McGill University
Department of Mechanical Engineering

MECH 541 Kinematic Synthesis **Class Test**

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Date and Time: **December 1st, 2009, from 8:35 a.m. to 9:55 a.m.**

1. The synthesis of a *symmetric* gripper based on a planar four-bar linkage, with $k_2 = k_3$ is revisited. This case is known to lead to a $m \times 2$ synthesis matrix $\mathbf{S}$, with a two-dimensional unknown vector $\mathbf{k} = [\,k_1,\, k_2\,]^T$ and a m-dimensional *right-hand side* vector $\mathbf{b}$. The synthesis of the linkage was conducted for the data given in Table 1, with $\overline{\phi}_i$ denoting the prescribed output-angle value, ϕ_i the *generated* value. Furthermore, the sensitivity values of the design error e_i w.r.t. the generated value ϕ_i are displayed in the fifth column of the table.

Table 1: Four data points equally spaced along line $\phi = 3\pi/2 - \psi$

ith point	ψ_i	$\overline{\phi}_i$	ϕ_i	$\mathrm{d}e_i/\mathrm{d}\phi_i$
1	0.5236	4.188	4.189	1.900
2	0.6981	4.015	4.014	1.948
3	0.8727	3.840	3.840	1.956
4	1.047	3.667	3.665	1.885

The corresponding synthesis matrix $\mathbf{S}$ and vector $\mathbf{b}$, both computed at the *prescribed* values of the output angle, along with the least-square approximation of the synthesis equation $\mathbf{Sk} = \mathbf{b}$ are given below[1]:

$$\mathbf{S} = \begin{bmatrix} 1 & -1.367 \\ 1 & -1.408 \\ 1 & -1.409 \\ 1 & -1.365 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} -.8666 \\ -.9847 \\ -.9848 \\ -.8670 \end{bmatrix}, \quad \mathbf{k} = \begin{bmatrix} 2.919 \\ 2.771 \end{bmatrix}$$

Letting $\mathbf{S}_g$ denote the synthesis matrix evaluated at the generated values, this was obtained as

$$\mathbf{S}_g = \begin{bmatrix} 1 & -1.366 \\ 1 & -1.409 \\ 1 & -1.409 \\ 1 & -1.366 \end{bmatrix}$$

(a) (10%) Find the structural error $\mathbf{s}$ of the linkage given by $\mathbf{k}$ above, and show that the linkage thus obtained does not minimize $\mathbf{s}$ at the given four-digit precision; moreover,

[1] These values are slightly different from those given in Class Test 1, as they were computed with additional digits.

(b) (40%) find an improvement to $\mathbf{k}$ that is expected to give a lower structural error, using *verbatim* the expression displayed in eq.(3.159) with a four-digit precision. *Hints:*

(*i*) *The product* $\mathbf{A}^T\mathbf{A}$*, with* $\mathbf{A}$ *of* $m \times 2$*, is a* 2×2 *matrix whose diagonal entries are the Euclidean norms-squared of the corresponding columns of* $\mathbf{A}$*, its identical off-diagonal entries being given by the scalar product of those two columns.*

(*ii*) *The inverse of a* 2×2 *matrix* $\mathbf{M}$ *is*[2]

$$\mathbf{M}^{-1} = \frac{1}{\det(\mathbf{M})}\begin{bmatrix} m_{22} & -m_{12} \\ -m_{21} & m_{11} \end{bmatrix}$$

2. Shown in Fig. 1 is a triangular workpiece RST that will undergo three steps of machining in a flexible manufacturing cell. As this operation will take place for a large batch of workpieces, the manufacturing engineer has decided to use a four-bar linkage to guide the workpiece through the different poses. Space limitations require that the grounded revolute joints have their centres at points $B(-1, 1)$ and $B^\star(1, 1)$.

(a) (40%) Compute the centre A_0 of the floating joint. *Hint: it will help speed up the computations if the synthesis equations are expressed in the form* $\mathbf{M}\mathbf{a}_0 = \mathbf{n}$*, where the row* $\mathbf{m}_j^T$ *of the* 2×2 *matrix* $\mathbf{M}$ *and the* n_j *entry of* $\mathbf{n}$ *are functions of* $\mathbf{b}$*,* $\mathbf{r}_j$ *and* $\mathbf{Q}_j$*. Hint (ii) of Problem 1 will also be helpful here.*

(b) (10%) The circular point of the second dyad $B^\star A_0^\star$ was found to be $A_0^\star(1 - \sqrt{2}/2, \sqrt{2})$. Assuming that the driving link is BA_0, determine the type of linkage this is, *double-crank*, *crank-rocker*, *rocker-crank* or *double-rocker*.

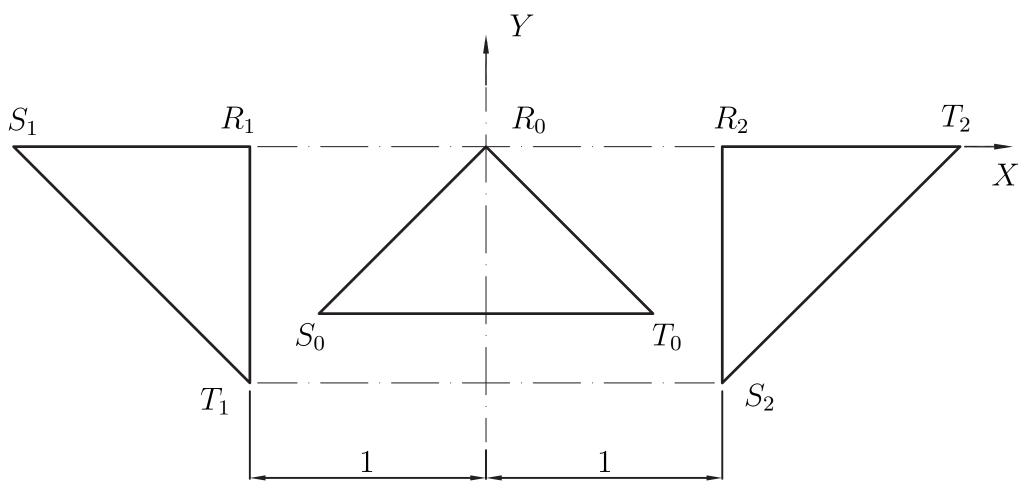

Figure 1: A triangular workpiece to undergo three machining steps in a flexible manufacturing cell, with lengths in m

[2]Caveat: the subscripts of the off-diagonal entries of the expression given in eq.(1.8) are flawed; entries should be swapped.