McGill

Book Number /*Cahier d'examen n°* ____ de /*of* ____

EXAMINATION BOOK/CAHIER D'EXAMEN

Last Name/*Nom* : ____________________

First Name/ *Prénom* : ____________________

McGill ID/*N° de matricule McGill* : __ __ __ __ __ __ __ __ __ __ __

Date of Exam/*Date de l'examen* : 2014/03/12 (yyyy/mm/dd) *(année/mois/jour)*

Subject and Course Code/*Sujet et code du cours* : MECH 541 Kinematic Synthesis

Section/*Section* : ________ Room/*Salle* : ________ Row and Seat Number/*Rangée et siège* : ________

INSTRUCTIONS:

1. Fill in the above carefully.
2. Write your answer on the right-hand side of the exam book. Use the left-hand side for rough work and calculations.
3. Do not write in the margins.
4. If a page is accidentally left blank, write "P.T.O." on it.
5. Do not tear pages from the exam book.
6. At the time of the examination, you must not have in your possession any cellphones, books, calculators, dictionaries, notes or any other extraneous material unless otherwise indicated on the Exam Paper Cover instructions.
7. Put additional books inside the first book when submitting your exam.
8. **This book cannot be taken from the examination room.**

DIRECTIVES :

1. Remplissez soigneusement la section ci-dessus.
2. Écrivez vos réponses dans la section de droite du cahier d'examen. Utilisez la section de gauche pour l'ébauche et le calcul.
3. N'écrivez pas dans les marges.
4. Si vous avez laissé involontairement une page blanche, veuillez y inscrire « voir page suivante ».
5. Aucune page du cahier d'examen ne doit être retirée.
6. Durant l'examen, vous ne pouvez avoir en votre possession de cellulaire, livre, calculatrice, dictionnaire, note ou tout matériel superflu à moins d'indication contraire dans les directives indiquées sur la couverture de l'examen.
7. Veuillez insérer les cahiers additionnels à l'intérieur du premier cahier au moment de remettre votre examen.
8. **Le présent cahier doit demeurer dans la salle d'examen.**

For Examiner's Use Only Section réservée à l'examinateur
1. 25
2. 35
3. 40
4.
5.
6.
7.
8.
9.
10.
11.
12.
13.
14.
15.

1) (a) $1 \overset{R}{\text{———}} 2 \overset{\Pi}{\text{———}} 3 \overset{\Pi}{\text{———}} 4 \overset{R}{\text{———}} 5$

(b) $\mathcal{L}(1,5) = \mathcal{R}(\mathcal{A}_1)\cdot \underbrace{\mathcal{D}_{\Pi}(\vec{h})\cdot \mathcal{D}_{\Pi}(\vec{h})}_{\mathcal{T}_2(\vec{h})} \cdot \mathcal{R}(\mathcal{A}_2)$

with $\mathcal{A}_2 \parallel \mathcal{A}_1$

$\mathcal{R}(\mathcal{A}_1)\cdot \mathcal{T}_2(\vec{h})$ = set of rigid-body displacements producing a translation of link 5 in 3D space accompanied by a parasitical rotation about a vertical axis.

$\mathcal{R}(\mathcal{A}_2)$ compensates for that parasitical translation and produces an arbitrary rotation about a vertical axis.

$\Rightarrow \mathcal{L}(1,5)$ is $\mathcal{X}(\vec{k})$, $\vec{k}$ = a unit vector in vertical dir.

Ans.

3

(c) First, we need a graph of the chain:

$1 \overset{R}{\text{———}} 2 \overset{P}{\text{———}} 3 \overset{H}{\text{———}} 1$

$\Rightarrow \mathcal{L}(1,3) = \mathcal{R}(\mathcal{A})\cdot \mathcal{P}(\vec{a})\cdot \mathcal{H}(\mathcal{A}, p)$

$\mathcal{A}$: axis in Fig. 1b

$\vec{a}$: unit vector $\parallel \mathcal{A}$

$\Rightarrow \mathcal{L}(1,3) = \mathcal{C}(\mathcal{A}) \Rightarrow d = 2$

As well, $l = 3$, $j = 3$, $r = 1$

$\Rightarrow f = 2\times(3-1) - 3\times 1 = 4 - 3 = 1$ Ans.

2. (a) $$\underset{\sim}{S}_r \equiv \begin{bmatrix} 1 & 1 & -1 \\ 1 & \sqrt{2}/2 & 1/2 \\ 1 & 0 & 1/2 \end{bmatrix}, \quad \vec{b}_r \equiv \begin{bmatrix} 1 \\ 0.2588 \\ -0.8660 \end{bmatrix}$$

Subtract first eqn from 2nd & 3rd eqns:

$$\underset{\sim}{S}_{rr} = \begin{bmatrix} \sqrt{2}/2 - 1 & 3/2 \\ -1 & 3/2 \end{bmatrix}, \quad \vec{b}_{rr} = \begin{bmatrix} -0.7412 \\ -1.8660 \end{bmatrix}$$

$$\Rightarrow \vec{k}_r \equiv \begin{bmatrix} k_2 \\ k_3 \end{bmatrix} = \underset{\sim}{S}_{rr}^{-1} \vec{b}_{rr}$$

$$\Delta = \det(\underset{\sim}{S}_{rr}) = \left(\frac{\sqrt{2}}{2} - 1\right)\frac{3}{2} + \frac{3}{2} = \frac{3}{2}\frac{\sqrt{2}}{2} = \frac{3\sqrt{2}}{4}$$

$$\Rightarrow \underset{\sim}{S}_{rr}^{-1} = \frac{4}{3\sqrt{2}} \begin{bmatrix} 3/2 & -3/2 \\ 1 & \frac{\sqrt{2}}{2} - 1 \end{bmatrix} = \frac{\cancel{4}^{2}\sqrt{2}}{\cancel{6}_{3}} \begin{bmatrix} 3/2 & -3/2 \\ 1 & \frac{\sqrt{2}}{2} - 1 \end{bmatrix}$$

$$= \begin{bmatrix} \sqrt{2} & -\sqrt{2} \\ \frac{2\sqrt{2}}{3} & \frac{2}{3} - \frac{2\sqrt{2}}{3} \end{bmatrix} = \begin{bmatrix} 1.414 & -1.414 \\ 0.9428 & -0.2761 \end{bmatrix}$$

$$\Rightarrow \vec{k}_r = \begin{bmatrix} 1.414 & -1.414 \\ 0.9428 & -0.2761 \end{bmatrix} \begin{bmatrix} -0.7412 \\ -1.8660 \end{bmatrix} = \begin{bmatrix} 1.5905 \\ -0.1836 \end{bmatrix} = \begin{bmatrix} k_2 \\ k_3 \end{bmatrix}$$

Back to first eqn: $k_1 + k_2 - k_3 = 1$

$$\Rightarrow k_1 = 1 - (k_2 - k_3) = 1 - k_2 + k_3 = -0.7741 \qquad \text{Ans.}$$

(b) For $\vec{k}$ as obtained in (a) to be the least-square solution of the approximate-synthesis problem given at the outset, the error in the approximation, $\vec{e} = \vec{b} - \underset{\sim}{S}\vec{k}$, must lie in the nullspace of $\underset{\sim}{S}^T$, i.e., $\underset{\sim}{S}^T \vec{e} \overset{!}{=} \vec{0}$

$$\vec{e} = \begin{bmatrix} 1 \\ 0.258 \\ -0.866 \\ 1 \end{bmatrix} - \underbrace{\begin{bmatrix} 1 & 1 & -1 \\ 1 & 0.7071 & 0.5 \\ 1 & 0 & 0.5 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} -0.7741 \\ 1.5905 \\ -0.1836 \end{bmatrix}}_{\begin{bmatrix} 1.0 \\ 0.2587 \\ -0.8660 \\ 1.0 \end{bmatrix}} = \begin{bmatrix} 0 \\ -0.0007 \\ 0 \\ 0 \end{bmatrix} \sim \vec{0}$$

The error is close to zero, thereby proving that Jean's conjecture was right.
The error being close to zero, it is bound to satisfy the normality condition, $\underset{\sim}{S}^T \vec{e} = \vec{0}$, thereby convincing the boss that Jean's solution is the least-square solution of the approximate synthesis problem.

3. (a) The IO eqn for the four-bar spherical linkage is recalled:

$$F(\psi, \phi) = k_1 + k_2 c\psi + k_3 c\psi c\phi - k_4 c\phi + s\psi s\phi = 0$$

In our case, $\alpha_1 = 90^\circ \Rightarrow k_3 \equiv c\alpha_1 = 0$

$$\alpha_2 = \alpha_4 \Rightarrow \lambda_4 = \lambda_2 \ \& \ \mu_4 = \mu_2$$

$$\Rightarrow k_4 = k_2$$

$$\Rightarrow F(\psi, \phi) = k_1 + k_2 c\psi - k_2 c\phi + s\psi s\phi = 0$$

$$\text{or } F(\psi, \phi) = k_1 + k_2 (c\psi - c\phi) + s\psi s\phi = 0$$

$$\Rightarrow \underset{\sim}{S}_r = \begin{bmatrix} 1 & c\psi_1 - c\phi_1 \\ 1 & c\psi_2 - c\phi_2 \\ 1 & c\psi_3 - c\phi_3 \end{bmatrix} = \begin{bmatrix} 1 & 1 - 0 \\ 1 & c67.5^\circ - c202.5^\circ \\ 1 & c135^\circ - c135^\circ \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 1.3066 \\ 1 & 0 \end{bmatrix}$$

$\underset{\sim}{S}$-reduced

$$\vec{b}_r = \begin{bmatrix} -s\psi_1 s\phi_1 \\ -s\psi_2 s\phi_2 \\ -s\psi_3 s\phi_3 \end{bmatrix} = \begin{bmatrix} 0 \\ -s67.5^\circ s\,202.5^\circ \\ -s135^\circ s\,135^\circ \end{bmatrix} = \begin{bmatrix} 0 \\ 0.3536 \\ -0.5000 \end{bmatrix}$$

$\vec{b}$-reduced

Ans.

(b) Using the normal equations, the synthesis problem can be cast as

$$\underset{\sim}{S}_r^T \underset{\sim}{S}_r \vec{k}_r = \underset{\sim}{S}_r^T \vec{b}_r$$

$$\vec{k}_r = [k_1 \quad k_2]^T$$

$$\underset{\sim}{S}_r^T \underset{\sim}{S}_r = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1.3066 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 1.3066 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 2.3066 \\ \text{sym} & 2.7072 \end{bmatrix}$$

$$\underset{\sim}{S}_r^T \vec{b}_r = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1.3066 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0.3536 \\ -0.5000 \end{bmatrix} = \begin{bmatrix} -0.1464 \\ 0.4620 \end{bmatrix}$$

$$\vec{k}_r = (\underset{\sim}{S}_r^T \underset{\sim}{S}_r)^{-1} \underset{\sim}{S}_r^T \vec{b}_r$$

$$\Delta = \det(\underset{\sim}{S}_r^T \underset{\sim}{S}_r) = 3 \times 2.7072 - 2.3066^2 = 2.8012$$

$$\Rightarrow (\underset{\sim}{S}_r^T \underset{\sim}{S}_r)^{-1} = \frac{1}{2.8012} \begin{bmatrix} 2.7072 & -2.3066 \\ -2.3066 & 3 \end{bmatrix} = \begin{bmatrix} 0.9664 & -0.8234 \\ -0.8234 & 1.0710 \end{bmatrix}$$

$$\Rightarrow \vec{k}_r = \begin{bmatrix} 0.9664 & -0.8234 \\ -0.8234 & 1.0710 \end{bmatrix} \begin{bmatrix} -0.1464 \\ 0.4620 \end{bmatrix} = \begin{bmatrix} -0.5219 \\ 0.6153 \end{bmatrix} = \begin{bmatrix} k_1 \\ k_2 \end{bmatrix}$$

$k_3 = 0, \quad k_4 = k_2 = 0.6153$

Ans.

(c) $\alpha_2 = \cot^{-1}(k_2) = \tan^{-1}(1/k_2) = 58.39^\circ$

$\alpha_1 = 90^\circ \Rightarrow \lambda_1 = 0, \quad \mu_1 = 1; \; k_2 = k_4 \Rightarrow \lambda_4 = \lambda_2 \;\&\; \mu_4 = \mu_2$

$(3.26) \Rightarrow k_1 = -\dfrac{\lambda_3}{\mu_2^2} \Rightarrow \underbrace{\lambda_3}_{c\alpha_3} = -\mu_2^2 k_1, \quad \mu_2 = \sin\alpha_2 = 0.8517$

$\Rightarrow \alpha_3 = \cos^{-1}(-\mu_2^2 k_1) = \cos^{-1}(-0.7253 \times (-0.5219)) = 67.76^\circ$

$\alpha_4 = \alpha_2 = 58.39^\circ$

Ans.