x} = \begin{bmatrix} \bar{a}_1 & \alpha_1 & \bar{a}_2 & \bar{b}_2 & \alpha_2 & \theta_2 & \cdots & \bar{a}_n & \bar{b}_n & \theta_n \end{bmatrix}^T \tag{8.6}$$

In this report, we restrict ourselves to th robots with six revolute joints, which leads to $n = 6$, and hence, a 6×6 homogeneous Jacobian matrix. The optimum design problem is then defined as

$$\min_{\mathbf{x}} \kappa_F(\mathbf{H}) \tag{8.7}$$

subject to the constraints

$$\begin{aligned} \|\mathbf{e}_i\| &= 1, \qquad i = 2, \dots, n && (8.8) \\ \mathbf{e}_i \cdot \bar{\mathbf{r}}_i &= 0, \qquad i = 1, 2, \dots, n && (8.9) \end{aligned}$$

The number of the first set of constraints is $n-1$, that of the second set is n, which leads to $2n-1$ constraints for a n-joint robot.

8.4 Method of Solution

In [3] the *direct method* was used to solve the optimum design problem. In this project, the application of a *gradient method*, like the *Orthogonal-Decomposition Algorithm*[4] is investigated. The problem is defined as:

$$f(\mathbf{x}) \rightarrow \min_{\mathbf{x}} \tag{8.10}$$

subject to the constraints

$$\mathbf{h}(\mathbf{x}) = 0 \tag{8.11}$$

where

$$f(\mathbf{x}) = \frac{1}{6}\sqrt{\mathrm{tr}(\mathbf{H}\mathbf{H}^T)\mathrm{tr}[(\mathbf{H}\mathbf{H}^T)^{-1}]} \tag{8.12}$$

Vector $\mathbf{h}$ includes the left-hand sides of constraints 8.8 and 8.9, namely,

$$\mathbf{h} = \begin{bmatrix} \|\mathbf{e}_2\|^2 - 1 \\ \vdots \\ \|\mathbf{e}_n\|^2 - 1 \\ \mathbf{e}_1 \cdot \bar{\mathbf{r}}_1 \\ \vdots \\ \mathbf{e}_n \cdot \bar{\mathbf{r}}_i \end{bmatrix} \tag{8.13}$$

The Orthogonal-Decomposition Algorithm requires the gradient and the Hessian of the objective function and the gradient of the vector $\mathbf{h}$. For the ensuing derivations the objective function can be simplified. For starters, this function is squared, for the function will be minimized when its square is minimized. Moreover, the factor 1/36 can be dropped, for it has no effect on the minimum. We thus have the new objective function

$$z = \mathrm{tr}(\mathbf{H}^T\mathbf{H})\mathrm{tr}[(\mathbf{H}^T\mathbf{H})^{-1}] \to \min_{\mathbf{x}} \tag{8.14}$$

which is factored into two functions, namley,

$$f_1 = \mathrm{tr}(\mathbf{H}^T\mathbf{H}) \tag{8.15}$$

$$f_2 = \mathrm{tr}[(\mathbf{H}^T\mathbf{H})^{-1}] \tag{8.16}$$

This leads to the gradient

$$\nabla z = f_2 \nabla f_1 + f_1 \nabla f_2 \tag{8.17}$$

8.4.1 Gradient of f_1

The gradient of f_1 with respect to a vector $\mathbf{x}$ is defined as

$$\frac{\partial}{\partial \mathbf{x}}\left[\mathrm{tr}(\mathbf{H}^T\mathbf{H})\right] \equiv \mathrm{tr}\left[\frac{\partial}{\partial \mathbf{x}}\left(\mathbf{H}^T\mathbf{H}\right)\right] \tag{8.18}$$

which thus leads to a third-rank tensor. As a means to avoid cumbersome third-rank tensors, we express the above gradient in the form

$$\frac{\partial}{\partial \mathbf{x}}\left[\mathrm{tr}(\mathbf{H}^T\mathbf{H})\right] = \begin{bmatrix} \partial \mathrm{tr}(\mathbf{H}^T\mathbf{H})/\partial x_1 \\ \partial \mathrm{tr}(\mathbf{H}^T\mathbf{H})/\partial x_2 \\ \vdots \\ \partial \mathrm{tr}(\mathbf{H}^T\mathbf{H})/\partial x_l \end{bmatrix} = \begin{bmatrix} \mathrm{tr}(\partial \mathbf{H}^T\mathbf{H}/\partial x_1) \\ \mathrm{tr}(\partial \mathbf{H}^T\mathbf{H}/\partial x_2) \\ \vdots \\ \mathrm{tr}(\partial \mathbf{H}^T\mathbf{H}/\partial x_l) \end{bmatrix} \tag{8.19}$$

For each entry of the design vector $\mathbf{x}$, a partial derivative of $\mathbf{H}^T\mathbf{H}$ with respect to a scalar has to be calculated, which yields a matrix. Since the trace of a matrix is a scalar, all array entries in eg. 8.19 are scalar. The product of the two matrices is a square, symmetric matrix, i.e.,

$$\mathbf{H}^T\mathbf{H} = \begin{bmatrix} \mathbf{e}_1^T\mathbf{e}_1 + (\mathbf{e}_1 \times \overline{\mathbf{r}}_1)^T(\mathbf{e}_1 \times \overline{\mathbf{r}}_1) & \cdots & \mathbf{e}_1^T\mathbf{e}_6 + (\mathbf{e}_1 \times \overline{\mathbf{r}}_1)^T(\mathbf{e}_6 \times \overline{\mathbf{r}}_6) \\ \vdots & \ddots & \vdots \\ \mathbf{e}_6\mathbf{e}_1^T + (\mathbf{e}_6 \times \overline{\mathbf{r}}_6)(\mathbf{e}_1 \times \overline{\mathbf{r}}_1)^T & \cdots & \mathbf{e}_6^T\mathbf{e}_6 + (\mathbf{e}_6 \times \overline{\mathbf{r}}_6)^T(\mathbf{e}_6 \times \overline{\mathbf{r}}_6) \end{bmatrix} \tag{8.20}$$

whose entries are all dimensionless, whence,

$$\operatorname{tr}\left(\mathbf{H}^T\mathbf{H}\right) = \|\mathbf{e}_1\|^2 + \|\mathbf{e}_1 \times \overline{\mathbf{r}}_1\|^2 + \cdots + \|\mathbf{e}_6\|^2 + \|\mathbf{e}_6 \times \overline{\mathbf{r}}_6\|^2 \tag{8.21}$$

Because the 2-norm of the unit vector e_i is 1 its derivative with respect to *any* variable vanishes. The second derivatives of terms $\|(e_i \times \overline{\mathbf{r}}_i)\|^2$ with respect to x_j are calculated as

$$\frac{\partial\|\mathbf{e}_i \times \overline{\mathbf{r}}_i\|^2}{\partial x_j} = \frac{\partial}{\partial x_j}\left[(\mathbf{e}_i \times \overline{\mathbf{r}}_i)^T(\mathbf{e}_i \times \overline{\mathbf{r}}_i)\right] \tag{8.22}$$

$$= \left[\frac{\partial(\mathbf{e}_i \times \overline{\mathbf{r}}_i)}{\partial x_j}\right]^T(\mathbf{e}_i \times \overline{\mathbf{r}}_i) + (\mathbf{e}_i \times \overline{\mathbf{r}}_i)^T\left[\frac{\partial(\mathbf{e}_i \times \overline{\mathbf{r}}_i)}{\partial x_j}\right] \tag{8.23}$$

$$= 2\left(\frac{\partial(\mathbf{e}_i \times \overline{\mathbf{r}}_i)}{\partial x_j}\right)^T \mathbf{e}_i \times \overline{\mathbf{r}}_i \tag{8.24}$$

$$\frac{\partial(\mathbf{e}_i \times \overline{\mathbf{r}}_i)}{\partial x_j} = \left(\frac{\partial\mathbf{e}_i}{\partial x_j}\right) \times \overline{\mathbf{r}}_i + \mathbf{e}_i \times \left(\frac{\partial\overline{\mathbf{r}}_i}{\partial x_j}\right) \tag{8.25}$$

In order to compute the gradient of the objective function, the partial derivatives of vectors $\mathbf{e}_i$ and $\mathbf{r}_i$ with respect to vector $\mathbf{x}$ have to be computed.

Partial Derivatives for $\mathbf{e}_i$

The vector $\mathbf{e}_i$ has to be expressed in the first frame $\mathcal{F}_1$. In the ith frame the vector is always $\mathbf{e}_i = [0,\ 0,\ 1]^T$, which leads to

$$[\mathbf{e}_i]_1 = \mathbf{Q}_1\mathbf{Q}_2\ldots\mathbf{Q}_{i-1}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \tag{8.26}$$

The derivative of $\mathbf{e}_i$ with respect to an arbitrary variable y representing any of the design parameters thus becomes

$$\begin{aligned}
\frac{\partial\left[\mathbf{e}_i\right]_1}{\partial y} &= \frac{\partial}{\partial y}\left(\mathbf{Q}_1\mathbf{Q}_2\ldots\mathbf{Q}_{i-1}\mathbf{e}_i\right) && (8.27)\\
&= \frac{\partial\mathbf{Q}_1}{\partial y}\mathbf{Q}_2\ldots\mathbf{Q}_{i-1}\mathbf{e}_i+\mathbf{Q}_1\frac{\partial\mathbf{Q}_2}{\partial y}\mathbf{Q}_3\ldots\mathbf{Q}_{i-1}\mathbf{e}_i+\ldots+\\
&\quad \mathbf{Q}_1\ldots\mathbf{Q}_{i-1}\frac{\partial\mathbf{e}_i}{\partial y} && (8.28)
\end{aligned}$$

This formula can be simplified for the given design variable. It is obvious that any derivative of the vector $\mathbf{e}_i(\equiv[\mathbf{e}_i]_i)$ is zero. Furthermore the matrices $\mathbf{Q}_i$ do not depend on the variables a and b, which leads to:

$$\begin{aligned}
\frac{\partial\mathbf{Q}_i}{\partial a_i} &= 0 && (8.29)\\
\frac{\partial\mathbf{Q}_i}{\partial b_i} &= 0 && (8.30)\\
\frac{\partial\mathbf{e}_i}{\partial\mathbf{y}} &= 0 && (8.31)
\end{aligned}$$

The transformation matrices reach from the first to the $(i-1)$st frame. Therefore, the design variables α and θ for the ith frame or any following frame have also no influence on the partial derivative of $\mathbf{e}_i$. The partial derivatives reduce to the two expressions below

$$\begin{aligned}
\frac{\partial\left[\mathbf{e}_i\right]_1}{\partial\alpha_j} &= \mathbf{Q}_1\ldots\mathbf{Q}_{j-1}\frac{\partial\mathbf{Q}_j}{\partial\alpha_j}\mathbf{Q}_{j+1}\ldots\mathbf{Q}_{i-1}\mathbf{e}_i && (8.32)\\
\frac{\partial\left[\mathbf{e}_i\right]_1}{\partial\theta_j} &= \mathbf{Q}_1\ldots\mathbf{Q}_{j-1}\frac{\partial\mathbf{Q}_j}{\partial\theta_j}\mathbf{Q}_{j+1}\ldots\mathbf{Q}_{i-1}\mathbf{e}_i && (8.33)
\end{aligned}$$

Where the derivatives for the matrix $\mathbf{Q}$ are defined as:

$$\begin{aligned}
\frac{\partial\mathbf{Q}_j}{\partial\alpha_j} &= \begin{bmatrix} 0 & \sin\alpha_j\sin\theta_j & \cos\alpha_j\sin\theta_j \\ 0 & -\sin\alpha_j\cos\theta_j & -\cos\alpha_j\cos\theta_j \\ 0 & \cos\alpha_j & -\sin\alpha_j \end{bmatrix} && (8.34)\\
\frac{\partial\mathbf{Q}_j}{\partial\theta_j} &= \begin{bmatrix} \cos\theta_j & -\cos\alpha_j\sin\theta_j & \sin\alpha_j\sin\theta_j \\ \sin\theta_j & \cos\alpha_j\cos\theta_j & -\sin\alpha_j\cos\theta_j \\ 0 & \sin\alpha_j & \cos\alpha_j \end{bmatrix} && (8.35)
\end{aligned}$$

Partial Derivatives for $\bar{\mathbf{r}}_i$

Similar to the position vector $\mathbf{e}_i$, the vector $\bar{\mathbf{r}}_i$ has to be expressed in the first frame:

$$[\bar{\mathbf{r}}_i]_1 \equiv \underbrace{\mathbf{Q}_1 \ldots \mathbf{Q}_{i-1}}_{\mathbf{Q}_{1\ldots(i-1)}} \underbrace{(\bar{\mathbf{a}}_i + \mathbf{Q}_i\bar{\mathbf{a}}_{i+1} + \mathbf{Q}_i\mathbf{Q}_{i+1}\bar{\mathbf{a}}_{i+2} + \ldots + \mathbf{Q}_i \ldots \mathbf{Q}_{n-1}\bar{\mathbf{a}}_n)}_{[\bar{\mathbf{r}}_i]_i = \bar{\mathbf{r}}_i} \tag{8.36}$$

With respect to an arbitrary variable y the partial derivative $[\bar{\mathbf{r}}_i]_1$,

$$\frac{\partial\,[\mathbf{r}_i]_1}{\partial y} = \left(\frac{\partial \mathbf{Q}_{1\ldots(i-1)}}{\partial y}\right)\mathbf{r}_i + \mathbf{Q}_{1\ldots(i-1)}\left(\frac{\mathbf{r}_i}{\partial y}\right) \tag{8.37}$$

$$\begin{aligned}
\frac{\partial \mathbf{Q}_{1\ldots(i-1)}}{\partial y} &= \frac{\partial \mathbf{Q}_1}{\partial y}\mathbf{Q}_2 \ldots \mathbf{Q}_{i-1} + \mathbf{Q}_1\frac{\partial \mathbf{Q}_2}{\partial y}\mathbf{Q}_3 \ldots \mathbf{Q}_{i-1} + \ldots + \\
&\quad \mathbf{Q}_1 \ldots \mathbf{Q}_{i-2}\frac{\mathbf{Q}_{i-1}}{\partial y} && (8.38)\\
\frac{\bar{\mathbf{r}}_i}{\partial y} &= \frac{\partial \bar{\mathbf{a}}_i}{\partial y} + \left(\frac{\partial \mathbf{Q}_i}{\partial y}\bar{\mathbf{a}}_{i+1} + \mathbf{Q}_i\frac{\partial \bar{\mathbf{a}}_{i+1}}{\partial y}\right) && (8.39)\\
&\quad + \left(\frac{\partial \mathbf{Q}_i}{\partial y}\mathbf{Q}_{i+1}\bar{\mathbf{a}}_{i+2} + \mathbf{Q}_i\frac{\partial \mathbf{Q}_{i+1}}{\partial y}\bar{\mathbf{a}}_{i+2} + \mathbf{Q}_i\mathbf{Q}_{i+1}\frac{\partial \bar{\mathbf{a}}_{i+2}}{\partial y}\right) + \ldots + \\
&\quad + \left(\frac{\partial \mathbf{Q}_i}{\partial y}\mathbf{Q}_{i+1} \ldots \mathbf{Q}_{n-1}\bar{\mathbf{a}}_n + \ldots + \mathbf{Q}_i \ldots \mathbf{Q}_{n-1}\frac{\bar{\mathbf{a}}_n}{\partial y}\right)
\end{aligned}$$

The influence of the design variables can be separated. As long as the variable x_j occurs in either $\mathbf{Q}_j$ or $\bar{\mathbf{a}}_j$, and $j < i$, it affects only the first term $\left[\left(\partial \mathbf{Q}_{1\ldots(i-1)}/\partial x_j\right)\bar{\mathbf{r}}_i\right]$ of eq. 8.37. The second term $\left[\mathbf{Q}_{1\ldots(i-1)}\left(\mathbf{r}_i/\partial x_j\right)\right]$ vanishing. When $j \geq i$ the first term vanishes and the influence occurs only in the second term. As explained in the previous subsection, the matrix $\mathbf{Q}$ does not depend of a and b. Also many terms of eq. 8.39 vanish for the different cases.

This leads to the partial-derivatives expressions below: $\mathbf{r}$:

for j < i

$$\begin{aligned}
\frac{\partial\,[\bar{\mathbf{r}}_i]_1}{\partial \alpha_j} &= \mathbf{Q}_1 \ldots \mathbf{Q}_{j-1}\frac{\partial \mathbf{Q}_j}{\partial \alpha_j}\mathbf{Q}_{j+1} \ldots \mathbf{Q}_{i-1}\bar{\mathbf{r}}_i\\
\frac{\partial\,[\bar{\mathbf{r}}_i]_1}{\partial \theta_j} &= \mathbf{Q}_1 \ldots \mathbf{Q}_{j-1}\frac{\partial \mathbf{Q}_j}{\partial \theta_j}\mathbf{Q}_{j+1} \ldots \mathbf{Q}_{i-1}\bar{\mathbf{r}}_i
\end{aligned}$$

for j = i

$$
\begin{aligned}
\frac{\partial \left[\overline{\mathbf{r}}_i\right]_1}{\partial a_j} &= \mathbf{Q}_{1\ldots(i-1)}\left(\mathbf{Q}_i\ldots\mathbf{Q}_{j-1}\frac{\partial \overline{\mathbf{r}}_j}{\partial a_j}\right)\\
\frac{\partial \left[\overline{\mathbf{r}}_i\right]_1}{\partial b_j} &= \mathbf{Q}_{1\ldots(i-1)}\left(\mathbf{Q}_i\ldots\mathbf{Q}_{j-1}\frac{\partial \overline{\mathbf{r}}_j}{\partial b_j}\right)\\
\frac{\partial \left[\overline{\mathbf{r}}_i\right]_1}{\partial \theta_j} &= \mathbf{Q}_{1\ldots(i-1)}\left(\frac{\partial \overline{\mathbf{a}}_i}{\partial \theta_j}+\frac{\partial \mathbf{Q}_i}{\partial \theta_j}\overline{\mathbf{a}}_{i+1}+\ldots+\frac{\partial \mathbf{Q}_i}{\partial \theta_j}\mathbf{Q}_{i+1}\ldots\mathbf{Q}_n\overline{\mathbf{a}}_n\right)\\
\frac{\partial \left[\overline{\mathbf{r}}_i\right]_1}{\partial \alpha_j} &= \mathbf{Q}_{1\ldots(i-1)}\left(\frac{\partial \mathbf{Q}_i}{\partial \alpha_j}\overline{\mathbf{a}}_{i+1}+\ldots+\frac{\partial \mathbf{Q}_i}{\partial \theta_j}\mathbf{Q}_{i+1}\ldots\mathbf{Q}_n\overline{\mathbf{a}}_n\right)
\end{aligned}
$$

for j > i

$$
\begin{aligned}
\frac{\partial \left[\overline{\mathbf{r}}_i\right]_1}{\partial a_j} &= \mathbf{Q}_{1\ldots(i-1)}\left(\mathbf{Q}_i\ldots\mathbf{Q}_{j-1}\frac{\partial \overline{\mathbf{a}}_j}{\partial a_j}\right)\\
\frac{\partial \left[\overline{\mathbf{r}}_i\right]_1}{\partial b_j} &= \mathbf{Q}_{1\ldots(i-1)}\left(\mathbf{Q}_i\ldots\mathbf{Q}_{j-1}\frac{\partial \overline{\mathbf{a}}_j}{\partial b_j}\right)\\
\frac{\partial \left[\overline{\mathbf{r}}_i\right]_1}{\partial \theta_j} &= \mathbf{Q}_i\ldots\mathbf{Q}_{j-1}\frac{\partial \overline{\mathbf{a}}_j}{\partial \theta_j}+\mathbf{Q}_i\ldots\mathbf{Q}_{j-1}\frac{\partial \mathbf{Q}_j}{\partial \theta_j}\overline{\mathbf{a}}_{j+1}+\ldots+\\
&\quad \mathbf{Q}_i\ldots\mathbf{Q}_{j-1}\frac{\partial \mathbf{Q}_j}{\partial \theta_j}\mathbf{Q}_{j+1}\ldots\mathbf{Q}_{n-1}\overline{\mathbf{a}}_n\\
\frac{\partial \left[\overline{\mathbf{r}}_i\right]_1}{\partial \alpha_j} &= \mathbf{Q}_i\ldots\mathbf{Q}_{j-1}\frac{\partial \mathbf{Q}_j}{\partial \alpha_j}\overline{\mathbf{a}}_{j+1}+\ldots+\mathbf{Q}_i\ldots\mathbf{Q}_{j-1}\frac{\partial \mathbf{Q}_j}{\partial \alpha_j}\mathbf{Q}_{j+1}\ldots\mathbf{Q}_{n-1}\overline{\mathbf{a}}_n
\end{aligned}
$$

8.4.2 Gradient of f_2

In functions f_2, the inverse of the matrix product comes into the picture, and hnece, the gradient of an inverse matrix A^{-1} is needed. This is readily calculated below: From we obtain, upon differentiation with respect to y, whence,

$$
\begin{aligned}
\mathbf{A}\mathbf{A}^{-1} &= 1 \qquad (8.40)\\
\frac{\partial}{\partial y}\left(\mathbf{A}\mathbf{A}^{-1}\right) &= \left(\frac{\partial \mathbf{A}}{\partial y}\right)\mathbf{A}^{-1}+\mathbf{A}\left(\frac{\partial \mathbf{A}^{-1}}{\partial y}\right)=\mathbf{O}\\
\Rightarrow \frac{\partial \mathbf{A}^{-1}}{\partial y} &= -\mathbf{A}^{-1}\left(\frac{\partial \mathbf{A}}{\partial y}\right)\mathbf{A}^{-1}
\end{aligned}
$$

The gradient of f_2 thus reduces to

$$\begin{aligned}
\mathrm{tr}\left[\frac{\partial(\mathbf{H}^T\mathbf{H})^{-1}}{\partial y}\right] &= -\mathrm{tr}\left[\left(\mathbf{H}^T\mathbf{H}\right)^{-1}\left(\mathbf{H}^T\mathbf{H}\right)^{-1}\frac{\partial(\mathbf{H}^T\mathbf{H})}{\partial y}\right] && (8.41)\\
&= -\mathrm{tr}\left[\left[\left(\mathbf{H}^T\mathbf{H}\right)^2\right]^{-1}\frac{\partial\left(\mathbf{H}^T\mathbf{H}\right)}{\partial y}\right] && (8.42)
\end{aligned}$$

All derivatives required in the above expression were defined in the previous subsections.

8.5 Conclusion and Recommendations for Further Work

During this project, only a first attempt to compute the gradient of the objective function was programmed. Because the final algorithm should be available in a MATLAB and implemented in RVS, MATLAB was used for the first developments. In order to minimize the changes between the MATLAB and the C codes, only methods are used that are basic C functions or available in the RVS mathematical library. The RVS mathematical library (**matrix** - folder) includes several function to calculate, for example, a cross product, matrix $\times$ vector multiplications or the Cholesky decomposition. Also, a function to compute the Jacobian matrix is implemented in RVS. However, as long as C does not support symbolic calculations every partial derivative has to be programmed. This could be managed in two ways:

1. The algorithm builds the partial derivative for each combination of $\mathbf{e}_i$ or $\overline{\mathbf{r}}_i$ and the design variables. This requires several `if` and `switch` statements for each equation, but the final calculations are minimized. (This method was used in the first attempt for the gradient calculation.)

2. Another possibility is to calculate first the partial derivative of matrices $\mathbf{Q}_i$ and vectors $\overline{\mathbf{a}}_i$ with respect to each design variable and use the general expressions for $\partial\left[\mathbf{e}_i\right]_1/\partial\mathbf{x}$ and $\partial\left[\mathbf{r}_i\right]_1/\partial\mathbf{x}$. In this case the number of `if` and `switch` statements would be less, but the number of calculations higher.

Implementation in RVS

Although the development of the algorithm is not very advanced at this stage, we suggest a list of the planned steps for RVS implementation:

1. Use the pre-saved DH-parameters as the initial guess

2. Show a rendering of the new architecture after each interaction

3. Create a new architecture file or overwrite the existing file. The overwrite function should be deactivated, when a fully rendered model is defined for the select architecture.

9 Conclusion and Recommendations for Further Work

Conclusion

The Robot Visualization System 2006 (RVS2006) is a free software package for the simulation of robots (Appendix B). With the program it is possible to render a skeleton model or a fully rendered model of a robotic architecture. For a skeleton model only an ASCII-file with the DH parameters is required. The fully rendered model has to be programmed for each robot individually.

In order to make RVS2006 operating system independent only cross-platform software packages are used. The program is written in C/C++ and OpenGL is used for all 3D renderings. In combination with the Fast and Light Toolkit, RVS2006 is available for Microsoft Windows, LINUX and MacOS. All main features from the original RVS are implemented in RVS2006. Some functions, e.g. reading data from an external floppy disk, are no longer required for the software. Therefore, these parts have not been implemented in RVS2006. The following list includes all features that are available in RVS2006:

- Create / load a new robot;
- edit an existing architecture;
- select rendering mode for a robot;
- choose different end-effectors;
- display frames;
- set joint limits;
- load saved postures;
- control each joint variable individual;
- cartesian configuration;
- trajectories on basis of joint variables;

- place obstacles;
- select different payloads;
- create a work environment; and
- export OpenGL scene.

Recommendations for Further Work

Although most of the functions work properly the source code still contains some bugs which could not be fixed during this project. This has to be done in the future. A list of known bugs is provided in Appendix C. In the original RVS the Cartesian Trajectory feature was mentioned. However, this function was not working in RVS and it is still not working in RVS2006. Only an interface for this feature has been created and some old functions were implemented. Currently the menu entry for this function is not shown.

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Appendix A

Architecture File: REDIESTRO

```
/*************************
   REDIESTRO
*************************/

geomtype = full

/** Link 1 **/ {
  a = 0.0
  b = 9.5229
  al = -58.3127
  th = 0.0
  limits = [ -170.0,  +95.0 ]
}
/** Link 2 **/
{
  a = 2.311300
  b = -0.2291
  al = -20.0289
  th = 20.0
  /** limits = [ -210.0,  +45.0 ] **/
}
/** Link 3 **/
{
  a = 0
  b = 0.369275
  al = 105.2568
  th = 40.0
  /** limits = [ -45.0,  +185.0] **/
}
/** Link 4 **/
{
  a = 3.9884
  b = 0
  al = 60.9094
  th = 20.0
  /** limits = [ -35.0,  +180.0] **/
}
/** Link 5 **/
{
  a = 0
  b = -4.71588
  al = 59.8823
  th = 40.0
}
/** Link 6 **/
{
  a = 1.35589
  b = 5.78206
  al =-75.4715
  th = 10.0
}
/** Link 7 **/
{
  a =  1.7835 /* 2.3445 */
  b = -1.4505
  al = 0
  th = 210.0
}

EndEffector = hand
```

Appendix B

RVS2006 Screenshot

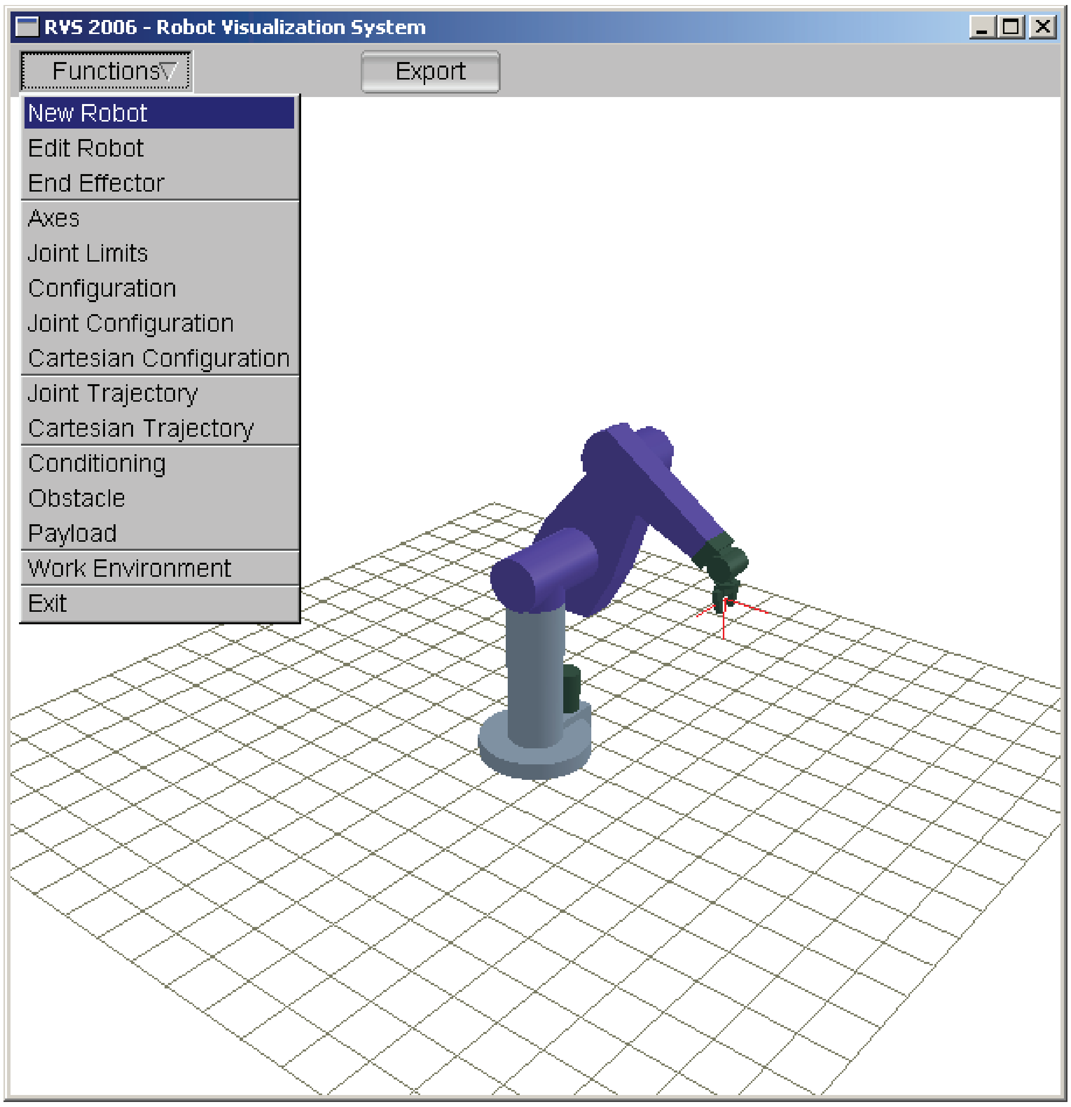

Figure B.1: RVS2006

Appendix C

Bugs in RVS

- Fully rendered model of TravPuma
- Missing labels on frame axes