Single Axis Rotations of a Cube

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1 The Given Cube

An origin centred cube with its edges parallel to principle Cartesian axes is given by its eight corners on the following points.

- The front face, viewed facing towards positive y, counterclockwise from the lower left corner contains points $A(-1,-1,-1)$, $B(1,-1,-1)$, $C(1,-1,1)$, $D(-1,-1,1)$. These points are joined in sequence to form a square when closed on A.
- The rear face, viewed facing towards positive y, counterclockwise from the lower left corner contains points $E(-1,1,-1)$, $F(1,1,-1)$, $G(1,1,1)$, $H(-1,1,1)$. These points are joined in sequence to form a square when closed on E.
- The remaining four edges, all parallel to the y-axis, are formed by joining A to E, B to F, C to G and D to H.

2 A Body Diagonal Rotated into Point View

By rotating a body diagonal of the cube, say the line on D and F, into point view when projected onto, say, plane $z = 0$, one obtains a view wherein the edges of the cube appear as sides of a regular hexagon and the six "spokes" radiating from the point image of DF to the six vertices of the hexagon. If this is done in two steps, first rotating about the positive z-axis through $\frac{\pi}{4}$ in the sense of a right hand screw, bringing the plane $DHFA$ normal to the x-axis, and then about the negative x-axis through $\cos^{-1}\frac{1}{\sqrt{3}}$ in the sense of a right hand screw, not only will the desired spoked hexagon appear but the x-axis will bisect two opposite sides and the y-axis will be on a pair of opposite vertices as shown in Fig. 1.

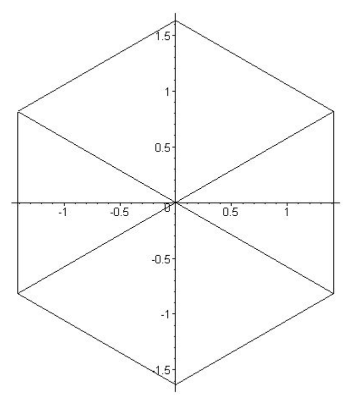

Figure 1: A Double Rotation Produces the Desired Orientation

3 How Can This Be Done with a Single Rotation?

It is known that any sequence of rotations, like the two rotations about fixed axes described above, can be effected by a single rotation but the axis and rotation angle required may not be obvious. This will be demonstrated by obtaining the end view of cube body diagonal line segment DF in two ways. First a right hand rotation of $\cos^{-1}\frac{1}{\sqrt{3}}$ about an axis with direction numbers $\{-\frac{1}{\sqrt{2}}:\frac{1}{\sqrt{2}}:0\}$ is illustrated in Fig. 2.

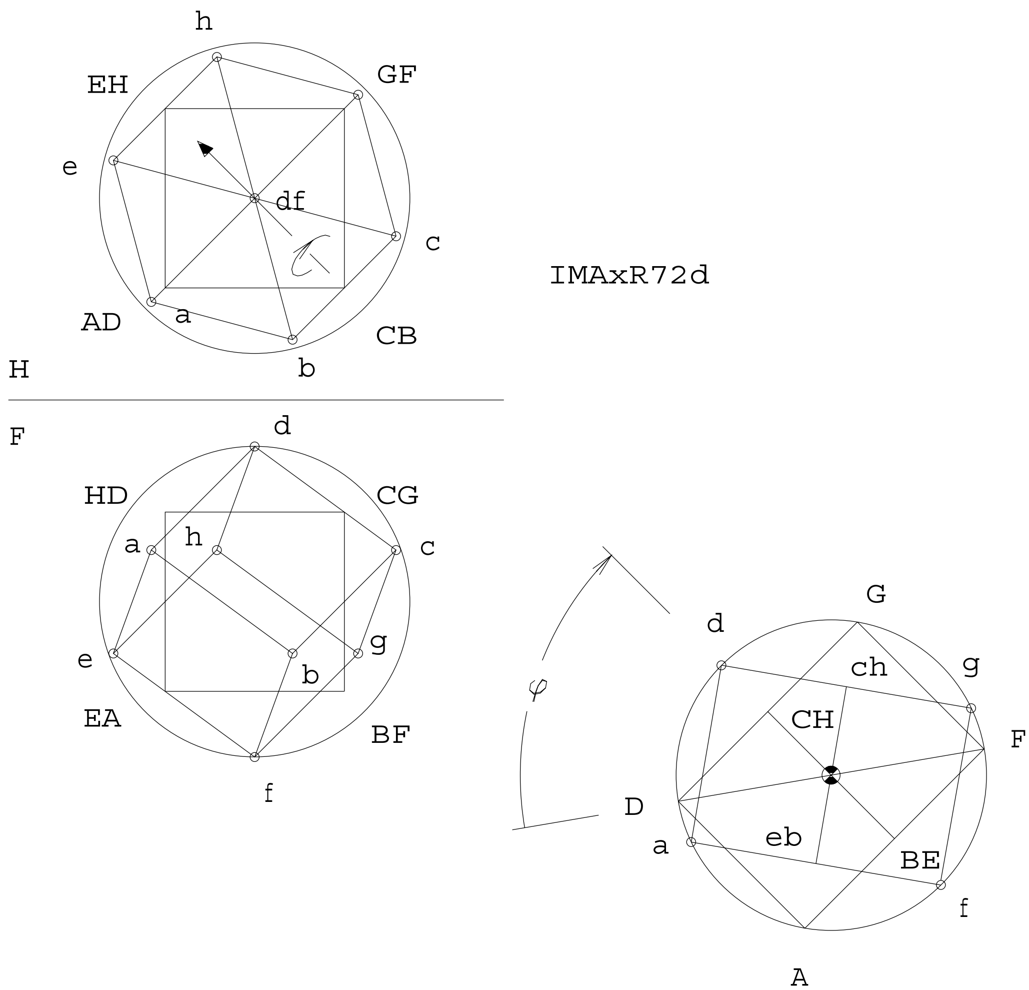

Figure 2: A Minimum Meridial Rotation

Note that, although the orientation of the rotated image in the plane $z = 0$ is symmetrical with respect to the rotation axis, no opposite corners or pairs of hexagon side midpoints are respectively oriented in the x- and y-directions. Nevertheless this is the *minimum* angle through which DF

can be rotated to achieve a point view. D and F have been moved along a great circle. The second demonstration will entail the greatest possible angle of rotation, *i.e.*, π. Here the line segments Dd and Ff are circle diameters and the rotation axis bisects the angle between the line on the cube centre and D and the z-axis. The result is shown in Fig. 3. Once again the hexagon has failed to achieve the desired orientation.

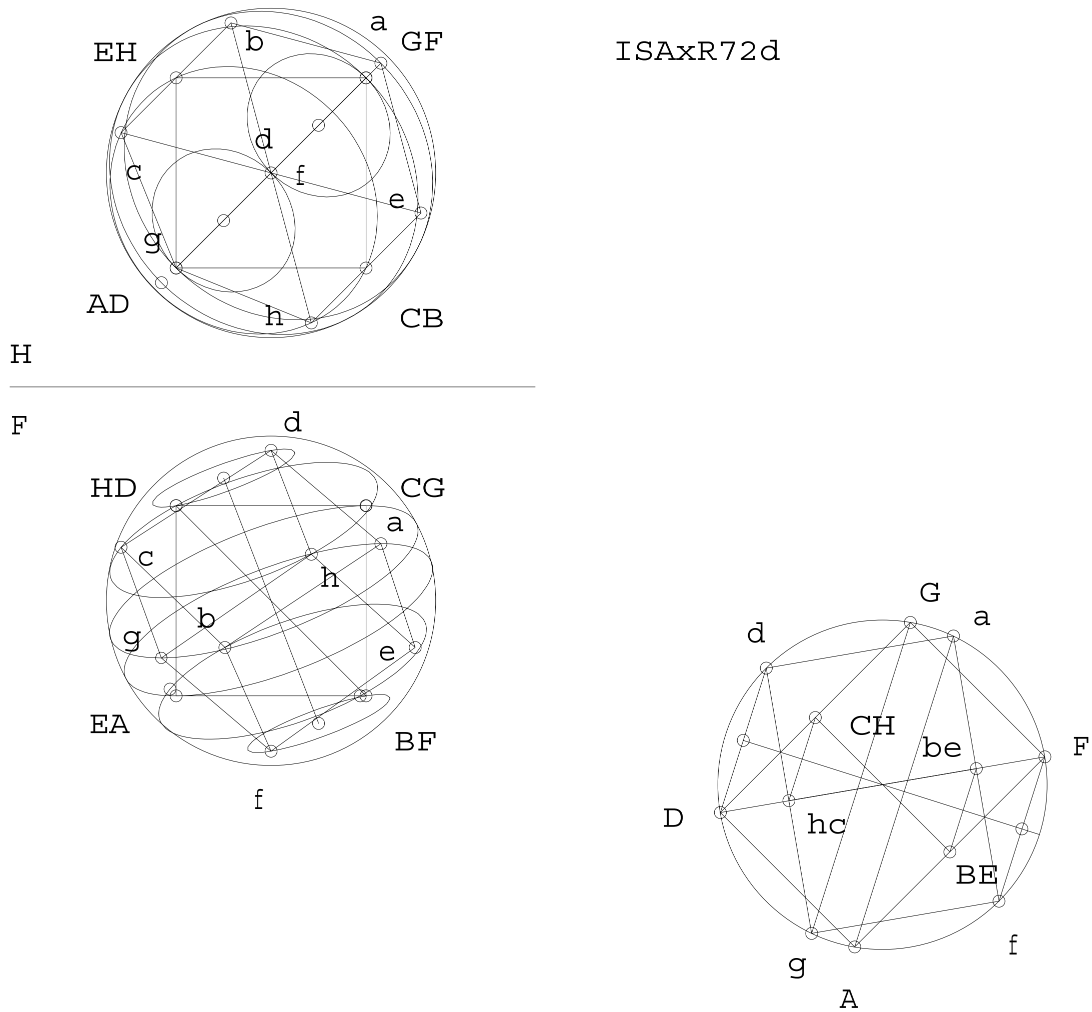

Figure 3: A Maximum Rotation through π

Somewhere in the plane on the two rotation axes, respectively chosen as shown in Fig. 2 and Fig. 3, lies the desired axis. Maybe it bears east-west or north-south. Using Maple (or some other appropriate mathematical software capable of vector and matrix manipulation) and the example worksheet, available via the link immediately after the one you used to access this document, validate or refute this conjecture together with some well-chosen statements supporting your conclusion. This exercise will constitute GCA 2. (MECH289)SAXRC72d.tex