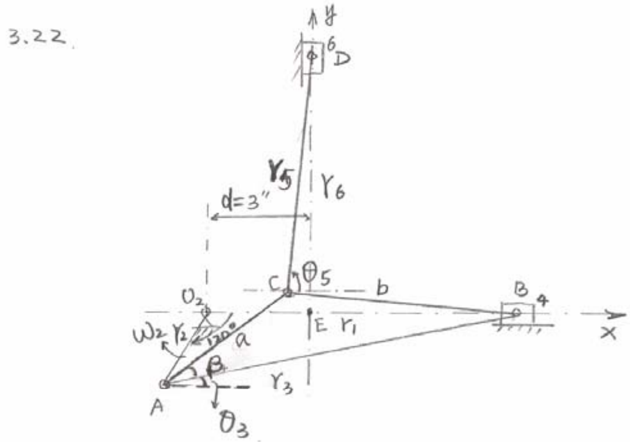
These are solutions to Problem Set 4.

3.22 ①



Given: $R_{AO_2} = 2\text{ in} = r_2$, $R_{BA} = 10\text{ in} = r_3$,
$R_{CA} = 4\text{ in} = a$, $R_{CB} = 7\text{ in.} = b$,
$R_{DC} = 8\text{ in.} = r_5$.

input: $\theta_2 = -120°$.
$\omega_2 = 42\text{ rad/s}$. cw.

Note: Some dimensions (position and Angle) are obtained by previous assignment solution.

Here, $\theta_3 = 10°$. $\beta = 34°$.
$\theta_5 = 83°$.

Direct differentiat method:

→ Loop I closure equation:

$$\vec{R}_{AO_2} + \vec{R}_{BA} - \vec{R}_{BO_2} = \vec{0}$$

↓

$$\left.\begin{aligned} r_2\cos\theta_2 + r_3\cos\theta_3 - r_1\cos\theta_1 &= 0. \\ r_2\sin\theta_2 + r_3\sin\theta_3 - r_1\sin\theta_1 &= 0. \end{aligned}\right\}$$

↓ Differentiate with time. $\theta_1 = 0$ also.

$$\left.\begin{aligned} -r_2\sin\theta_2\cdot\dot{\theta}_2 - r_3\sin\theta_3\cdot\dot{\theta}_3 - \dot{r}_1 &= 0 \\ r_2\cos\theta_2\cdot\dot{\theta}_2 + r_3\cos\theta_3\cdot\dot{\theta}_3 &= 0 \end{aligned}\right\} \rightarrow$$

$$\begin{bmatrix} -r_3\sin\theta_3 & -1 \\ r_3\cos\theta_3 & 0 \end{bmatrix}\begin{bmatrix} \dot{\theta}_3 \\ \dot{r}_1 \end{bmatrix} = \begin{bmatrix} r_2\sin\theta_2 \\ -r_2\cos\theta_2 \end{bmatrix}\dot{\theta}_2 \rightarrow \text{invert the coefficient Matrix.}$$

$$\rightarrow \begin{bmatrix} \dot{\theta}_3 \\ \dot{r}_1 \end{bmatrix} = \begin{bmatrix} 0 & \dfrac{1}{r_3\cos\theta_3} \\ -1 & \dfrac{\sin\theta_3}{\cos\theta_3} \end{bmatrix}\begin{bmatrix} r_2\sin\theta_2 \\ -r_2\cos\theta_2 \end{bmatrix}\dot{\theta}_2$$

$$\rightarrow \dot{\theta}_3 = \underbrace{-\frac{r_2\cos\theta_2}{r_3\cos\theta_3}}_{\theta_3' = \frac{d\theta_3}{d\theta_2}}\dot{\theta}_2 = \underbrace{0.1015}_{\theta_3'}(-42\text{ rad/s}) = -4.26\text{ rad/s}.$$

$$\dot{r}_1 = \underbrace{\left(-r_2\sin\theta_2 + \frac{\sin\theta_3}{\cos\theta_3}\cdot r_2\cos\theta_2\right)}_{r_1' = \frac{dr_1}{d\theta_2}}\dot{\theta}_2 = \underbrace{1.556}_{r_1'}\text{ m/rad} \times (-42\text{ rad/s}) = -65.3\text{ m/s}.$$

$\therefore \ \vec{V}_B = 65.3\ m/s \ \leftarrow$.

$\vec{\omega}_3 = 4.26\ rad/s$ cw.

Loop 2 $\rightarrow$ Closure Equation:

$$\vec{R}_{AO_2} + \vec{R}_{CA} + \vec{R}_{D_6C} - \underbrace{\vec{R}_{D_6O_2}}_{-\vec{R}^x_{D_6O_2} + \vec{R}^y_{D_6O_2}} = \vec{0}$$

$\left(\text{Assume} \quad r_6 = R^y_{D_6O_2}, \quad d = R^x_{D_6O_2} \right)$

$$\downarrow$$

$$\left.\begin{aligned} r_2 \cos\theta_2 + a\cos(\theta_3+\beta) + r_5\cos\theta_5 - \underbrace{d}_{3''} &= 0 \\ r_2\sin\theta_2 + a\sin(\theta_3+\beta) + r_5\sin\theta_5 - r_6 &= 0 \end{aligned}\right\}$$

$\downarrow$ differentiate with time $\frac{d}{dt}$. Note: β is a constant.

$$\left.\begin{aligned} -r_2\sin\theta_2\cdot\dot\theta_2 - a\cdot\sin(\theta_3+\beta)\cdot\dot\theta_3 - r_5\sin\theta_5\cdot\dot\theta_5 &= 0 \\ r_2\cos\theta_2\,\dot\theta_2 + a\cos(\theta_3+\beta)\,\dot\theta_3 + r_5\cos\theta_5\,\dot\theta_5 - \dot r_6 &= 0 \end{aligned}\right\} \rightarrow$$

$$\begin{bmatrix} r_5\sin\theta_5 & 0 \\ r_5\cos\theta_5 & -1 \end{bmatrix}\begin{bmatrix} \dot\theta_5 \\ \dot r_6 \end{bmatrix} = \begin{bmatrix} -r_2\sin\theta_2\,\dot\theta_2 - a\sin(\theta_3+\beta)\dot\theta_3 \\ -r_2\cos\theta_2\,\dot\theta_2 - a\cos(\theta_3+\beta)\dot\theta_3 \end{bmatrix} \xrightarrow{\text{invert matrix}}$$

$$\begin{bmatrix} \dot\theta_5 \\ \dot r_6 \end{bmatrix} = \begin{bmatrix} \frac{1}{r_5\sin\theta_5} & 0 \\ \frac{\cos\theta_5}{\sin\theta_5} & -1 \end{bmatrix}\begin{bmatrix} -r_2\sin\theta_2\,\dot\theta_2 - a\sin(\theta_3+\beta)\,\dot\theta_3 \\ -r_2\cos\theta_2\dot\theta_2 - a\sin(\theta_3+\beta)\dot\theta_3 \end{bmatrix}$$

$$\therefore \ \dot\theta_5 = \frac{1}{r_5\sin\theta_5}\cdot(-r_2\sin\theta_2\cdot\dot\theta_2 - a\cdot\sin(\theta_3+\beta)\dot\theta_3) = -7.67\ rad/s.$$

$$\dot r_6 = r_2\cos\theta_2\dot\theta_2 + a\cos(\theta_3+\beta)\cdot\dot\theta_3 + \frac{\cos\theta_5}{\sin\theta_5}(-r_2\sin\theta_2\,\dot\theta_2 - a\sin(\theta_3+\beta)\dot\theta_3)$$

$$= 22.25\ m/s.$$

$\therefore \ \omega_5 = 7.67\ rad/s$. cw, $\quad V_D = 22.25\ m/s, \uparrow$

→ Loop 3 for Point C:

$$\vec{R}_{AO_2} + \vec{R}_{CA} - \vec{R}_{CO_2} = 0.$$

$$\downarrow$$

$$\left.\begin{aligned} r_2 \cos\theta_2 + a\cos(\theta_3+\beta) - x_c = 0 \\ r_2 \sin\theta_2 + a\sin(\theta_3+\beta) - y_c = 0 \end{aligned}\right\}$$

$$\downarrow \frac{d}{dt}$$

$$-r_2 \sin\theta_2 \cdot \dot{\theta}_2 - a\sin(\theta_3+\beta)\cdot\dot{\theta}_3 - \dot{x}_c = 0$$

$$r_2 \cos\theta_2 \cdot \dot{\theta}_2 + a\cos(\theta_3+\beta)\dot{\theta}_3 - \dot{y}_c = 0$$

$$\therefore \quad \dot{x}_c = -r_2 \sin\theta_2 \cdot \dot{\theta}_2 - a\sin(\theta_3+\beta)\cdot\dot{\theta}_3 = -60.89 \text{ m/s}.$$

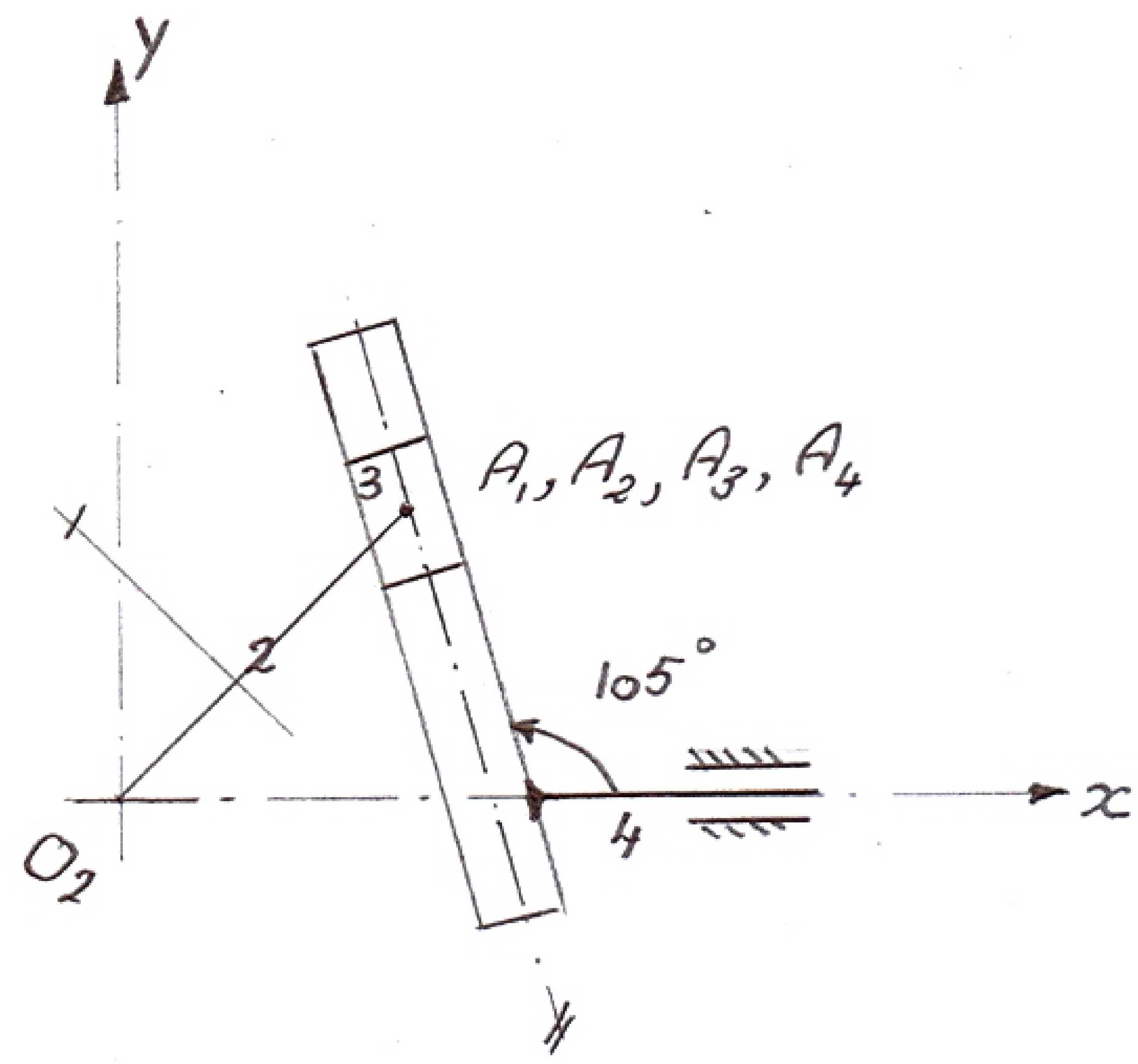
$$\dot{y}_c = r_2 \cos\theta_2 \dot{\theta}_2 + a\cos(\theta_3+\beta)\dot{\theta}_3 = 29.73 \text{ m/s}.$$

$$\therefore \quad V_c = \sqrt{\dot{x}_c^2 + \dot{y}_c^2} = 67.76 \text{ m/s}. \nwarrow$$

Problem (2) : P4.33 page 192

Given: $R_{AO_2} = 0.25\,m$

$\omega_2 = 36$ rad/s (cw)



Position Scale: 1cm : 10 cm

Velocity Scale: 1cm : 1m/s

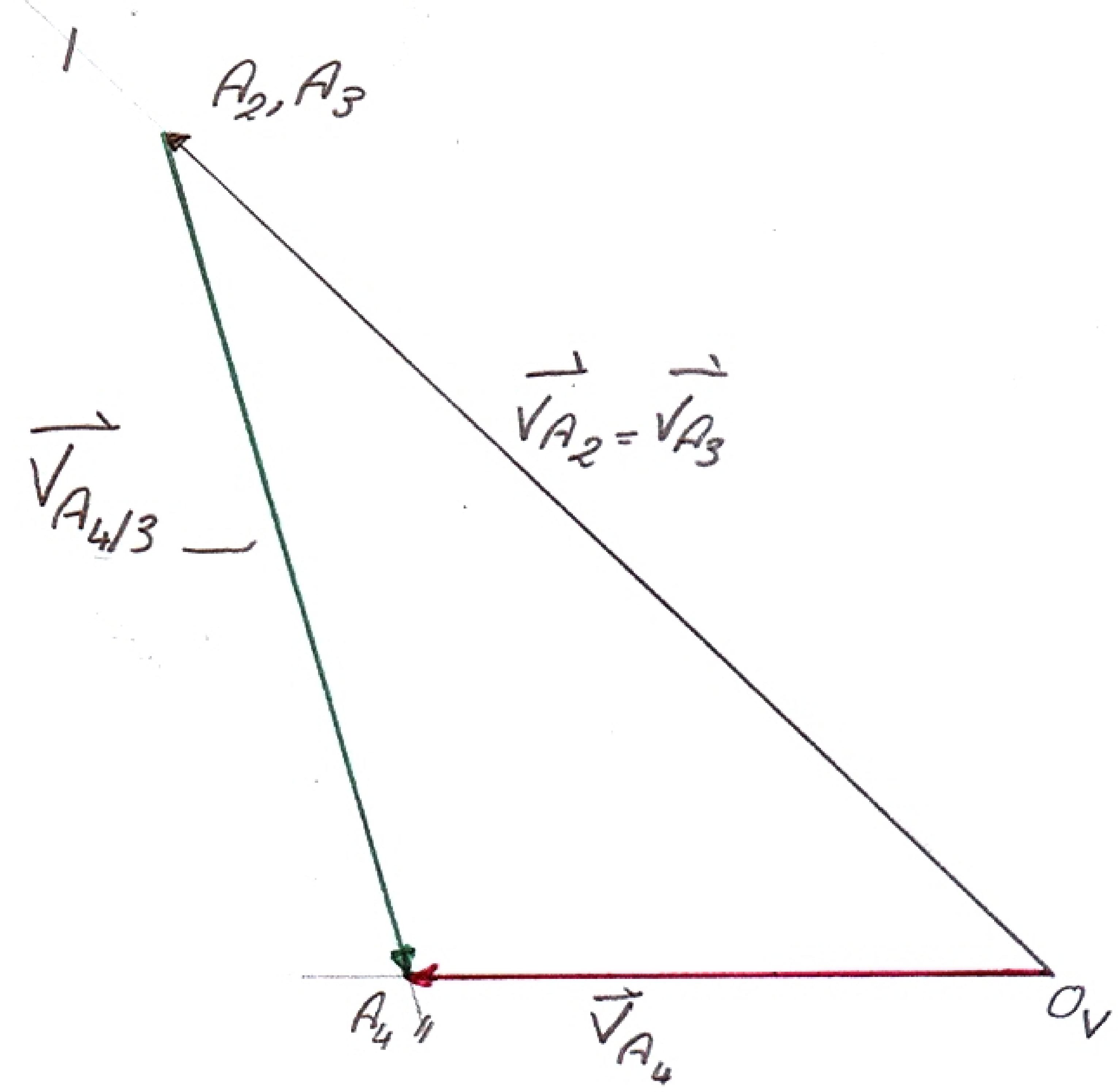
Acceleration Scale: 1cm : 50 m/s²

Velocity Solution:

$$\vec{V}_{A_3} = \vec{V}_{A_2} = \cancel{\vec{V}_{O_2}}^{\,0} + \underbrace{\vec{\omega}_2 \times \vec{R}_{AO_2}}_{\perp \vec{R}_{AO_2}} \qquad (V_{A_2} = \omega_2 R_{AO_2} = 9\,m/s)$$

$$\vec{V}_{A_4} = \vec{V}_{A_3} + \underbrace{\vec{V}_{A_4/3}}_{//\ \text{link } 4} \Rightarrow \boxed{V_{A_4/3} = 6.45\,m/s} \searrow$$

$$\vec{V}_{A_4} = \cancel{\vec{V}_{A_1}}^{\,0} + \underbrace{\vec{V}_{A_4/1}}_{//\ x\text{-dir}} \Rightarrow \boxed{V_{A_4} = 4.7\,m/s} \leftarrow$$

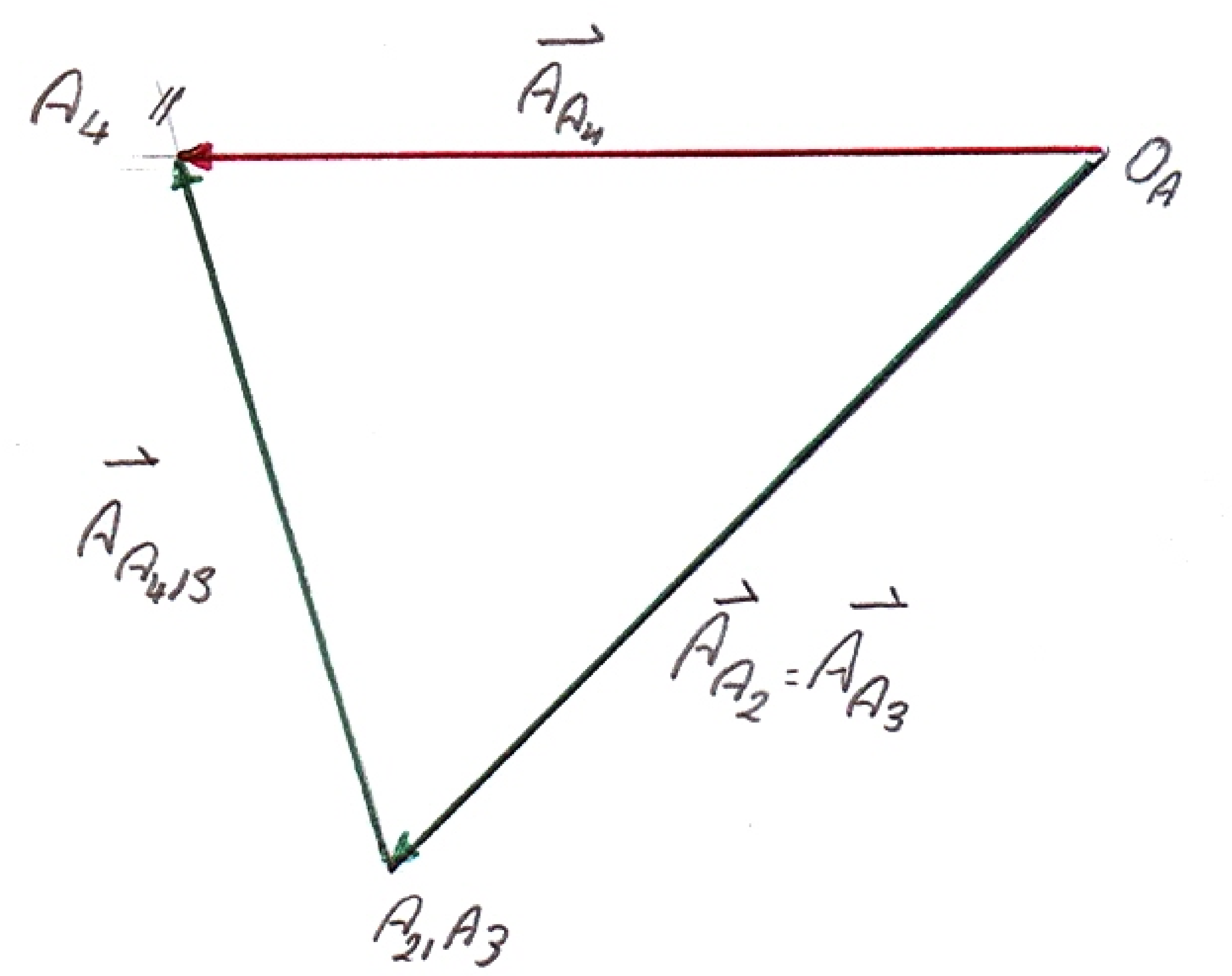


Acceleration Solution:

$$\vec{A}_{A_3} = \vec{A}_{A_2} = \cancel{\vec{A}_{O_2}}^{\,0} + \underbrace{\vec{A}^{\,n}_{AO_2}}_{-\omega_2^2 \vec{R}_{AO_2}} + \underbrace{\vec{A}^{\,t}_{AO_2}}_{\cancel{\vec{\alpha}_2} \times \vec{R}_{AO_2}} \qquad (A^n_{AO_2} = 324\,m/s^2)$$

$$\vec{A}_{A_4} = \vec{A}_{A_3} + \underbrace{\cancel{\vec{A}^{\,c}_{A_4A_3}}^{\,0}}_{(\omega_3 = 0)} + \underbrace{\vec{A}_{A_4/3}}_{\text{Line of action // Link 4}}$$

$$\vec{A}_{A_4} = \vec{A}_{A_1} + \underbrace{\vec{A}_{A_4/1}}_{//\ x\text{-dir}} \Rightarrow \boxed{A_{A_4} = 295\,m/s^2} \leftarrow$$



4.35

1 cm = 2 in

Given

$R_{AB} = 38.47\,in$

$R_{O_4A} = 26\,in$

$R_{BO_4} = 13\,in$

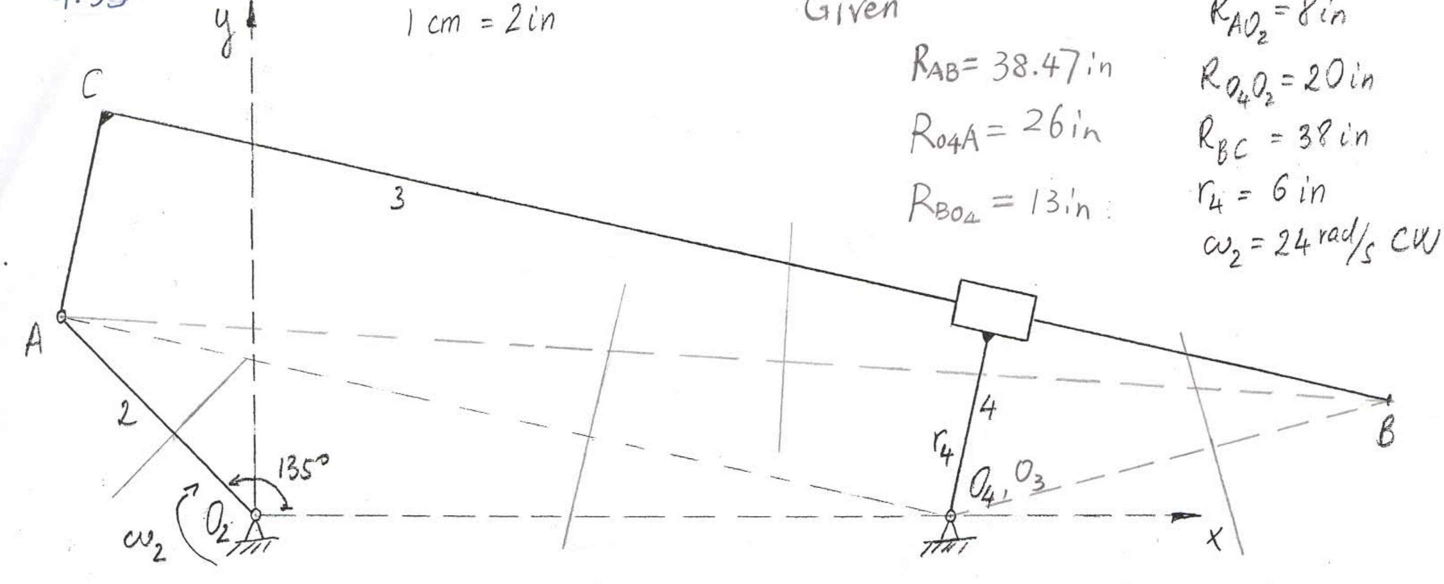
$R_{AO_2} = 8\,in$

$R_{O_4O_2} = 20\,in$

$R_{BC} = 38\,in$

$r_4 = 6\,in$

$\omega_2 = 24\ rad/s$ CW



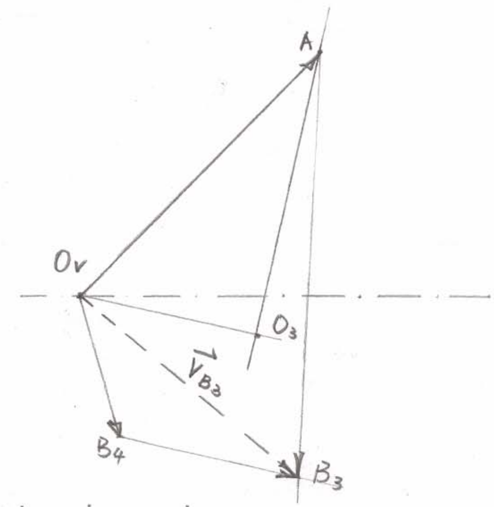
→ Velocity Analysis has been done in previous Assignment. Briefly shown as below:

$\vec{V}_A = 192\,in/s \perp \vec{R}_{AO_2}$, $\vec{V}_{O_3} = 104\,in/s = \vec{V}_{O_3/4}$

$\omega_3 = \omega_4 = 6.3\ rad/s$

$\vec{V}_{B_3} = 158\,in/s$ ↘ (absolute)

Velocity Scale: 1 cm = 40 in/s



→ Acceleration Analysis:

$$\vec{A}_A = \underbrace{\vec{A}_{O_2}}_{0} - \underbrace{\omega_2^2 \vec{R}_{AO_2}}_{\omega_2^2 R_{AO_2} = 4608\,in/s^2} + \underbrace{\vec{\alpha}_2 \times \vec{R}_{AO_2}}_{0}$$

By same fashion in Velocity Analysis, consider Point O_3 attached on body 3, coincide at O_4.

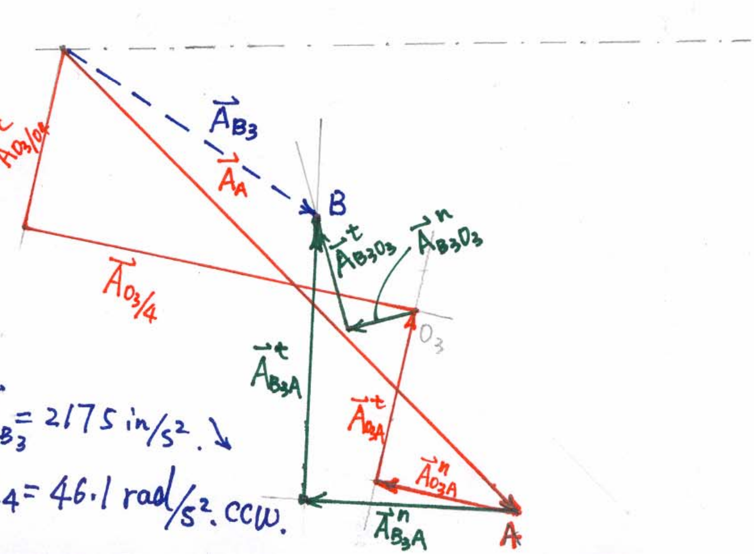
Acceleration scale: 1 cm = 500 in/s²

$$\begin{cases} \vec{A}_{O_3} = \vec{A}_A - \underbrace{\omega_3^2 \vec{R}_{O_3A}}_{A^n_{O_3A} = 6.3^2 \times 26 = 1032\,in/s^2} + \underbrace{\vec{\alpha}_3 \times \vec{R}_{O_3A}}_{A^t_{O_3A}} \\ \vec{A}_{O_3} = \underbrace{\vec{A}_{O_4}}_{0} + \underbrace{\vec{A}^c_{O_3O_4}}_{2\omega_4 V_{O_3/4} = 1310\,in/s^2} + \vec{A}_{O_3/4} \parallel \vec{R}_{BC} \end{cases}$$

Obtain α_3 and α_4: $A^t_{O_3A} = 1200\,in/s^2$ ↑

$\alpha_3 = 46.1\ rad/s^2$ ccw.

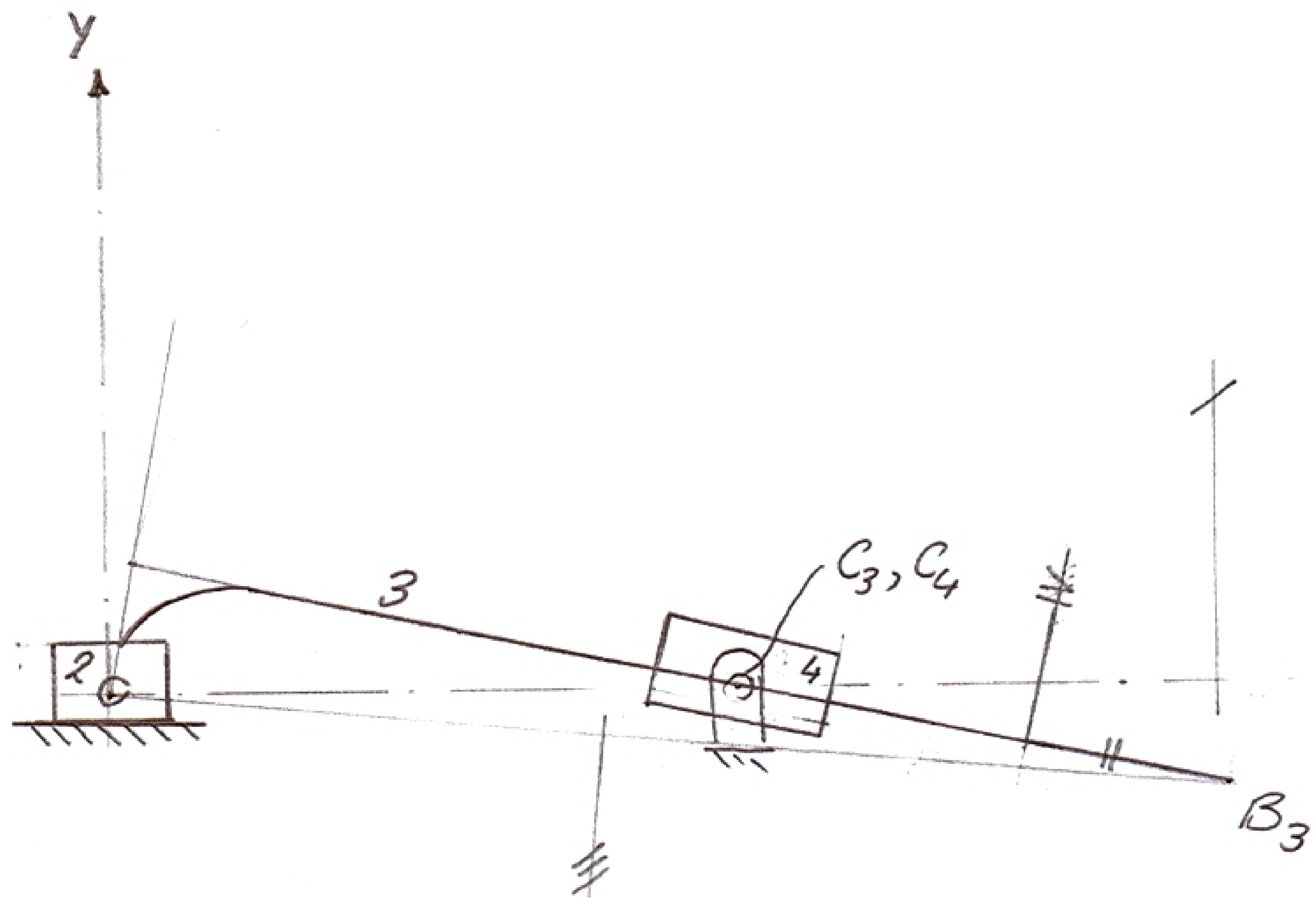
$$\begin{cases} \vec{A}_{B_3} = \vec{A}_A - \underbrace{\omega_3^2 \vec{R}_{B_3A}}_{A^n_{B_3A} = 1527\,in/s^2} + \underbrace{\vec{\alpha}_3 \times \vec{R}_{BA}}_{A^t_{B_3A}} \\ \vec{A}_{B_3} = \vec{A}_{O_3} - \underbrace{\omega_3^2 \vec{R}_{B_3O_3}}_{A^n_{B_3O_3} = 515.97\,in/s^2} + \underbrace{\vec{\alpha}_3 \times \vec{R}_{B_3O_3}}_{A^t_{B_3O_3}} \end{cases}$$



∴ $\vec{A}_{B_3} = 2175\,in/s^2$ ↘

$\alpha_4 = 46.1\ rad/s^2$ ccw.

Problem (4): P4.36 (page 192)



Given: $V_A = 1\ ft/s$ (const.)

Unknowns: $\vec{A}_B$, $\vec{\alpha}_3$, $\vec{\alpha}_4$

Position scale: 1cm : 2in

Velocity scale: 1cm : 2in/s

Acceleration scale: 1cm : 1in/s²

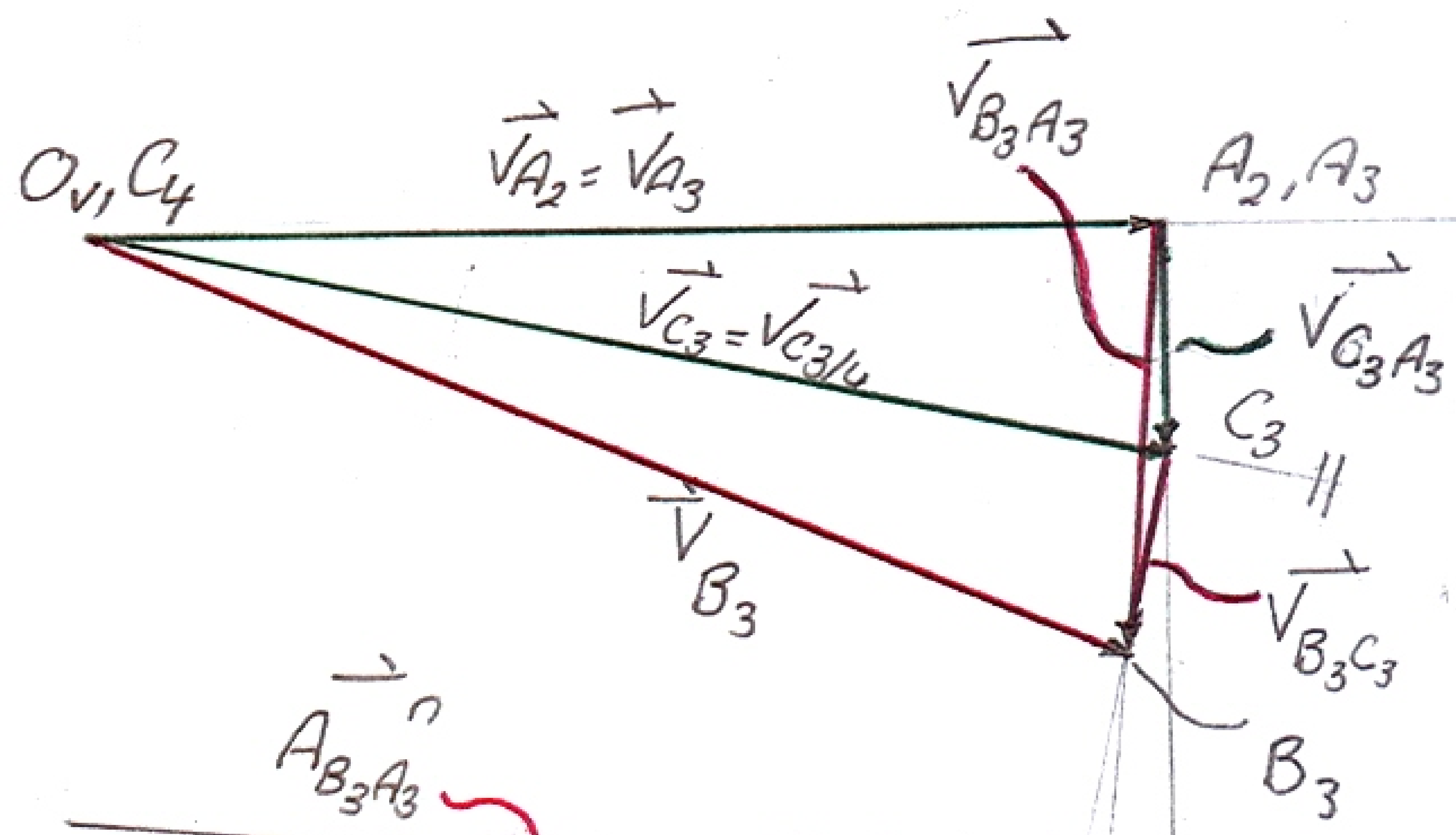
Velocity Solution

$\vec{V}_{A_3} = \vec{V}_{A_2}$ $\qquad (V_{A_3} = V_{A_2} = 12\,in/s)$

$\vec{V}_{C_3} = \vec{V}_{A_3} + \underbrace{\vec{\omega}_3 \times \vec{R}_{CA}}_{\perp R_{CA}}$ $\quad \Rightarrow \omega_3 = \frac{V_{C_3A_3}}{R_{CA}} = 0.31\,rad/s\,(cw)$

$\vec{\omega}_3 = \vec{\omega}_4$

$\vec{V}_{C_3} = \cancel{\vec{V}_{C_4}} + \vec{V}_{C_{3/4}}$ $\quad \Rightarrow V_{C_3} = V_{C_{3/4}} = 12.40\,in/s \searrow$



$\vec{V}_{B_3} = \vec{V}_A + \underbrace{\vec{\omega}_3 \times \vec{R}_{BA}}_{\perp \vec{R}_{BA}}$

$\vec{V}_{B_3} = \vec{V}_C + \underbrace{\vec{\omega}_3 \times \vec{R}_{BC}}_{\perp \vec{R}_{BC}}$ $\quad \Rightarrow V_B = 12.5\,in/s \searrow$

Acceleration Analysis

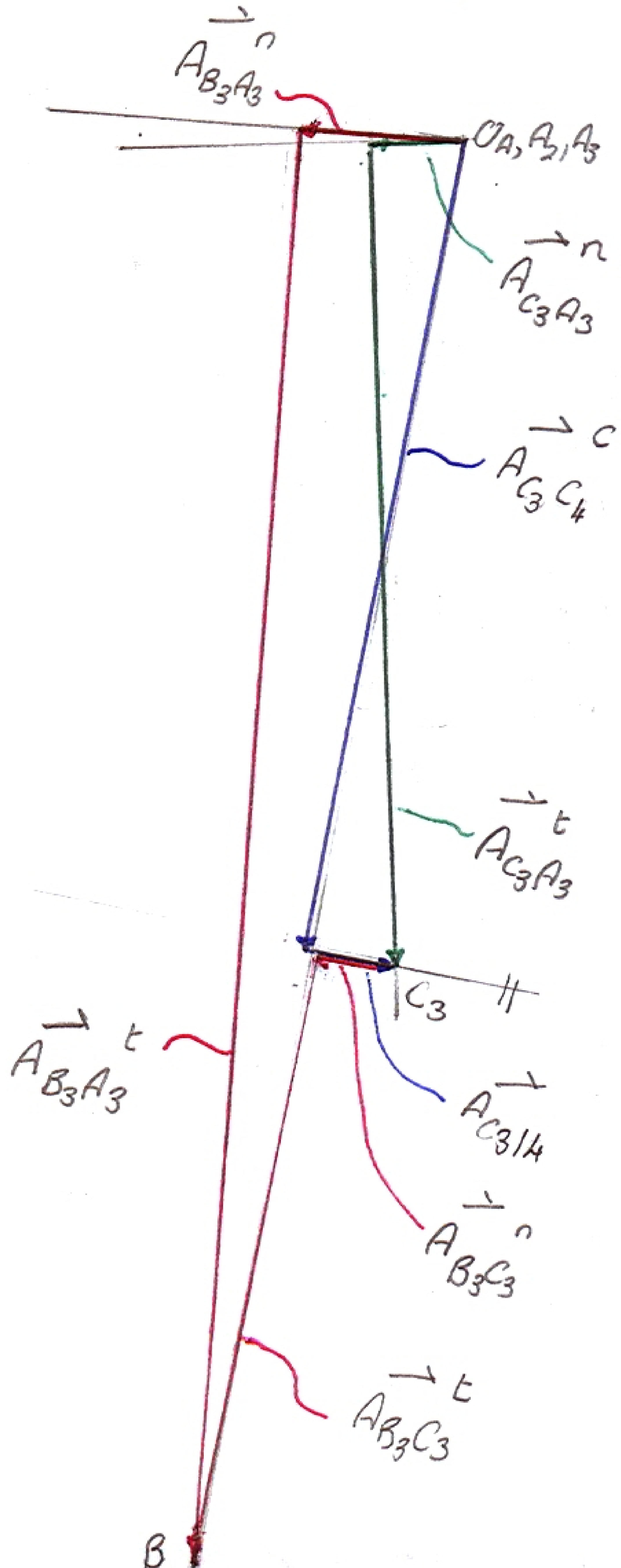
$A_{A_3} = A_{A_2} = 0$

$\vec{A}_{C_3} = \cancel{\vec{A}_{A_3}} - \underbrace{\omega_3^2 \vec{R}_{CA}}_{\vec{A}^n_{C_3A_3}} + \underbrace{\vec{\alpha}_3 \times \vec{R}_{CA}}_{\vec{A}^t_{C_3A_3}}$ $\quad (A^n_{C_3A_3} = 0.865\,in/s^2)$

$\vec{A}_{C_3} = \cancel{\vec{A}_{C_4}} + \underbrace{\vec{A}^c_{C_3C_4}}_{2\vec{\omega}_4 \times \vec{V}_{C_{3/4}}} + \vec{A}_{C_{3/4}}$ → line of action // link 3

$(A^c_{C_3C_4} = 7.6855\,in/s^2)$

$\alpha_3 = \alpha_4 = \frac{A^t_{C_3A_3}}{R_{CA}} = 0.86\,\frac{rad}{s^2}\,(cw)$



Problem (4): continued

$$\begin{cases} \vec{A}_{B_3} = \vec{A}_{A_3} - \underbrace{\omega_3^2 \vec{R}_{BA}}_{\vec{A}^n_{B_3A_3}} + \underbrace{\vec{\alpha}_3 \times \vec{R}_{BA}}_{\vec{A}^t_{B_3A_3}} \quad (A^n_{B_3A_3} = 1.56\ in/s^2) \\ \vec{A}_{B_3} = \vec{A}_{C_3} - \underbrace{\omega_3^2 \vec{R}_{BC}}_{\vec{A}^n_{B_3C_3}} + \underbrace{\vec{\alpha}_3 \times \vec{R}_{BC}}_{\vec{A}^t_{B_3C_3}} \quad (A^n_{B_3C_3} = 0.694\ in/s^2) \end{cases}$$

$\Rightarrow A_B = 13.6\ in/s^2$ ✓