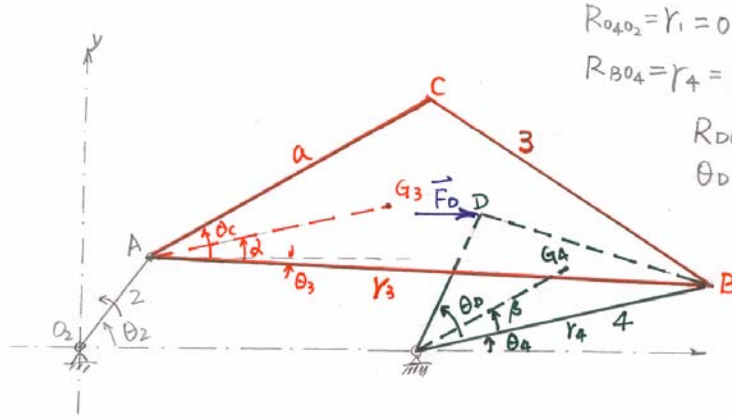
These are solutions to Problem Set 6.

Problem 1. (Problem 15.8, Page 512 From the book J.J Uicker, et.al.)

Note: Use the principle of Superposition.

Scale: 1 cm = 0.15 m.

Given: $\theta_2 = 53°$. $R_{AO_2} = r_2 = 0.3\,m$.
$R_{O_4O_2} = r_1 = 0.9\,m$. $R_{BA} = r_3 = 1.5\,m$.
$R_{BO_4} = r_4 = 0.8\,m$. $R_{CA} = a = 0.85\,m$.
$R_{DO_4} = 0.4\,m$. $\theta_C = 33°$.
$\theta_D = 53°$. $R_{G_2O_2} = 0$.



$R_{G_3A} = 0.65\,m$, $\alpha = 16°$.
$R_{G_4O_4} = 0.45\,m$, $\beta = 17°$.
$m_2 = 5.2\,kg$.
$I_{G_2} = 2.3\,kg \cdot m^2$
$m_3 = 65.8\,kg$
$I_{G_3} = 4.2\,kg\,m^2$.
$m_4 = 21.8\,kg$
$I_{G_4} = 0.51\,kg \cdot m^2$

Kinematic Analysis: $\omega_2 = 12\hat{k}$ rad/s ccw,

Graphically, $\theta_3 = -2.7°$, $\theta_4 = 12.2°$,

$\vec{R}_{AO_2} = 0.24\hat{i} + 0.18\hat{j}$ (m). $\vec{R}_{BO_4} = 0.78\hat{i} + 0.17\hat{j}$ (m)

$\vec{R}_{BA} = 1.498\hat{i} - 0.071\hat{j}$ (m)

$$\vec{R}_{G_3A} = R_{G_3A}\cos\underbrace{(\alpha+\theta_3)}_{13.3°}\hat{i} + R_{G_3A}\sin\underbrace{(\alpha+\theta_3)}_{13.3°}\hat{j} = 0.63\hat{i} + 0.15\hat{j}\ (m)$$

$$\vec{R}_{G_4O_4} = R_{G_4O_4}\cos\underbrace{(\beta+\theta_4)}_{29.2°}\hat{i} + R_{G_4O_4}\sin\underbrace{(\beta+\theta_4)}_{29.2°}\hat{j} = 0.39\hat{i} + 0.22\hat{j}\ (m)$$

$$\vec{R}_{DO_4} = R_{DO_4}\cos\underbrace{(\theta_D+\theta_4)}_{70°}\hat{i} + R_{DO_4}\sin\underbrace{(\theta_D+\theta_4)}_{70°}\hat{j} = 0.137\hat{i} + 0.376\hat{j}\ (m)$$

Dynamic given condition:

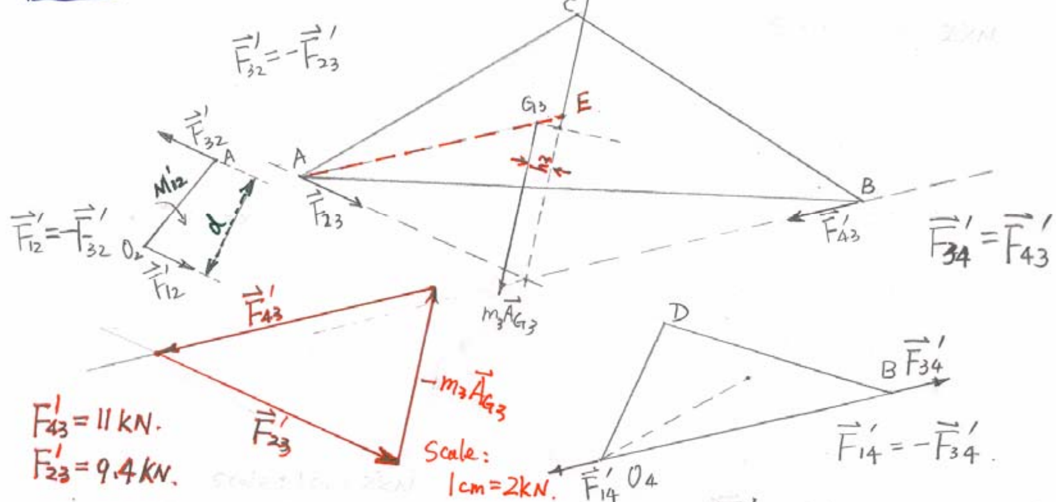
$\vec{\alpha}_3 = 85.6$ rad/s² cw, $\vec{A}_{G_3} = 96.4\angle 259°$ m/s² $= -18.4\hat{i} - 94.6\hat{j}$ (m/s²)

$\vec{\alpha}_4 = 172$ rad/s² ccw, $\vec{A}_{G_4} = 97.8\angle 270°$ m/s² $= -97.8\hat{i}$ (m/s)

$\vec{F}_D = 12\angle 0°$ kN $= 12\hat{i}$ kN.

Note: Inertia force of Link 2 has no effect on $\vec{M}_{12}$ since $\begin{cases} \vec{R}_{G_2O_2} = \vec{0} \\ \vec{\alpha}_2 = 0 \end{cases}$.

$\boxed{\text{Step 1}}$ Inertia force of Link 3:



$m_3 A_{G_3} = 65.8 \times 96.4 = 6.343$ KN

$I_3\alpha_3 = 4.2 \times 85.6 = 359.2$ N·m. cw

$h_3 = \frac{I_3\alpha_3}{m_3 A_{G_3}} = 0.057$ m.

→ Graphically $\begin{cases} F'_{32} = 9.4 \text{ KN.} \\ d = 0.28 \text{ m.} \\ M'_{12} = dF'_{32} = 2.608 \text{ KN·m. cw.} \end{cases}$

Link 4 is a two-force number. we have: $\underline{-m_3\vec{A}_{G_3} + \vec{F}'_{23} + \vec{F}'_{43} = 0}$.

The moment balance at point A:

$$0 = \vec{M}'_A = \vec{R}_{EA} \times (-m\vec{A}_{G_3}) + \vec{R}_{BA} \times \vec{F}'_{43}$$

$$= \left[R_{EA}\cos(\theta_3+\alpha)\hat{i} + R_{EA}\sin(\theta_3+\alpha)\right] \times \left[-mA_{G_3}(\cos 259^\circ\hat{i} + \sin 259^\circ\hat{j})\right] + \vec{R}_{BA} \times \vec{F}'_{43}$$

$$= (0.69\hat{i} + 0.164\hat{j}) \times (1.21\hat{i} + 6.23\hat{j}) + (1.498\hat{i} - 0.071\hat{j}) \times (F'_{43}\cos\theta_4\hat{i} + F'_{43}\sin\theta_4\hat{j})$$

$$= 4.1\hat{k} + 0.39 F'_{43}\hat{k}. \ (\text{KN·m})$$

$\therefore \ F'_{43} = -10.5$ KN. $\quad \vec{F}'_{43} = -10.3\hat{i} - 2.22\hat{j}$ (KN)

$\therefore \ \vec{F}'_{23} = m_3\vec{A}_{G_3} - \vec{F}'_{43} = 9.09\hat{i} - 4.01\hat{j}$ (KN)

$\vec{F}'_{34} = -\vec{F}'_{43} = 10.3\hat{i} + 2.22\hat{j}$. (KN)

$\vec{F}'_{14} = -\vec{F}'_{34} = -10.3\hat{i} - 2.22\hat{j}$ (KN).

$$\vec{F}_{32}' = -\vec{F}_{23}' = -9.09\hat{i} + 2.22\hat{j} \ (KN)$$

$$\vec{F}_{12}' = -\vec{F}_{32}' = 9.09\hat{i} - 2.22\hat{j} \ (KN).$$

$$\vec{M}_{12}' = -\vec{R}_{AO_2} \times \vec{F}_{32}' = -(0.24\hat{i} + 0.18\hat{j}) \times (-9.09\hat{i} + 2.22\hat{j})$$

$$= -2.169 \ \hat{k} \ KN \cdot m.$$

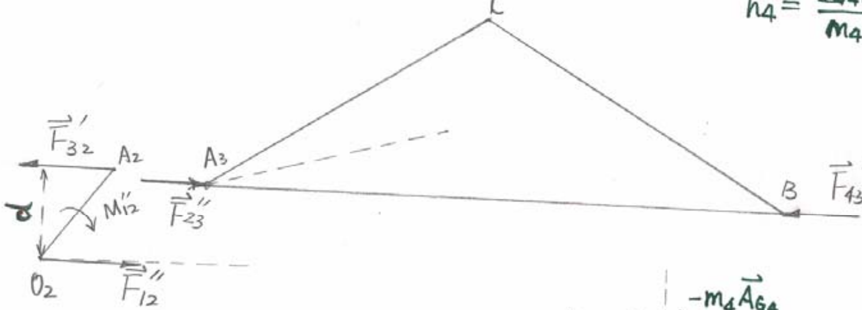
or, $\quad M_{12}' = 2.169 \ KN \cdot m \ . \ CW.$

$\boxed{\text{step 2}}$ Inertia force of Link 4.

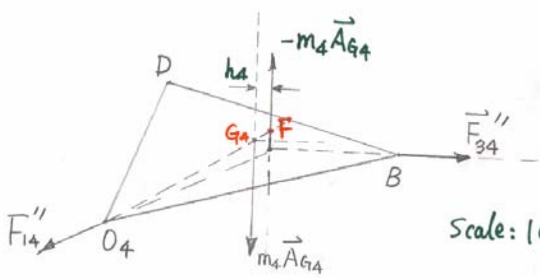
$\rightarrow m_4 A_{G4} = 2.132$ KN.

$I_{G4}\alpha_4 = 87.72$ N·m.

$$h_4 = \frac{I_{G4}\alpha_4}{m_4 A_{G4}} = 0.041 \ m.$$



Now, Link 3 is two-force Number.



Scale: 1 cm = 0.5 Kı

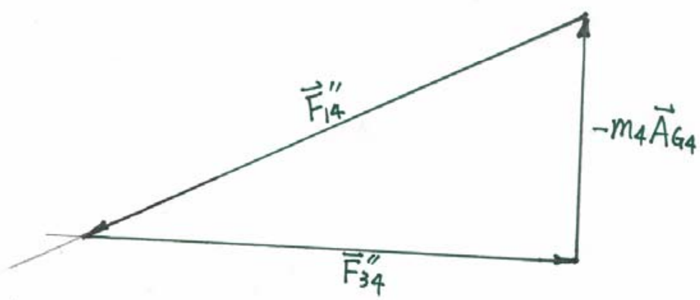
$$\vec{F}_{34}'' = -\vec{F}_{43}''$$

$$\vec{F}_{23}'' = -\vec{F}_{43}''$$

$$\vec{F}_{32} = -\vec{F}_{23}''$$

$d = 0.173$ m.

$M_{12} = d F_{32}'' = 0.82$ KN·m CW.



$F_{34}'' = 4.75$ KN

$F_{14}'' = 5.2$ KN.

Analytically, the link 4 force balance, $-m_4\vec{A}_{G4} + \vec{F}_{14}'' + \vec{F}_{34}'' = 0$

The moment balance at O_4:

$$\vec{0} = \vec{M}_{O4}'' = \vec{R}_{FO_4} \times (-m_4 \vec{A}_{G4}) + \vec{R}_{BO4} \times \vec{F}_{34}''$$

$$= \left[R_{FO_4}\cos(\beta+\theta_4)\hat{i} + \vec{R}_{FO_4}\sin(\beta+\theta_4)\hat{j}\right] \times (-m_4\vec{A}_{G4})$$

$$+ (R_{BO4}\cos\theta_4\hat{i} + R_{BO4}\sin\theta_4\hat{j}) \times (F_{34}\cos\theta_3\hat{i} + F_{34}\sin\theta_3\hat{j})$$

$$= [0.431\hat{i} + 0.22\hat{j}] \times (2.132\hat{j}) + (0.78\hat{i} + 0.17\hat{j}) \times F_{34}(0.999\hat{i} - 0.05\hat{j})$$

$$= 0.919\hat{k} - 0.209 F_{34}\hat{k} \quad KN \cdot m.$$

$\therefore \quad F_{34}'' = 4.4$ KN or $\vec{F}_{34}'' = 4.395\hat{i} - 0.21\hat{j}$ KN.

$\vec{F}_{14}'' = -4.395\hat{i} - 1.922\hat{j}$ KN.

Thus, $\vec{F}_{43}'' = -\vec{F}_{34}'' = -4.395\hat{i} + 0.21\hat{j}$ KN

$\vec{F}_{23}'' = -\vec{F}_{43}'' = 4.395\hat{i} - 0.21\hat{j}$ KN.

$\vec{F}_{32}'' = -\vec{F}_{23}'' = -4.395\hat{i} + 0.21\hat{j}$ KN

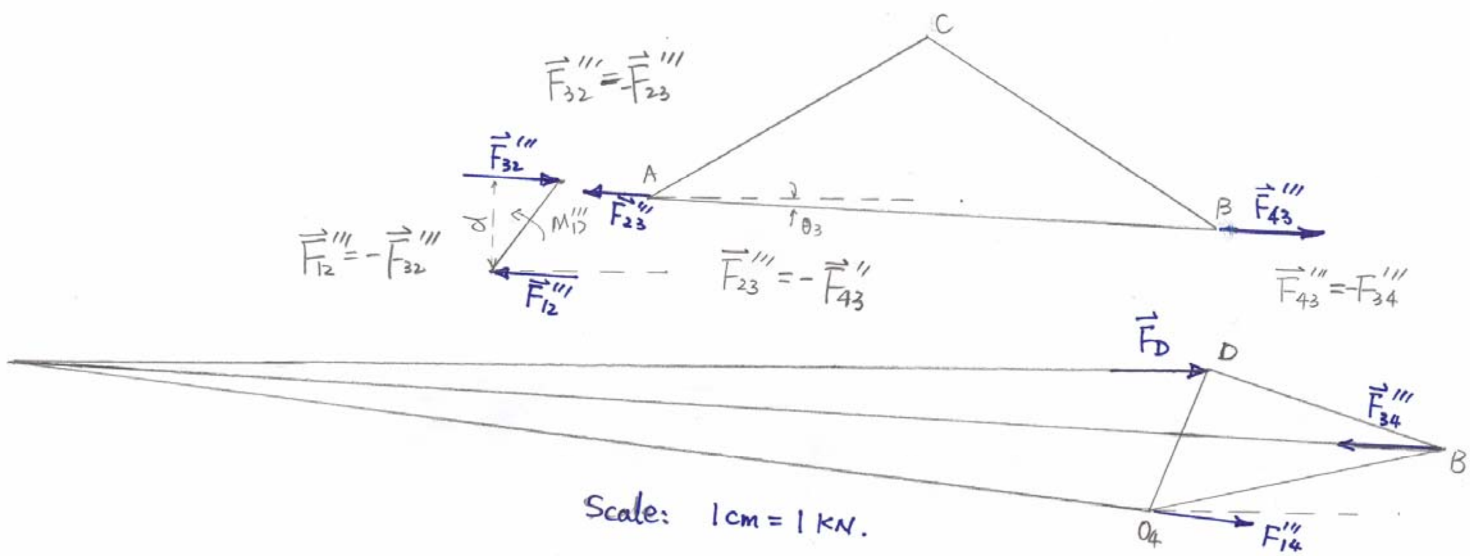
$\vec{F}_{12}'' = -\vec{F}_{32}'' = 4.395\hat{i} - 0.21\hat{j}$ KN.

$$\vec{M}_{12}'' = -\vec{R}_{AO_2} \times \vec{F}_{32}'' = -(0.24\hat{i} + 0.18\hat{j}) \times (-4.395\hat{i} + 0.21\hat{j})$$

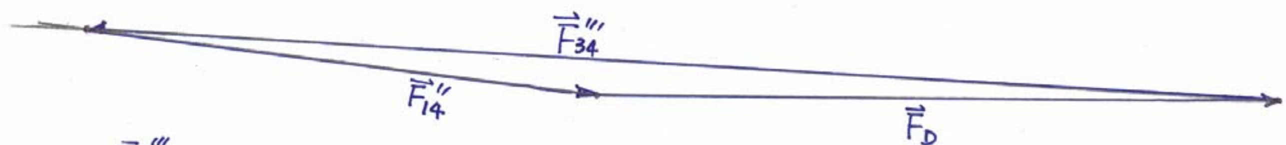
$$= -0.8415\hat{k} \, KN \cdot m.$$

or, $M_{12} = 0.8415$ KN·m CW.

step 3: external force $\vec{F}_D = 12\ KN\ \hat{i}$ at point D.



Scale: 1cm = 1 KN.



$F_{34}''' = 21$ KN.

$F_{14}''' = 9.1$ KN.

$d = 0.173$ m

$M_{12}''' = d \cdot F_{34}''' = 3.633$ KN.m ccw.

On Link 3, the force balance: $\vec{F}_D + \vec{F}_{34}''' + \vec{F}_{14}''' = 0$.

The moment balance at point O_4:

$$0 = \vec{M}_{O4}''' = \vec{R}_{DO_4} \times \vec{F}_D + \vec{R}_{BO_4} \times \vec{F}_{34}'''$$

$$= (0.137\hat{i} + 0.376\hat{j}) \times (12\hat{i}) + (0.78\hat{i} + 0.17\hat{j}) \times F_{34}'''(\cos\theta_3\hat{i} + \sin\theta_3\hat{j})$$

$$= -4.512\hat{k} - 0.209 F_{34}''' \hat{k} \quad (\text{kN}\cdot\text{m}).$$

$\therefore \quad F_{34}''' = -21.59$ kN. $\quad$ or $\vec{F}_{34}''' = -21.56\hat{i} + 1.02\hat{j}$ kN.

$\vec{F}_{14}''' = 9.59\hat{i} - 1.02\hat{j}$ kN.

$\vec{F}_{43}''' = -\vec{F}_{34}''' = 21.56\hat{i} - 1.02\hat{j}$ kN.

$\vec{F}_{23}''' = -\vec{F}_{43}''' = -21.56\hat{i} + 1.02\hat{j}$ kN,

$\vec{F}_{32}''' = -\vec{F}_{23}''' = 21.56\hat{i} - 1.02\hat{j}$ kN.

$\vec{F}_{12}''' = -\vec{F}_{32}''' = -21.56\hat{i} + 1.02\hat{j}$ kN.

$$\vec{M}_{12}''' = -\vec{R}_{AO_2} \times \vec{F}_{32}''' = -(0.24\hat{i} + 0.18\hat{j}) \times (21.56\hat{i} - 1.02\hat{j})$$

$$= 4.126\ \hat{k}\ \text{kN}\cdot\text{m}.$$

Or. $\vec{M}_{12}''' = 4.126$ kN·m CCW.

Superpositioning the three steps:

$$\vec{M}_{12} = \vec{M}_{12}' + \vec{M}_{12}'' + \vec{M}_{12}''' = 1.115 \text{ kN}\cdot\text{m CCW}.$$

$$\vec{F}_{12} = \vec{F}_{12}' + \vec{F}_{12}'' + \vec{F}_{12}''' = -8.08\hat{i} - 1.41\hat{j} \text{ kN}.$$

$$\vec{F}_{14} = \vec{F}_{14}' + \vec{F}_{14}'' + \vec{F}_{14}''' = -5.105\hat{i} - 5.162\hat{j} \text{ kN}.$$

Also: $\vec{F}_{23} = -\vec{F}_{32} = \vec{F}_{23}' + \vec{F}_{23}'' + \vec{F}_{23}''' = -8.075\hat{i} - 1.41\hat{j}$ kN.

$\vec{F}_{34} = -\vec{F}_{43} = \vec{F}_{34}' + \vec{F}_{34}'' + \vec{F}_{34}''' = -6.865\hat{i} + 3.03\hat{j}$ kN.

Assignment #6: (MECH 314).

Problem 1. solution: [10.2 in the book of J. J. Uicker].

By given conditions, we have, (see figure in the book).

$$\theta'_{5/2} = \left(-\frac{R_4}{R_5}\right)\left(-\frac{R_2}{R_3}\right) = \left(-\frac{7}{15}\right)\cdot\left(-\frac{9}{30}\right) = \frac{7}{50}.$$

$$\therefore\ \omega_5 = \theta'_{5/2}\cdot\omega_2 = \frac{7}{50}\cdot 120 = 16.8 \text{ rev/min. cw.}$$

$$\theta'_{7/2} = \left(-\frac{R_6}{R_7}\right)\left(-\frac{R_5}{R_6}\right)\left(-\frac{R_4}{R_5}\right)\left(-\frac{R_2}{R_3}\right)$$

$$= \frac{\cancel{R_6}}{R_7}\cdot\frac{R_5}{\cancel{R_6}}\cdot\theta'_{5/2} = \frac{30}{16}\cdot\frac{7}{50} = \frac{21}{80}.$$

$$\therefore\ \omega_7 = \theta'_{7/2}\cdot\omega_2 = \frac{21}{80}\cdot 120 = 31.5 \text{ rev/min. cw.}$$

Problem 2 Solution: [10.6 in the book J. J. Uicker]

By given Conditions in the book, $\omega_2 = 0$ (Gear 2 is fixed).

$$\theta^*_{7/2} = \frac{N_6}{N_7}\cdot\left(-\frac{\cancel{N_4}}{N_5}\right)\cdot\left(-\frac{N_2}{\cancel{N_4}}\right) = \frac{36}{154}\cdot\frac{20}{18} = \frac{20}{77}.$$

Also,
$$\theta^*_{7/2} = \frac{\omega_7 - \omega_3}{\omega_2 - \omega_3}.$$

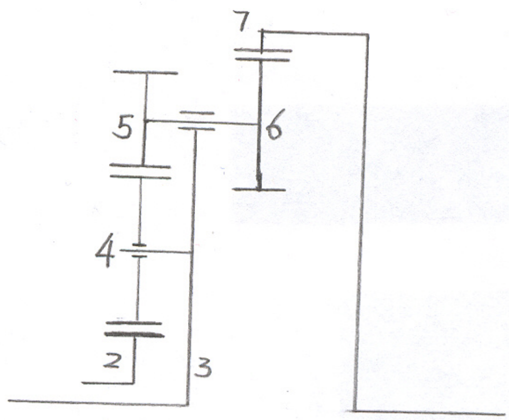
$$\theta^*_{7/2}(\omega_2 - \omega_3) = \omega_7 - \omega_3$$

$$\therefore\ \omega_3 = \frac{\omega_7 - \theta^*_{7/2}\cdot\omega_2}{1 - \theta^*_{7/2}} = \frac{60}{1 - \frac{20}{77}} = \frac{60\times 77}{57} = 81.5 \text{ rev/min.}$$

ccw.

Problem 3. (10.12 in the book J.J Uicker)

The Lévai type-L is shown as follows,



When Gear 7 is fixed, $\omega_7 = 0$. and $\omega_2 = -100$ rev/min. CW.

$$\theta^*_{7/2} = \frac{N_6}{N_7} \cdot \left(-\frac{N_4}{N_5}\right)\left(-\frac{N_2}{N_4}\right) = \frac{24}{95} \cdot \left(-\frac{19}{17}\right)\left(-\frac{16}{19}\right) = \frac{384}{1615}.$$

Also,
$$\theta^*_{7/2} = \frac{\omega_7 - \omega_3}{\omega_2 - \omega_3}.$$

$$\therefore \omega_3 = \frac{\omega_7 - \theta^*_{7/2} \cdot \omega_2}{1 - \theta^*_{7/2}} = \frac{-\left(\frac{384}{1615}\right)(-100)}{1 - \frac{384}{1615}}$$

$= 31.19$ rev/min. CCW.