1.3 Definitions for "machine," "mechanism," and "linkage":

Machine

- Definitions in Merriam Webster's Collegiate Dictionary (on-line, 2002):
  - (archaic): a constructed thing whether material or immaterial.
  - an assemblage of parts that transmit forces, motion, and energy one to another in a predetermined manner
  - an instrument (as a lever) designed to transmit or modify the application of power, force, or motion
  - a mechanically, electrically, or electronically operated device for performing a task (a calculating machine, a card-sorting machine)

  Comment: comprehensive definitions when considered as a whole
- *An apparatus* for transformation of power, materials, and information to substitute or simplify physical or intellectual work (Frolov, 1987). *Comment: a comprehensive definition, that includes computers*
- *Mechanical system* that performs a specific task, such as the forming of material, and the transference and transformation of *motion* and *force*, Vol. 38, Nos. 7–10 (2003) of *Mechanism and Machine Theory* on Standardization of Terminology. *Comment: leaves computers out*
- An *apparatus* for applying mechanical power, having several parts, each with definite function (The Concise Oxford Dictionary). *Comment: same as above*
- An *apparatus* consisting of interrelated parts with separate functions, used in the performance of some kind of work (The Random House College Dictionary). *Comment: ditto*
- Any *system* in which a specific correspondence exists between an input form of energy or information and the corresponding ones at the output (Loosely translated from Le Petit Robert). *Comment: as comprehensive as Frolov's*

Mechanism

- A piece of machinery (Merriam Webster's Collegiate Dictionary (on-line, 2002)). *Comment: too vague*
- Definitions in Vol. 38, Nos. 7–10 (2003) of *Mechanism and Machine Theory* on Standardization of Terminology.
  - *System* of bodies designed to convert *motions* of, and *forces* on, one or several bodies into constrained motions of, and *forces* on, other bodies. *Comment: English could be terser, but idea is fine.*
  - *Kinematic chain* with one of its components (*link* or *joint*) connected to the *frame*. *Comment: confuses mechanism with its representation as a kinematic chain*
- Structure, adaptation of parts of machine; system of mutually adapted parts working together (as) in machine (The Concise Oxford Dictionary)
- An assembly of moving parts performing a complete functional motion (The Random House College Dictionary)
- A combination layout of pieces or elements, assembled with the goal of (producing) an operation as a unit (Loosely translated from Le Petit Robert)

Comment: In all above definitions, the concept of goal or task is present

Linkage

- Definitions in Merriam Webster's Collegiate Dictionary (on-line, 2002):
  - a system of links. *Comment: concise and comprehensive*
  - a system of links or bars which are jointed together and more or less constrained by having a link or links fixed and by means of which straight or nearly straight lines or other point paths may be traced. *Comment: unnecessarily cumbersome and limited to path-generating linkages*
- *Kinematic chain* whose *joints* are equivalent to *lower pairs* only (Vol. 38, Nos. 7–10 (2003) of *Mechanism and Machine Theory* on Standardization of Terminology). *Comment: confuses linkage with its representation*

The Concise Oxford Dictionary of Current English, 1995, Clarendon Press, Oxford

Frolov, K. V., (editor), 1987, *Teoriya Mechanismov i Mashin* (Theory of Mechanisms and Machines), Vyschaya Shkola, Moscow (in Russian)

Mechanism and Machine Theory on Standardization of Terminology, 2003, Vol. 38, Nos. 7–10

Le Petit Robert, 1994, Dictionnaires Le Robert, Paris

Random House Webster's College Dictionary, 1997, Random House, New York

Merriam Webster's Collegiate Dictionary, on-line 2002

1.10 Here we want to estimate the time required to add two floating-point numbers. Since this time is very small, it is more suitable to perform a bunch of additions, say 10^7, which should amount to a total time of the order of seconds.

- The Microsoft Visual C++6.0 program below was run on a Pentium IV 2.0:

```
#include <iostream>
    #include <time.h>
    int main()
    {
        clock_t start, end;
        start = clock();
        float a, i, CLOCKS_PER_mSEC=CLOCKS_PER_SEC/1000;
        for(i=1; i<=10000000; i++)
        a = 5+5;
        std::cout << "a= "<< a << std::endl;
        end = clock();
        long duration = (long)(end-start)/CLOCKS_PER_mSEC;
        std::cout << "Time: "<< duration<<"ms"<<std::endl;
        return 0;
    }
```

  A Pentium IV 2.0 processor required 125 ms to perform the 10^7 additions, which amounts to $1.25 \times 10^{-}8$ s/add, or 8×10^7 add/s.

- The C program below was run for the same purpose on the CLUMEQ supercomputer, AMD Athlon 1900+ cluster:

```
#include "stdio.h"
#include "math.h"
#include <sys/time.h>
```

```
#include <sys/resource.h>
#define RUSAGE_SELF  0    /* calling process */
main()
{
    int getrusage(int who, struct rusage *rusage);
    long diffsec, diffmsec;
    float a;
    int i;
    struct rusage time_begin, time_end;
    struct rusage *pb, *pe;
    pb=&time_begin;
    getrusage(RUSAGE_SELF, pb);
    for(i=1; i<=10000000; i++)
    {
    a = 5+5;
    }

    printf("a=%f \t\n",a);
    pe=&time_end;
    getrusage(RUSAGE_SELF,pe);

    diffsec=time_end.ru_utime.tv_sec-time_begin.ru_utime.tv_sec;
    diffmsec=time_end.ru_utime.tv_usec-time_begin.ru_utime.tv_usec;
    printf("User time used:%d microsec\n",diffmsec);
}
```

The CLUMEQ supercomputer required 40 ms to perform the 10^7 additions, which amounts to $4 \times 10^{-}9$ add/mult, or 2.5×10^8 add/s.

1.11 What we must find here is the largest floating-point number ϵ that, when added to a given number a, leaves this number unchanged, i.e.,

$$a + \epsilon = a$$

- The Microsoft Visual C++6.0 program below was run on a Pentium IV 2.0 processor at 550 MHz:

```
#include <iostream>
    //#include <time.h>
    int main()
    {
        long i;
            //double a=1;
            float a=1;
            for(i=1; i<=10000000; i++)
            {
                a = a/10;
            if (a==0)
                break;
            }

        std::cout << "smallest floating-point"<< i-1 << std::endl;
        return 0;
    }
```

The value of ϵ reported in single-precision arithmetic was $1.0e-045$; in double-precision...... $1.0e-323$.

- The program below was run on the CLUMEQ supercomputer, AMD Athlon 1900+ cluster:

```
#include "stdio.h"
#include "math.h"
#include <sys/resource.h>
main()
{
    int i;
        float a=1;
        for(i=1; i<=10000000; i++)
        {
            a=a/10;
        if (a==0)
            break;
        }

    printf("smallest floating-point=%d \n",i-1);
}
```

This machine reported exactly the same values as its Pentium counterpart.

2.7 The determinant of a matrix is equal to the product of its eigenvalues. Thus

$$\det(\mathbf{1}+\mathbf{u}\mathbf{v}^T)=\det(\mathbf{T})=\lambda_1\lambda_2\lambda_3=1+\mathbf{v}^T\mathbf{u}=1+\mathbf{u}\cdot\mathbf{v}$$

2.14 We have, from Cayley's Theorem,

$$\mathbf{Q}=(\mathbf{1}-\mathbf{S})(\mathbf{1}+\mathbf{S})^{-1}$$

where $\mathbf{1}+\mathbf{S}$ is invertible. Indeed, if we write $\text{vect}(\mathbf{S})=\mathbf{s}\equiv\sigma\mathbf{e}$, with $\sigma>0$ denoting the Euclidean norm of $\mathbf{s}$, then $\mathbf{S}=\sigma\mathbf{E}$, in which $\mathbf{E}$ is the cross-product matrix of the unit vector $\mathbf{e}$. Hence, the eigenvalues of $\mathbf{S}$ are, from Exercise 2.10, 0, σj, and $-\sigma j$. Consequently, the eigenvalues of $\mathbf{1}+\mathbf{S}$ are 1, $1+\sigma j$, and $1-\sigma j$, none of which is zero, thereby showing that $\mathbf{1}+\mathbf{S}$ is indeed invertible. Multiplying both sides of this equation by $(\mathbf{1}+\mathbf{S})$, from the right, we obtain

$$\mathbf{Q}(\mathbf{1}+\mathbf{S})=(\mathbf{1}-\mathbf{S})$$

i.e.,

$$(\mathbf{1}+\mathbf{Q})\mathbf{S}=\mathbf{1}-\mathbf{Q}$$

and hence,

$$\mathbf{S}=(\mathbf{1}+\mathbf{Q})^{-1}(\mathbf{1}-\mathbf{Q})\ .$$

For this factoring to be possible, then, matrix $\mathbf{1}+\mathbf{Q}$ should be invertible, i.e., nonsingular. Therefore, the above-mentioned factoring is not possible if one of the eigenvalues of $\mathbf{Q}$ is -1, which happens iff the angle of rotation is π.

2.10 Let λ be an eigenvalue of $\mathbf{E}$, the cross-product matrix of the unit vector $\mathbf{e}$, and $\mathbf{u}$ the corresponding unit eigenvector, i.e.,

$$\mathbf{E}\mathbf{u} = \lambda\mathbf{u} \tag{1}$$

Multiplying eq.(1) by $\mathbf{E}$, we obtain

$$\mathbf{E}^2\mathbf{u} = \lambda^2\mathbf{u} \tag{2}$$

and multiplying a second time by $\mathbf{E}$,

$$\mathbf{E}^3\mathbf{u} = \lambda^3\mathbf{u} \tag{3}$$

However,

$$\mathbf{E}^2 = -\mathbf{1} + \mathbf{e}\mathbf{e}^T$$

and

$$\mathbf{E}^3 = \mathbf{E}(-\mathbf{1} + \mathbf{e}\mathbf{e}^T) = -\mathbf{E} + \underbrace{(\mathbf{E}\mathbf{e})}_{\mathbf{0}}\mathbf{e}^T = -\mathbf{E} \tag{4}$$

Substituting eq.(4) into eq.(3), we obtain

$$-\mathbf{E}\mathbf{u} = \lambda^3\mathbf{u} \tag{5}$$

Moreover, from eq.(1) we have $\mathbf{E}\mathbf{u} = \lambda\mathbf{u}$, and thus eq.(5) reduces to

$$\lambda(1 + \lambda^2)\mathbf{u} = \mathbf{0} \tag{6}$$

However, $\|\mathbf{u}\| = 1$ and hence, $\mathbf{u} \neq \mathbf{0}$. Therefore, we have from eq.(6)

$$P(\lambda) \equiv \lambda(1 + \lambda^2) = 0 \tag{7}$$

Equation (7) is the characteristic equation of $\mathbf{E}$, and $P(\lambda)$ its characteristic polynomial. The three roots of eq.(7) are thus

$$\lambda_1 = 0, \quad \lambda_2 = j, \quad \lambda_3 = -j$$

The eigenvector $\mathbf{u}_1$ corresponding to λ_1 is, obviously, $\mathbf{e}$, for $\mathbf{E}\mathbf{e} = \mathbf{0}$. Now, from eqs.(2) and (3), note that $\mathbf{E}$ and its integer powers share the same set of eigenvectors.

Now, to find $\mathbf{u}_2$, corresponding to λ_2, let us substitute these into eq.(2), along with $\mathbf{E}^2 = -\mathbf{1} + \mathbf{e}\mathbf{e}^T$,

$$(-\mathbf{1} + \mathbf{e}\mathbf{e}^T)\mathbf{u}_2 = -\mathbf{u}_2 \quad \text{or} \quad \mathbf{u}_2 + (\mathbf{e}^T\mathbf{u}_2)\mathbf{e} = -\mathbf{u}_2$$

which implies that

$$\mathbf{e}^T\mathbf{u}_2 = 0$$

Thus, $\mathbf{u}_2$ is any real vector normal to $\mathbf{e}$, which means that $\mathbf{u}_2$ lies in the plane normal to $\mathbf{e}$. Moreover, since $\mathbf{E}^2$ is symmetric, its three eigenvectors are mutually orthogonal, $\mathbf{u}_3$ thus being, simply,

$$\mathbf{u}_3 = \mathbf{e} \times \mathbf{u}_2$$

In conclusion, $\mathbf{u}_2$ and $\mathbf{u}_3$ are any two mutually orthogonal unit vectors lying in the complex plane perpendicular to $\mathbf{e}$, e.g.

$$\mathbf{u}_2 = [0 \quad \alpha \quad \beta]^T$$

where α and β are two complex numbers, so that

$$\|\mathbf{u}_2\|^2 \equiv \mathbf{u}_2^*\mathbf{u}_2 = [0 \quad \bar{\alpha} \quad \bar{\beta}]\begin{bmatrix} 0 \\ \alpha \\ \beta \end{bmatrix} = \bar{\alpha}\alpha + \bar{\beta}\beta = |\alpha|^2 + |\beta|^2 = 1$$

where $(\cdot)^*$ is the *transpose conjugate* of the complex vector $(\cdot)$, $\bar{(\cdot)}$ denotes conjugate of $(\cdot)$, and $|\cdot|$ the module of $(\cdot)$. Thus

$$\mathbf{u}_2^*\mathbf{u}_3 = 0$$