CS-417 INTRODUCTION TO ROBOTICS AND INTELLIGENT SYSTEMS

Ultrasonic Sensing and Mapping
Sonar sensing

"The sponge"

0

a “chirp” is emitted into the environment

75µs

typically when reverberations from the initial chirp have stopped

sonar timeline

the transducer goes into “receiving” mode and awaits a signal...

.5s

after a short time, the signal will be too weak to be detected

Polaroid sonar emitter/receivers

Why is sonar sensing limited to between ~12 in. and ~25 feet?
Sonar effects

(a) Sonar providing an accurate range measurement

(b-c) Lateral resolution is not very precise; the closest object in the beam’s cone provides the response

(d) Specular reflections cause walls to disappear

(e) Open corners produce a weak spherical wavefront

(f) Closed corners measure to the corner itself because of multiple reflections --> sonar ray tracing resolution: time / space
Sonar modeling

- Initial time response
- Accumulated responses
- Blanking time
- Cone width
- Spatial response
Sonar Modeling

response model (Kuc)

\[ h_R(t, z, a, \alpha) = \frac{2c \cos \alpha}{\pi a \sin \alpha} \sqrt{1 - \frac{c^2(t - 2z/c)^2}{a^2 \sin^2 \alpha}} \]

- Models the response, \( h_R \), with:
  - \( c \) = speed of sound
  - \( a \) = diameter of sonar element
  - \( t \) = time
  - \( z \) = orthogonal distance
  - \( \alpha \) = angle of environment surface

- Then, add noise to the model to obtain a probability:
  \[ p(S \mid o) \]

  chance that the sonar reading is \( S \), given an obstacle at location \( O \)
Using sonar to create maps

What should we conclude if this sonar reads 10 feet?
Using sonar to create maps

What should we conclude if this sonar reads 10 feet?

Local Map
- unoccupied
- occupied

there isn’t something here

there is something somewhere around here

10 feet
What should we conclude if this sonar reads 10 feet?

- There isn’t something here
- There is something around here
- No information

Local Map
- Unoccupied
- No information
- Occupied

Or ...
Using sonar to create maps

What should we conclude if this sonar reads 10 feet...

and how do we add the information that the next sonar reading (as the robot moves) reads 10 feet, too?
Combining sensor readings

- The key to making accurate maps is combining lots of data.
- But combining these numbers means we have to know what they are!

What should our map contain?

- small cells
- each represents a bit of the robot’s environment
- larger values => obstacle
- smaller values => free

what is in each cell of this sonar model / map?
What is it a map of?

Several answers to this question have been tried:

- It’s a map of occupied cells.

Each cell is either occupied or unoccupied -- this was the approach taken by the Stanford Cart.

What information should this map contain, given that it is created with sonar?
What is it a map of?

Several answers to this question have been tried:

- **pre ‘83**
  - It’s a map of occupied cells.
  - $O_{xy}$: cell $(x,y)$ is occupied
  - $\overline{O}_{xy}$: cell $(x,y)$ is unoccupied

- **‘83 - ‘88**
  - It’s a map of probabilities:
    - $p(o | S_{1..i})$: The certainty that a cell is **occupied**, given the sensor readings $S_1, S_2, ..., S_i$
    - $p(\overline{o} | S_{1..i})$: The certainty that a cell is **unoccupied**, given the sensor readings $S_1, S_2, ..., S_i$

- maintaining related values separately?
- initialize all certainty values to zero
- contradictory information will lead to both values near 1
- combining them takes some work...
A Geometric (non-probabilistic) Approach

Arc-Carving
Combining probabilities

How to combine two sets of probabilities into a single map?
What is it a map of?

Several answers to this question have been tried:

pre '83

It's a map of occupied cells.

\[ o_{xy} \text{ cell (x,y) is occupied} \]

\[ \overline{o}_{xy} \text{ cell (x,y) is unoccupied} \]

'83 - '88

It's a map of probabilities:

\[ p(o \mid S_{1...i}) \quad \text{The certainty that a cell is occupied, given the sensor readings } S_1, S_2, ..., S_i \]

\[ p(\overline{o} \mid S_{1...i}) \quad \text{The certainty that a cell is unoccupied, given the sensor readings } S_1, S_2, ..., S_i \]

It's a map of odds. The odds of an event are expressed relative to the complement of that event.

The odds that a cell is occupied, given the sensor readings \( S_1, S_2, ..., S_i \)

\[ \text{odds}(o \mid S_{1...i}) = \frac{p(o \mid S_{1...i})}{p(\overline{o} \mid S_{1...i})} \]
An example map

Evidence grid of a tree-lined outdoor path

lighter areas: lower odds of obstacles being present
darker areas: higher odds of obstacles being present

how to combine them?
Conditional probability

Some intuition...

\[ p(o \mid S) = \]

The probability of event \( o \), given event \( S \).

The probability that a certain cell \( o \) is occupied, given that the robot sees the sensor reading \( S \).

\[ p(S \mid o) = \]

The probability of event \( S \), given event \( o \).

The probability that the robot sees the sensor reading \( S \), given that a certain cell \( o \) is occupied.

• What is really meant by conditional probability?
• How are these two probabilities related?
Bayes Rule

- Conditional probabilities

\[ p(o \wedge S) = p(o | S) \cdot p(S) \]
Bayes Rule

- Conditional probabilities

\[ p(o \land S) = p(o \mid S) p(S) \]
Bayes Rule

- Conditional probabilities

\[ p(o \land S) = p(o \mid S) p(S) \]

- Bayes rule relates conditional probabilities

\[ p(o \mid S) = \frac{p(o \mid S) p(o)}{p(S)} \]

Bayes rule
Bayes Rule

- Conditional probabilities

\[ p(o \land S) = p(o \mid S) p(S) \]

- Bayes rule relates conditional probabilities

\[
p(o \mid S) = \frac{p(o \mid S) p(o)}{p(S)}
\]

Bayes rule

- So, what does this say about \( \text{odds}(o \mid S_2 \land S_1) \)? Can we update easily?
Combining evidence

So, how do we combine evidence to create a map?

What we want --

\[ \text{odds}( o \mid S_2 \wedge S_1) \]

the new value of a cell in the map after the sonar reading \( S_2 \)

What we know --

\[ \text{odds}( o \mid S_1) \]

the old value of a cell in the map (before sonar reading \( S_2 \))

\[ p( S_i \mid o ) \ \& \ p( S_i \mid \overline{o} ) \]

the probabilities that a certain obstacle causes the sonar reading \( S_i \)
Combining evidence

\[
\text{odds}(o \mid S_2 \land S_1) = \frac{p(o \mid S_2 \land S_1)}{p(\bar{o} \mid S_2 \land S_1)}
\]
Combining evidence

\[
\text{odds}(o \mid S_2 \land S_1) = \frac{p(o \mid S_2 \land S_1)}{p(\bar{o} \mid S_2 \land S_1)} = \frac{p(S_2 \land S_1 \mid o) p(\bar{o})}{p(S_2 \land S_1 \mid \bar{o}) p(o)}
\]
Combining evidence

\[
\text{odds}(o \mid S_2 \land S_1) = \frac{p(o \mid S_2 \land S_1)}{p(\overline{o} \mid S_2 \land S_1)}
\]

\[
= \frac{p(S_2 \land S_1 \mid o) p(\overline{o})}{p(S_2 \land S_1 \mid \overline{o}) p(o)}
\]

\[
= \frac{p(S_2 \mid o) p(S_1 \mid o) p(\overline{o})}{p(S_2 \mid \overline{o}) p(S_1 \mid \overline{o}) p(o)}
\]

definition of odds

Bayes’ rule (⁺)
Combining evidence

\[ \text{odds}(o \mid S_2 \land S_1) = \frac{p(o \mid S_2 \land S_1)}{p(\overline{o} \mid S_2 \land S_1)} \]

\[ = \frac{p(S_2 \land S_1 \mid o) p(\overline{o})}{p(S_2 \land S_1 \mid \overline{o})} \]

\[ = \frac{p(S_2 \mid o) p(S_1 \mid o) p(\overline{o})}{p(S_2 \mid \overline{o}) p(S_1 \mid \overline{o})} \]

\[ = \frac{p(S_2 \mid o) p(o \mid S_1)}{p(S_2 \mid \overline{o}) p(\overline{o} \mid S_1)} \]

Definition of odds

Bayes’ rule (+)

Conditional independence of \( S_1 \) and \( S_2 \)

Bayes’ rule (+)
Combining evidence

\[
odds(o \mid S_2 \land S_1) = \frac{p(o \mid S_2 \land S_1)}{p(\overline{o} \mid S_2 \land S_1)}.
\]

\[
= \frac{p(S_2 \land S_1 \mid o) p(\overline{o})}{p(S_2 \land S_1 \mid \overline{o}) p(o)}
\]

\[
= \frac{p(S_2 \mid o) p(S_1 \mid o) p(\overline{o})}{p(S_2 \mid \overline{o}) p(S_1 \mid \overline{o}) p(o)}
\]

\[
= \frac{p(S_2 \mid o) p(o \mid S_1)}{p(S_2 \mid \overline{o}) p(\overline{o} \mid S_1)}
\]

definition of odds
Bayes’ rule (+)
conditional independence of \(S_1\) and \(S_2\)
Bayes’ rule (+)

precomputed values
the sensor model
previous odds

Update step = multiplying the previous odds by a precomputed weight.
Evidence grids

hallway with some open doors  lab space

known map and estimated evidence grid
The sonar model depends dramatically on the environment -- we’d like to learn an appropriate sensor model rather than hire Roman Kuc to develop another one...
Learning the Sensor Model

The sonar model depends dramatically on the environment -- we’d like to *learn* an appropriate sensor model rather than hire Roman Kuc to develop another one...
Learning the Sensor Model

the idealized model

the mapping results of a model that had an even better match score (against the ideal map)

part of the learned model
Sensor fusion

Incorporating data from other sensors -- e.g., IR rangefinders and stereo vision...

(1) create another sensor model
(2) update along with the sonar