Data-Conditioning in the Optimization of Function-Generating Linkages

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Overview

- Introduction
- Rank-Deficiencies (Singularities) in the Synthesis Matrix
- Elimination of Singularities
- Condition-Number Minimization
- Conclusion

Introduction

- Synthesis of function-generating four-bar linkages
  - Given $\{\psi_i,\ \phi_i\}_1^m$, find the linkage (n parameters) that will approximate the set with least-square error
  - $m \leq n$: exact synthesis; $m > n$: approximate synthesis
  - Linkage optimization: nonlinear programming
- Optimization procedure
  - Design-error minimization

    Solve overdetermined linear system $\mathbf{Sx} = \mathbf{b}$

    $\mathbf{S}$: $m \times n$ synthesis matrix

    $\mathbf{x}$: n-dimensional design vector

    $\mathbf{b}$: m-dimensional data vector

$$\mathbf{d} = \mathbf{b} - \mathbf{Sx}$$

  - Structural-error minimization

$$s_i = \varphi_i - \phi_i, \quad i = 1, \ldots, m$$

φ_i: actual value of output angle

ϕ_i: desired value of output angle

s_i: structural error

Introduction (cont'd)

$$\boldsymbol{\Sigma} = \left(\frac{\partial \mathbf{d}}{\partial \mathbf{u}}\right)^{-1} \mathbf{S}$$

$\boldsymbol{\Sigma}$: sensitivity matrix of structural error to design vector, $\boldsymbol{\Sigma} \equiv d\mathbf{s}/d\mathbf{k}$

$\mathbf{u} = [\varphi_1, \ldots, \varphi_m]^T$; φ_i : ith generated output angle

- The numerical conditioning of **S**
  - affects the optimum solution directly
  - depends on the given I/O pairs $\{\psi_i,\ \phi_i\}_1^m$
- Previous work: Freudenstein (1959) and Hall (1961)
  Limitations:
  - conditioning guidelines drawn from experience
  - focus only on planar linkages
- Objective of this work
  - Identify $\{\psi_i,\ \phi_i\}_1^m$ pairs leading to rank-deficient **S** matrix
  - Consider all types of linkages (planar, spherical & spatial)
  - Elimination of ill-conditioning and rank-deficiencies in **S**

Rank-Deficiencies in the Synthesis Matrix

- Condition number

$$\kappa \equiv \text{cond}(\mathbf{S}) = \frac{\tau_M}{\tau_m}$$

where τ_M and τ_m are the largest and the smallest singular values of $\mathbf{S}$

$$1 \leq \kappa < \infty$$

which is unbounded from above. Define

$$\gamma = \frac{1}{\kappa}, \quad 0 \leq \gamma \leq 1$$

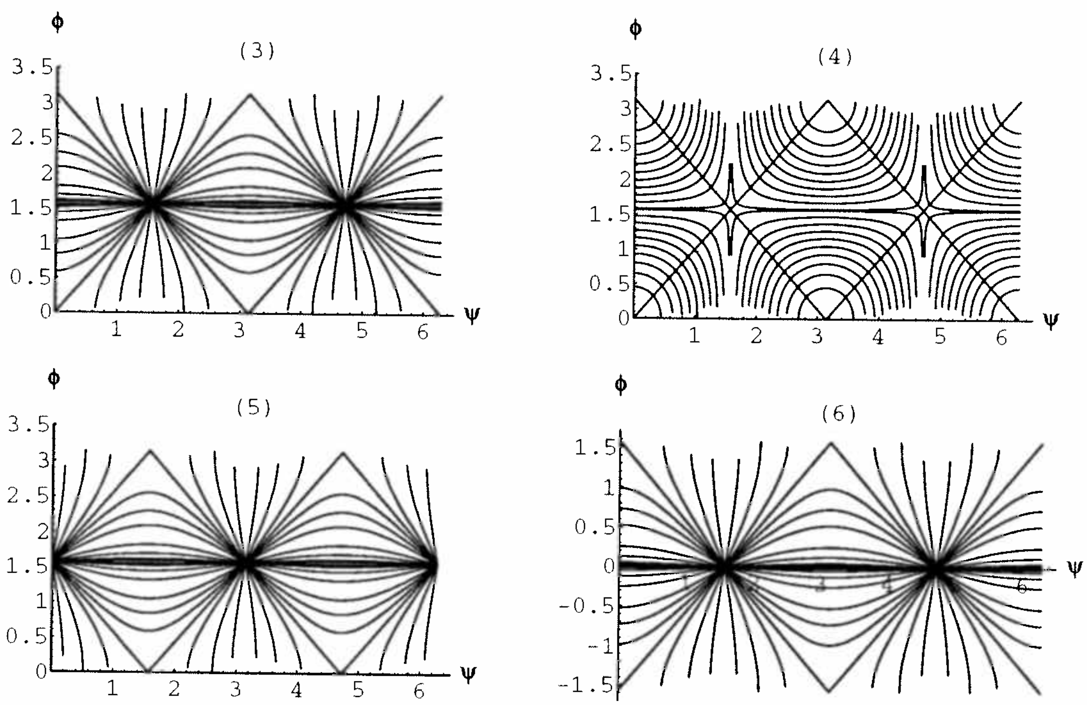
which is bounded from below and from above. $\Rightarrow \gamma$ is better behaved than κ.

- Singularity curves
  - Singularity: rank-deficiency in $\mathbf{S}$
  - Improper selection of $\{\psi_i,\ \phi_i\}_1^m$ leads to singularity
  - Location in ψ-ϕ plane: *singularity curves*
  - Ten cases of singularities found

Linkage Type	Possible Singularities
Planar $RRRR$	1, 2, 3
Spherical $RRRR$	1, 2, 3, 4
Spatial $RSSR$	1, 2, 3, 4, 5, 6, 7, 8, 9
Spatial $RCCC$	1, 2, 9, 10

Rank-deficiencies in the synthesis matrix (cont'd)

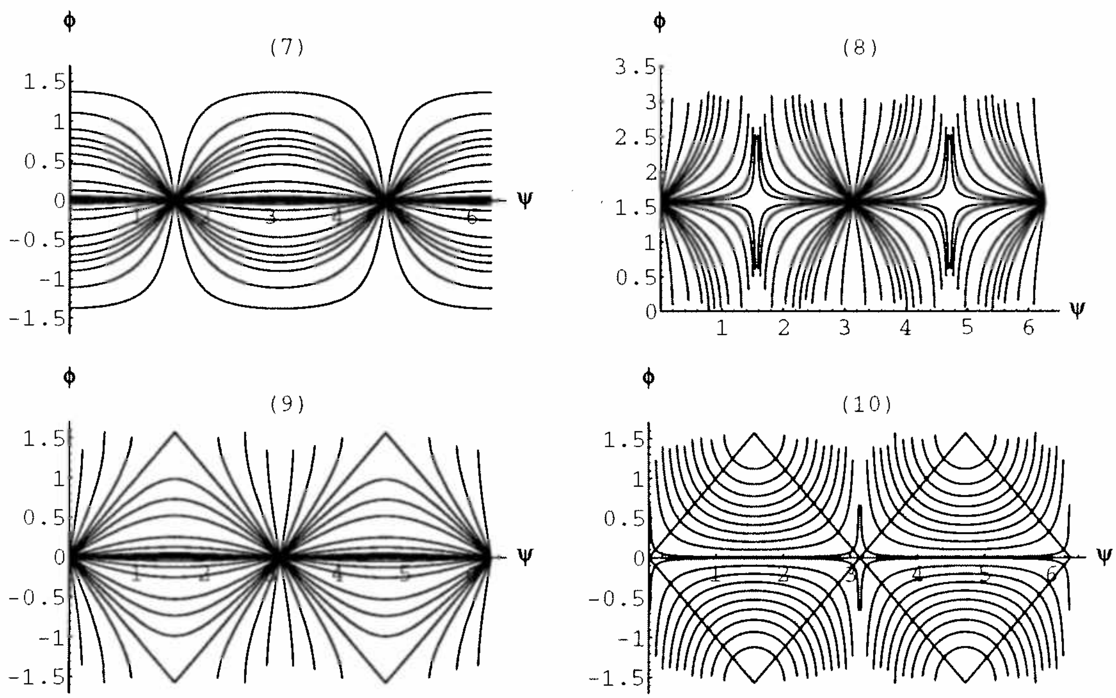
- Singularity curves (cont'd)
  Singularity 1: ϕ = Constant
  Singularity 2: ψ = Constant
  Singularities 3 — 6



Rank-deficiencies in the synthesis matrix (cont'd)

- Singularity curves (cont'd)

  Singularities 7 — 10



Elimination of Singularities

- Adding supplementary data points
  Purpose: change the overall distribution of I/O data in ψ-ϕ plane

Table 1: I/O pairs (in deg.) leading to singularity 3

ψ	15.51	29.02	42.58	55.81	68.02	81.01	97.89	112.25
ϕ	36.59	43.22	52.15	62.08	71.83	82.52	96.57	108.39

Example:

Adding two points: $(145^\circ,\ 85^\circ)$ and $(155^\circ,\ 55^\circ)$ to Table 1

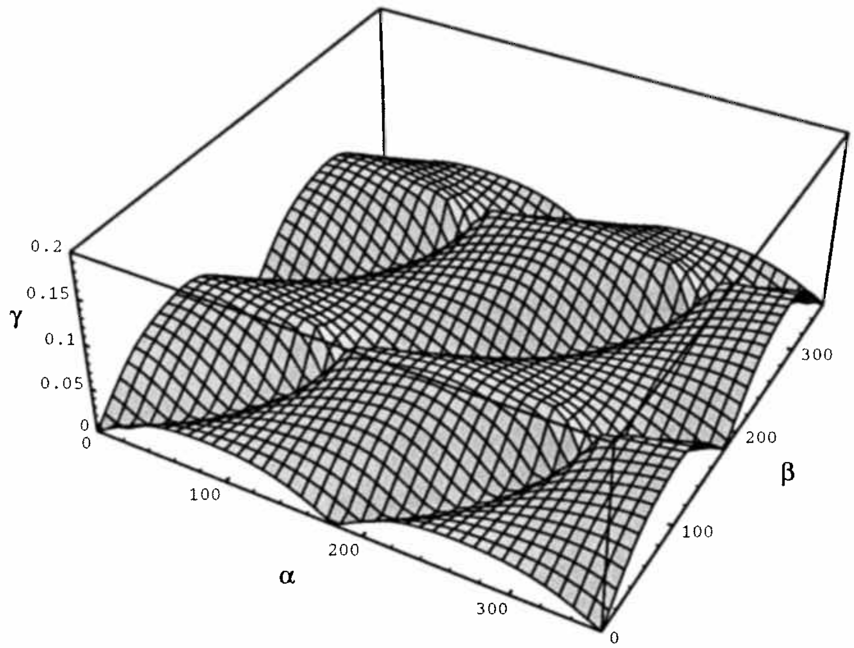
Linkages	Condition number with data as given	Condition number with added data
Planar $RRRR$	2.87×10^5	4.14
Spherical $RRRR$	3.05×10^5	8.39
Spatial $RSSR$	4.29×10^5	95.79

Elimination of Singularities (cont'd)

- Changing the zeros of the input and output dials
  Purpose: move I/O data away from a certain singularity curve

  $$\psi_i = \alpha + \Delta\psi_i, \qquad \phi_i = \beta + \Delta\phi_i, \qquad i = 1, \ldots, m$$

  Example:
  Planar linkage, singularity 3 (Table 1)



Note: This method does not apply to $RSSR$ Linkages

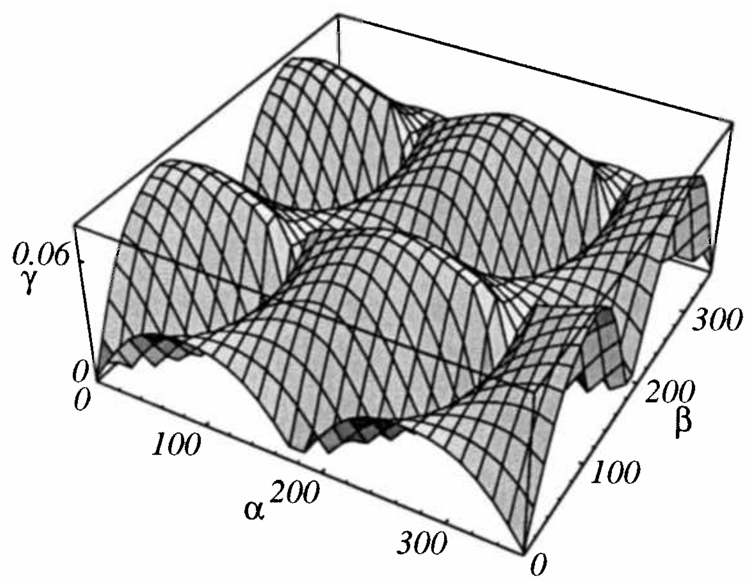
Elimination of Singularities (cont'd)

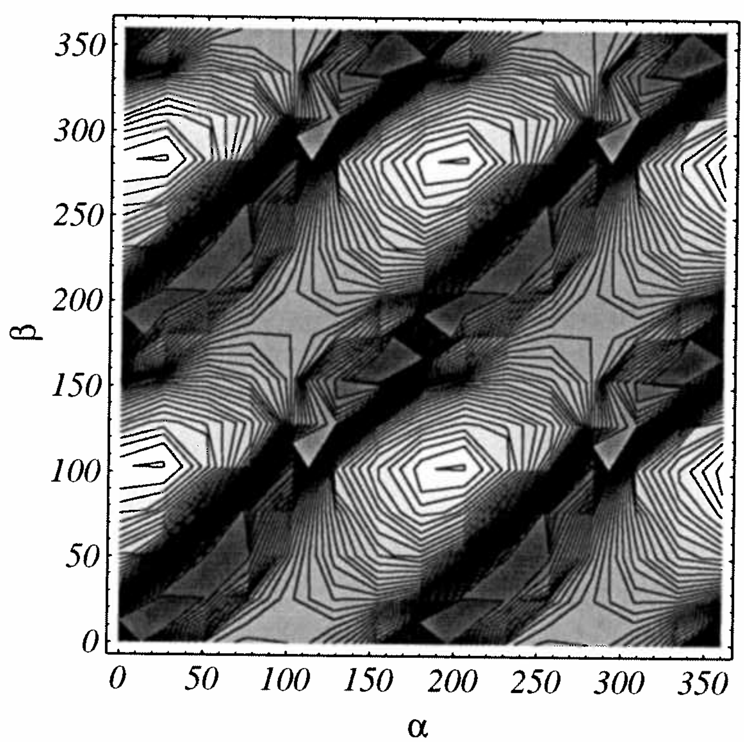
- Isoconditioning Contour

  The contours of constant condition number lie in the α-β plane

  Examples (with I/O pairs in Table 1) :

  Planar linkage :

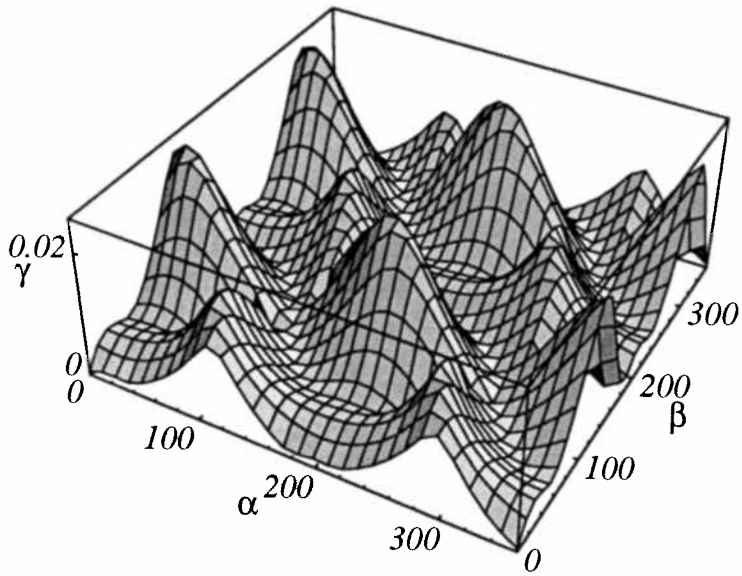


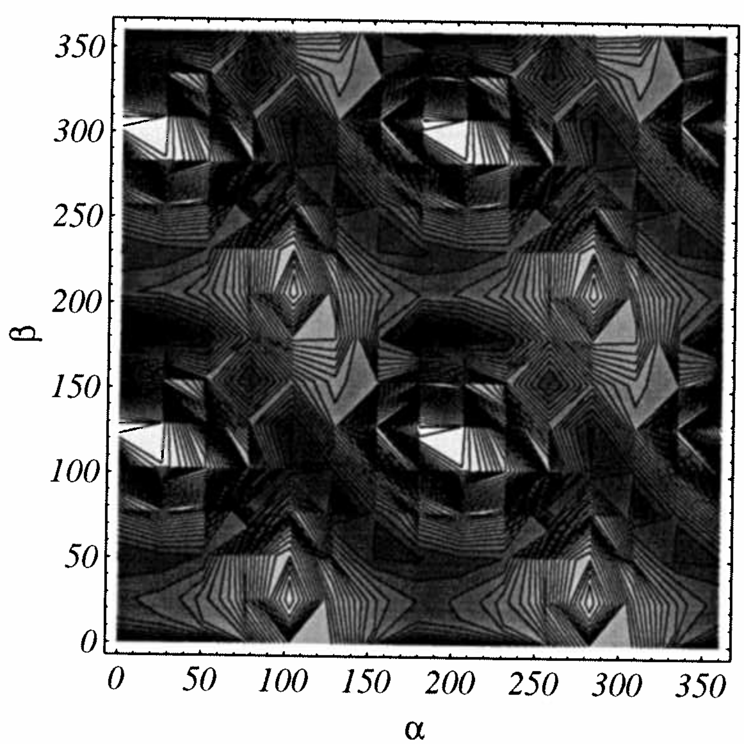


Elimination of Singularities (cont'd)

- Isoconditioning Contour (cont'd)

  Spherical linkage:

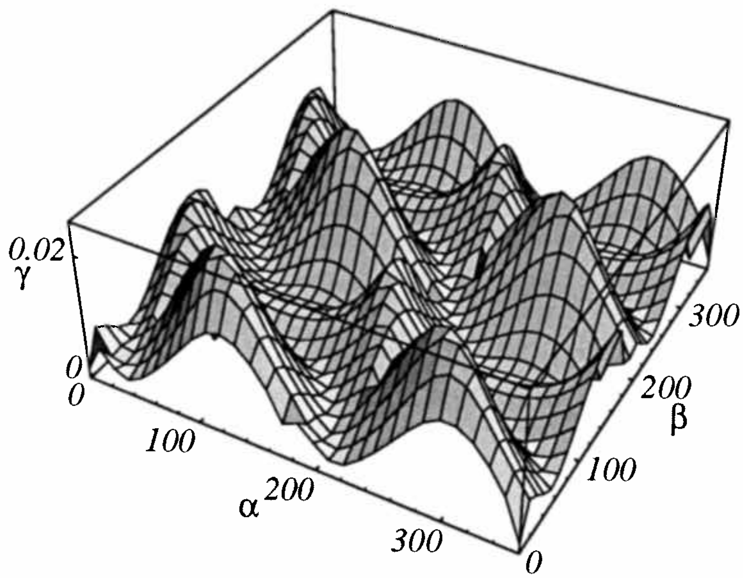


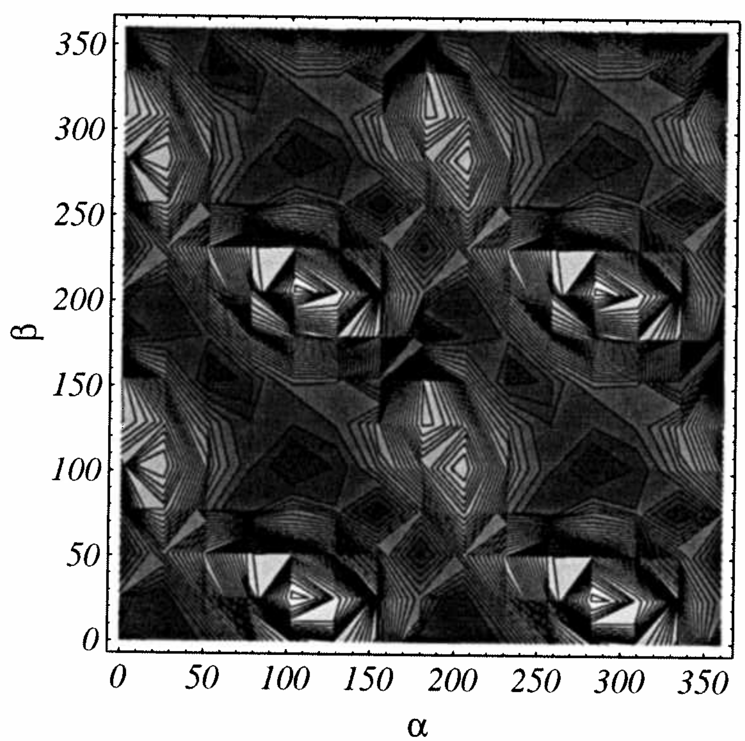


Elimination of Singularities (cont'd)

- Isoconditioning Contour (cont'd)

  RCCC linkage:





Condition-Number Minimization

- Formulation

$$\mathbf{S} = \mathbf{S}(\alpha,\ \beta)$$

and hence,

$$\kappa = \kappa(\alpha,\ \beta)$$

$$\min_{\alpha,\ \beta} \kappa(\alpha,\ \beta)$$

Polytope algorithm (direct search)

Example: using I/O pairs in Table 1

Linkages	Initial Guess $\alpha_0,\ \beta_0$	Optimum Values $\alpha_{opt},\ \beta_{opt}$	Minimum Condition Number κ_{min}
Planar *RRRR*	23°, 0°	102.9°, 16.6°	13.4
Spherical *RRRR*	195°, 115°	216.8°, 99.5°	62.7

Conclusions

- We showed that rank-deficiency of **S** is caused by the distribution of $\{\psi_i,\ \phi_i\}_1^m$ in the ψ-ϕ plane
- We identified singularity loci for all types of linkages
- We proposed effective methods for eliminating singularities
- We plotted contours of constant condition number for the linkages studied here
- We found values of α and β that minimize $\kappa(\tilde{S})$