# McGill University

## Department of Mechanical Engineering

**MECH 541 Kinematic Synthesis** **Class Test**

**OPEN BOOK. ONLY FACULTY STANDARD CALCULATORS ALLOWED**

Date and Time: October 8th, 2009, from 8:35 a.m. to 9:55 a.m.

**1.** The synthesis of a planar four-bar linkage, to be used as a *symmetric* gripper, requires $k_2 = k_3$, and hence, leads to a synthesis matrix $\mathbf{S}$ of $m \times 2$, with a 2-dimensional unknown vector $\mathbf{k} = [\,k_1,\, k_2\,]^T$ and a *right-hand side* $m$-dimensional vector $\mathbf{b}$. The synthesis of the linkage was conducted for the data given in Table 1, which represent four equally spaced points along the line $\phi = 3\pi/2 - \psi$.

Table 1: Four data points equally spaced along line $\phi = 3\pi/2 - \psi$

| $i$th point | $\psi_i$ | $\phi_i$ |
|---|---|---|
| 1 | 0.5236 | 4.189 |
| 2 | 0.6981 | 4.014 |
| 3 | 0.8727 | 3.840 |
| 4 | 1.047 | 3.665 |

The corresponding synthesis matrix $\mathbf{S}$ and vector $\mathbf{b}$, along with the least-square approximation of the synthesis equation $\mathbf{Sk} = \mathbf{b}$ are given below:

$$\mathbf{S} = \begin{bmatrix} 1 & -1.366 \\ 1 & -1.409 \\ 1 & -1.409 \\ 1 & -1.366 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} -.8660 \\ -.9848 \\ -.9848 \\ -.8660 \end{bmatrix}, \quad \mathbf{k} = \begin{bmatrix} 2.906 \\ 2.762 \end{bmatrix}$$

(a) (20%) Verify whether $\mathbf{k}$ indeed leads to the least-square approximation of the synthesis equations; and

(b) (30%) set up the *normal equations* of the same problem, and compute their solution with four digits. Comment on the results. *Hint: Feel free to use the expression for the inverse of a* $2 \times 2$ *matrix that appears in the Lecture Notes. From this expression follows that the inverse of a* $2 \times 2$ *matrix is a replica of the given one, with its diagonal entries swapped, its off-diagonal entries with the signs reversed, and the whole matrix thus resulting divided by the determinant of the given matrix.*

**2.** Shown in Fig. 1 is a serial kinematic chain of the HHΠΠ type, with the screw pairs of parallel axes and different pitches, while the two Π-joints lie in the same plane, which is parallel to the axes of the H pairs.

(a) (20%) If the links are numbered from 1 to 5, with 1 being the base and 5 the end-link, find the kinematic bond $\mathcal{L}(1,5)$ of the kinematic chain, and determine the degree of freedom of the chain.

(b) (30%) Find the kinematic bond resulting from the parallel array of two identical chains of the type of Fig. 1, with the planes of their Π-joints at right angles. That is, the two chains share the same base and end-links.

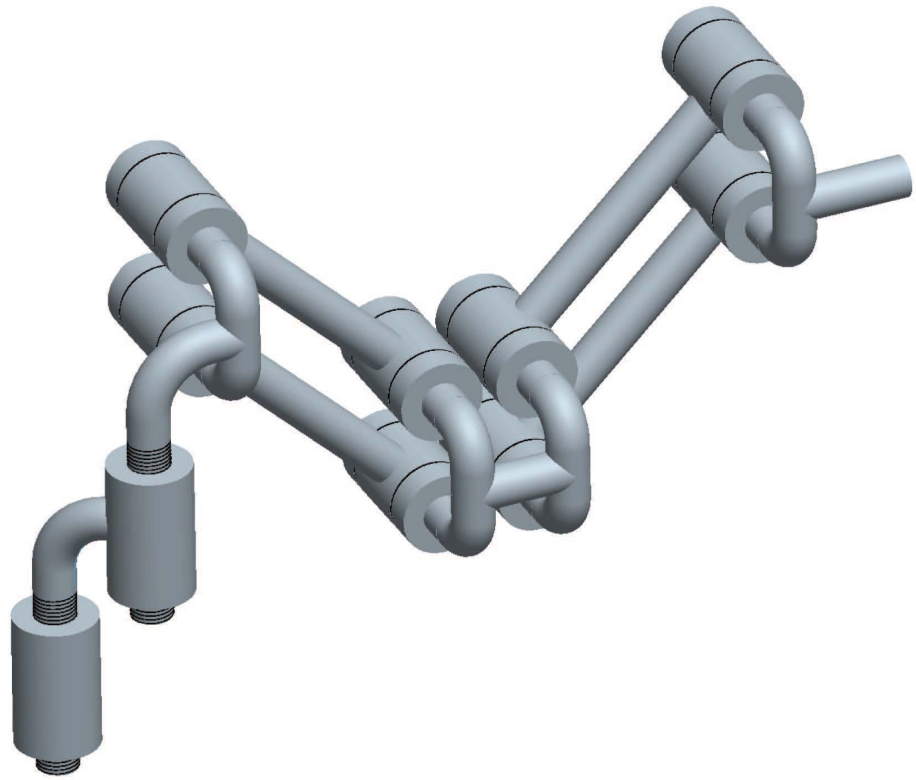

Figure 1: A serial chain of the HHΠΠ type with screw pairs of parallel axes and distinct pitches, the planes of the Π joints being parallel to the screw axes