These are solutions to Problem Set 3

3.14

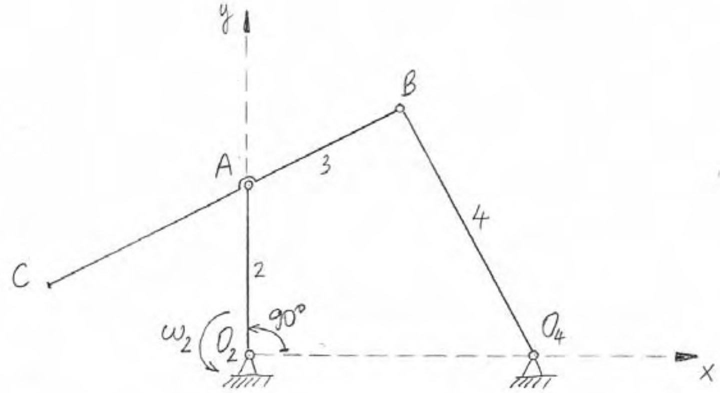
$1\,cm = 2\,in$

$R_{AO_2} = R_{BA} = 6\,in$

$R_{O_4O_2} = R_{BO_4} = 10\,in$

$R_{CA} = 8\,in$

$\omega_2 = 60\,rad/s$ CCW



$\vec{V}_A = \vec{\omega}_2 \times \vec{R}_{AO_2} \rightarrow V_A = 360\,in/s \leftarrow$

$1\,cm = 50\,in/s$

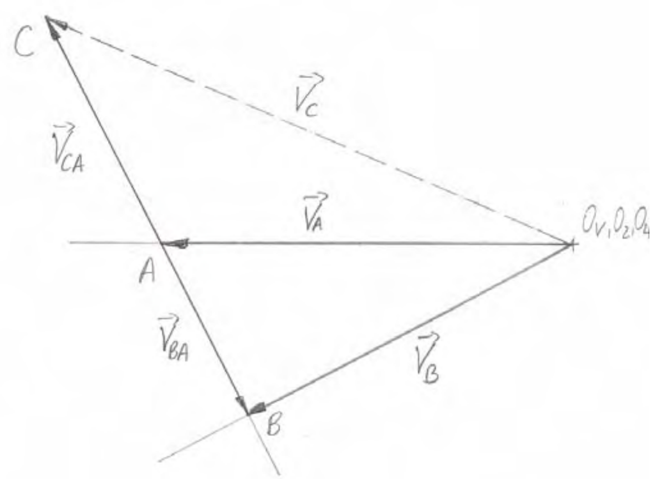
$$\left.\begin{aligned}\vec{V}_B &= \vec{V}_A + \vec{\omega}_3 \times \vec{R}_{BA} \\ \vec{V}_B &= \cancel{\vec{V}_{O_4}} + \vec{\omega}_4 \times \vec{R}_{BO_4}\end{aligned}\right\} \quad \begin{aligned}\vec{V}_{BA} &= \vec{\omega}_3 \times \vec{R}_{BA} \perp \vec{R}_{BA} \\ \vec{V}_{BO_4} &= \vec{\omega}_4 \times \vec{R}_{BO_4} \perp R_{BO_4}\end{aligned}$$

Based on similarity:

$$\frac{R_{BA}}{R_{CA}} = \frac{V_{BA}}{V_{CA}} = \frac{6}{8}$$

$V_{BA} = 170\,in/s$

$$V_{CA} = \frac{8 * V_{BA}}{6} = 226.7\,in/s$$

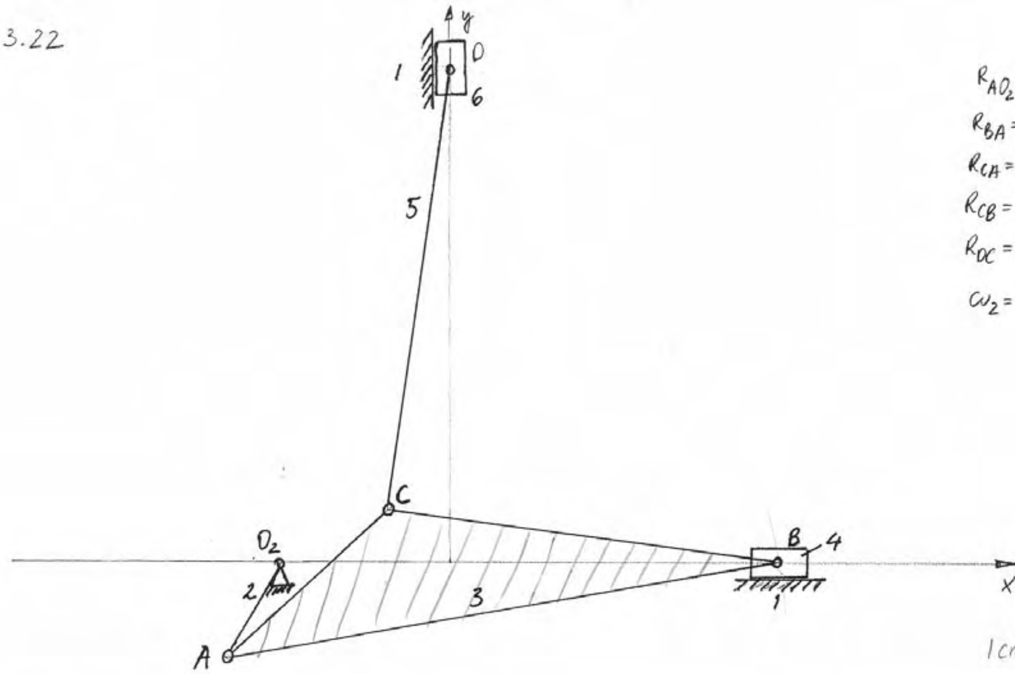


$V_C = 525\,in/s$ ↖

$$\omega_3 = \frac{V_{BA}}{R_{BA}} = \frac{170}{6} = 28.3\,\frac{rad}{s}\ CW$$

$$\omega_4 = \frac{V_{BO_4}}{R_{BO_4}} = \frac{V_B}{R_{BO_4}} = \frac{322.5}{10} = 32.25\,\frac{rad}{s}\ CCW$$

3.22



$R_{AO_2} = 2$

$R_{BA} = 10$

$R_{CA} = 4$

$R_{CB} = 7$

$R_{DC} = 8$

$\omega_2 = 42\ \text{rad/s CW}$

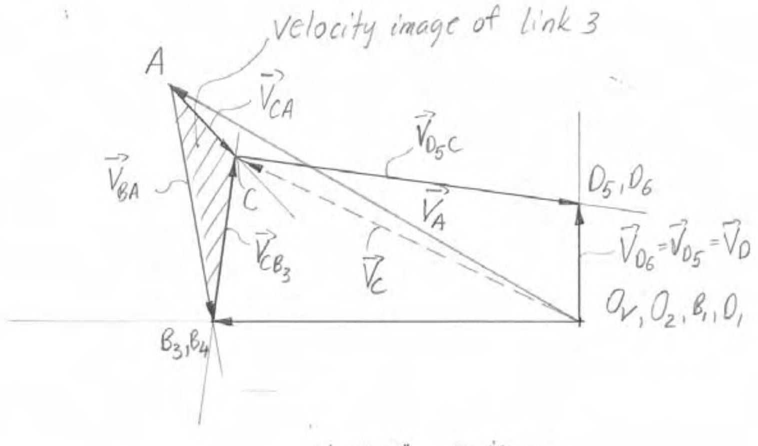
1 cm = 10 in/s

$\vec{V}_A = \vec{\omega}_2 \times \vec{R}_{AO_2} \quad \perp \vec{R}_{AO_2} \quad V_A = 84\ \text{in/s}$

$$\left.\begin{aligned}
\vec{V}_B &= \vec{V}_A + \underbrace{\vec{\omega}_3 \times \vec{R}_{BA}}_{\vec{V}_{BA} \perp \vec{R}_{BA}} \\
\vec{V}_B &= \vec{V}_{B_3} = \vec{V}_{B_4} \\
\vec{V}_{B_4} &= \cancel{\vec{V}_{B_1}} + \vec{V}_{B_4/1}
\end{aligned}\right\}$$

$$\left.\begin{aligned}
\vec{V}_C &= \vec{V}_A + \underbrace{\vec{\omega}_3 \times \vec{R}_{CA}}_{\vec{V}_{CA} \perp \vec{R}_{CA}} \\
\vec{V}_C &= \vec{V}_{B_3} + \underbrace{\vec{\omega}_3 \times \vec{R}_{CB}}_{\vec{V}_{CB_3} \perp \vec{R}_{CB}}
\end{aligned}\right\}$$

$$\left.\begin{aligned}
\vec{V}_{D_6} &= \vec{V}_{D_5} = \vec{V}_C + \underbrace{\vec{\omega}_5 \times \vec{R}_{DC}}_{\vec{V}_{D_5C} \perp \vec{R}_{DC}} = \vec{V}_D \\
\vec{V}_{D_6} &= \cancel{\vec{V}_{D_1}} + \vec{V}_{D_6/1}
\end{aligned}\right\}$$



$V_B = V_{B_3} = V_{B_4} = 65\ \text{in/s}$ ←

$V_C = 68.5\ \text{in/s}$ ↖

$V_D = V_{D_5} = V_{D_6} = 21\ \text{in/s}$ ↑

$\omega_5 = \dfrac{V_{D_5C}}{R_{DC}} = \dfrac{61}{8} = 7.63\ \text{rad/s CW}$

3.24.

$1\text{ cm} = 2\text{ in}$

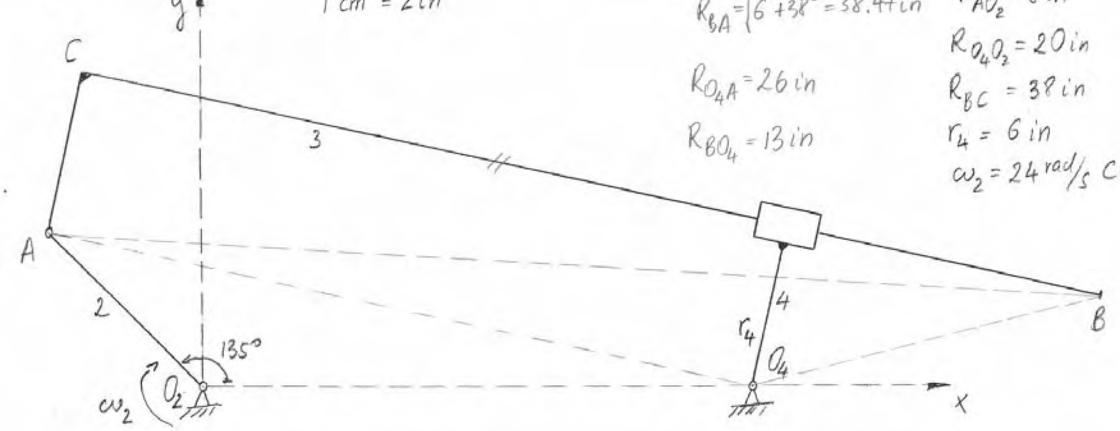
$R_{BA} = \sqrt{6^2+38^2} = 38.47\text{ in}$ $R_{AO_2} = 8\text{ in}$

$R_{O_4A} = 26\text{ in}$ $R_{O_4O_2} = 20\text{ in}$

$R_{BO_4} = 13\text{ in}$ $R_{BC} = 38\text{ in}$

$r_4 = 6\text{ in}$

$\omega_2 = 24\text{ rad/s CW}$



$\vec{V}_A = \vec{\omega}_2 \times \vec{R}_{AO_2} \rightarrow V_A = 192\text{ in/s} \perp \vec{R}_{AO_2}$

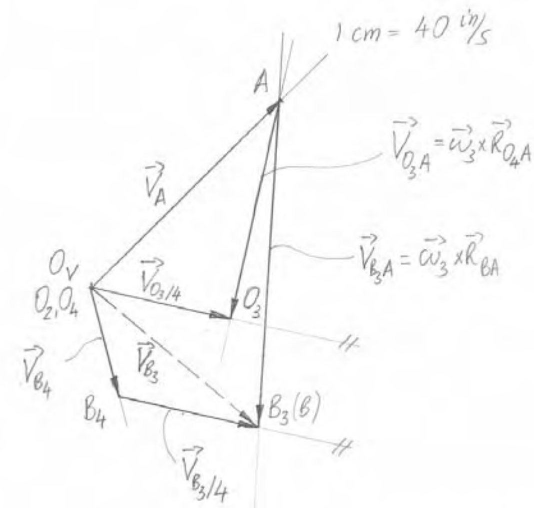
Consider a pair of points: O_3 and O_4 instantaneously coincident at O_4

O_3: attached to link 3

O_4: attached to link 4

$$\left.\begin{aligned} &\vec{V}_{O_4} = \vec{0} \\ &\vec{V}_{O_3} = \vec{V}_A + \vec{\omega}_3 \times \vec{R}_{O_4A} \\ &\vec{V}_{O_3} = \cancel{\vec{V}_{O_4}} + \vec{V}_{O_3/4} \end{aligned}\right\} \qquad \vec{\omega}_3 \times \vec{R}_{O_4A} \rightarrow \vec{V}_{O_3A} \perp \vec{R}_{O_4A}$$

$$\omega_3 = \omega_4 = \frac{V_{O_3A}}{R_{O_4A}} = \frac{164}{26} = 6.3\,\frac{\text{rad}}{\text{s}}\text{ CW}$$



Consider another pair of points: $B_3(B)$ and B_4 instantaneously coincident at B

$B_3(B)$: attached to link 3

B_4 : attached to link 4

$\vec{V}_{B_4} = \vec{\omega}_4 \times \vec{R}_{BO_4} \rightarrow V_{B_4} = 13 \times 6.3 = 81.9\text{ in/s} \perp \vec{R}_{BO_4}$ ↘

$$\left.\begin{aligned} &\vec{V}_{B_3} = \vec{V}_A + \vec{\omega}_3 \times \vec{R}_{BA} \\ &\vec{V}_{B_3} = \vec{V}_{B_4} + \vec{V}_{B_3/4} \end{aligned}\right\} \qquad \vec{\omega}_3 \times \vec{R}_{BA} = \vec{V}_{B_3A} \perp \vec{R}_{BA}$$

$V_{B_3} = 158\text{ in/s}$ ↘ (absolute velocity)

$V_{B_3/4} = 104\text{ in/s}$ ← (apparent velocity to link 4)