McGill University
Department of Mechanical Engineering

MECH 541 Kinematic Synthesis **Class Test**

OPEN BOOK. NO CALCULATORS ALLOWED

Date and Time: March 15th, 2012, from 8:35 a.m. to 9:55 a.m.
N.B.: question weights are based on time to finish, rather than difficulty.

1. (50%) While working on the design of a spherical four-bar linkage that would produce a constant velocity ratio of 1:1 in the interval $\psi_1 \leq \psi \leq \psi_2$, a clever junior engineer proposes to impose four conditions: $\psi_1 = -30°$, $\phi_1 = 30°$; $\psi_2 = 30°$, $\phi_2 = 90°$; $\dot{\psi} = \dot{\phi}$ at both (ψ_1, ϕ_1) and (ψ_2, ϕ_2). With these conditions, a 4×4 synthesis matrix $\mathbf{H}$ and a 4-dimensional right-hand side vector $\mathbf{h}$—these variable names have been chosen to avoid confusion with the usual $\mathbf{S}$ and $\mathbf{b}$—are obtained, namely,

$$\mathbf{H} = \begin{bmatrix} 1 & \sqrt{3}/2 & 3/4 & -\sqrt{3}/2 \\ 1 & \sqrt{3}/2 & 0 & 0 \\ 0 & -1/2 & 0 & -1/2 \\ 0 & 1/2 & \sqrt{3}/2 & -1 \end{bmatrix}, \quad \mathbf{h} = \begin{bmatrix} -1/4 \\ 1/2 \\ 0 \\ \sqrt{3}/2 \end{bmatrix}$$

Compute the solution $\mathbf{k}$ to the *synthesis equations* $\mathbf{Hk} = \mathbf{h}$, and, **without computing the Denavit-Hartenberg parameters**—angles α_i, for $i = 1, \ldots, 4$—determine whether the linkage is *feasible. Hint: reciprocal bases are strongly recommended here, to do the computation swiftly and safely. To use reciprocal bases, you will have to decouple one equation from the other three.*

2. (15%) Shown in Fig. 1 is a serial kinematic chain of the RHHR type, with the screw pairs of parallel axes and different pitches, while each H joint is coaxial with its corresponding R joint. Find the degree of freedom of the mechanism and describe the motion undergone by link 3 with respect to link 1.

3. A spherical four-bar linkage has been designed with the Denavit-Hartenberg parameters $\alpha_1 = 120°$, $\alpha_2 = \alpha_3 = \alpha_4 = 90°$. The R&D Department has approved the production of the linkage, as it was found *feasible*.

(a) (5%) Compute its Freudenstein parameters.

(b) (15%) Determine whether its input link is a crank. *Hint: recall the rationale behind the mobility analysis in planar linkages, and state whether it can be applied here, giving reasons, while taking into account that the linkage is feasible.*

(c) (15%) What about the output link?

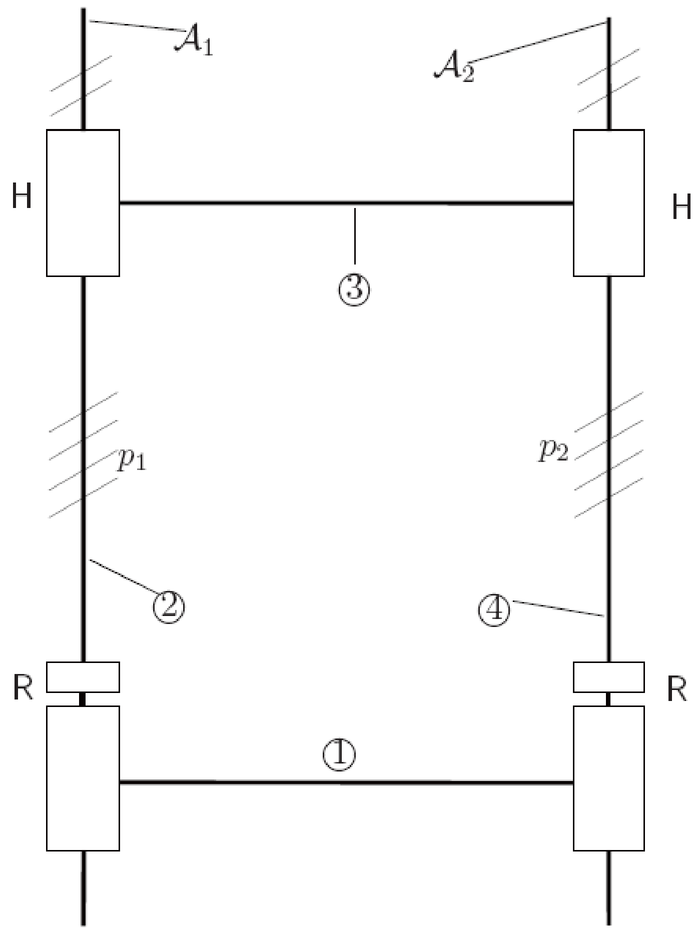

Figure 1: A RHHR mechanism