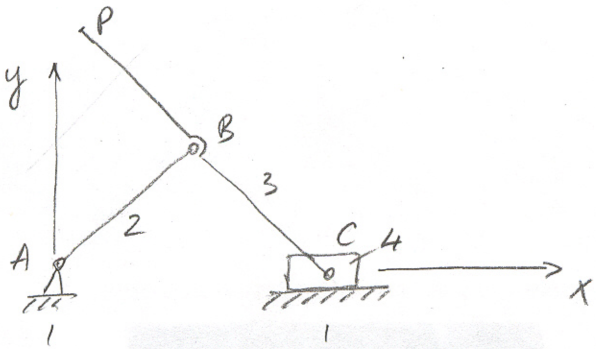
These are solutions to Problem Set 2

Problem 1:



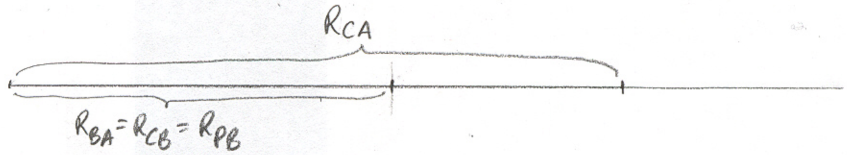
$R_{BA} = R_{CB} = R_{PB} = 25\ mm$

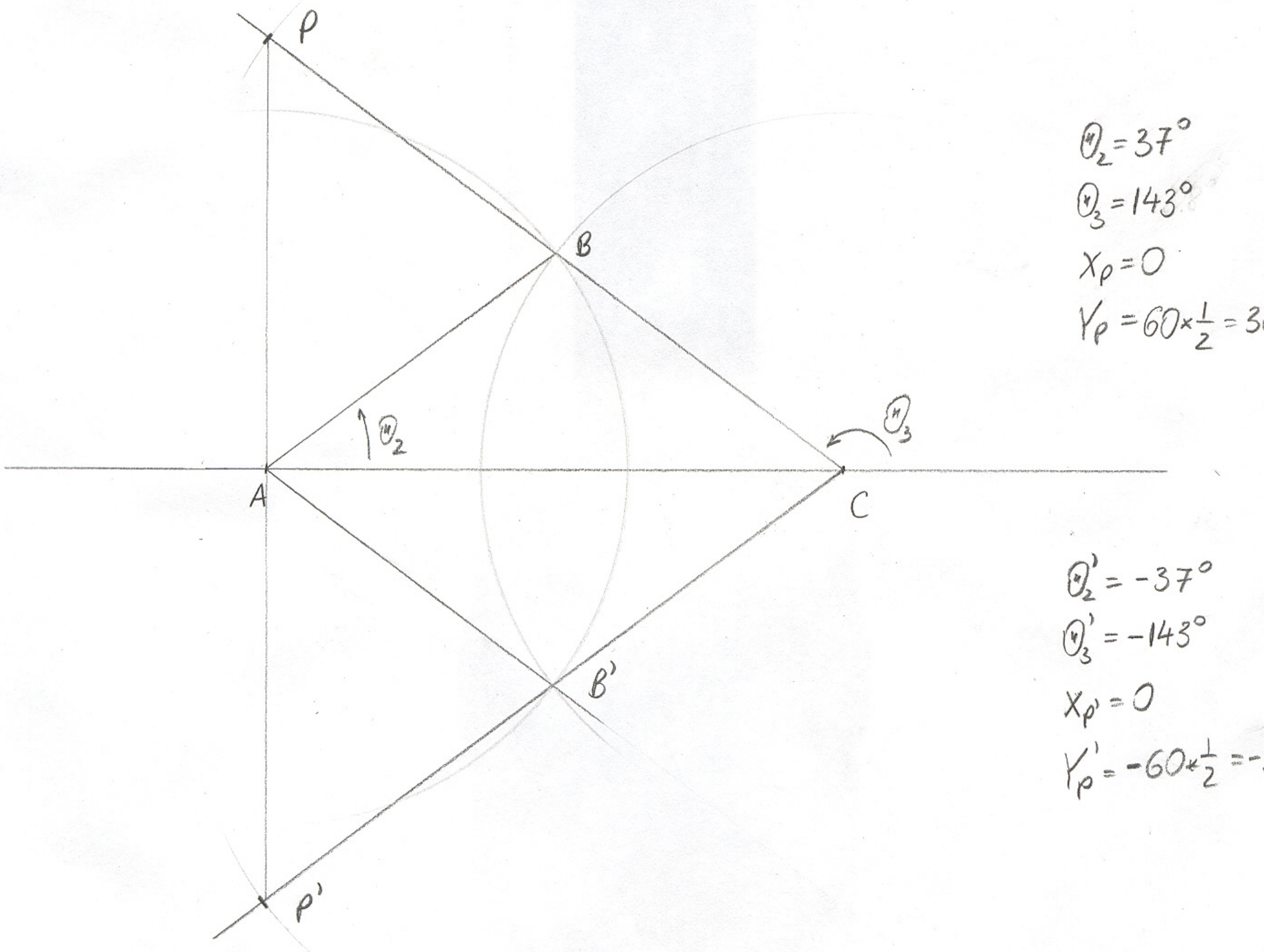
$R_{CA} = 40\ mm$ /input/

Loop closure equation: $\vec{R}_{CA} + \vec{R}_{BC} = \vec{R}_{BA}$ (or $\vec{R}_C + \vec{R}_{BC} = \vec{R}_B$)

(a) Graphical solution:

scale: 2cm/1cm





$\theta_2 = 37°$

$\theta_3 = 143°$

$X_P = 0$

$Y_P = 60 \times \frac{1}{2} = 30\ mm$

$\theta_2' = -37°$

$\theta_3' = -143°$

$X_{P'} = 0$

$Y_P' = -60 \times \frac{1}{2} = -30\ mm$

(b) Analytical solution using complex numbers

$$\vec{R}_{CA} + \vec{R}_{BC} = \vec{R}_{BA}$$

$$R_{CA}e^{j\theta_1} + R_{BC}e^{j\theta_3} = R_{BA}e^{j\theta_2} \qquad / e^{j\theta_1} \qquad (\text{but } \theta_1 = 0)$$

$$R_{CA} + R_{BC}e^{j\theta_3} = R_{BA}e^{j\theta_2} \longrightarrow \left.\begin{array}{l} R_{CA} + R_{BC}\cos\theta_3 = R_{BA}\cos\theta_2 \\ R_{BC}\sin\theta_3 = R_{BA}\sin\theta_2 \end{array}\right\}$$

$$R_{CA}^2 + R_{BC}^2 + 2R_{CA}R_{BC}\cos\theta_3 = R_{BA}^2 \longrightarrow \theta_3 = \cos^{-1}\left(\frac{R_{BA}^2 - R_{CA}^2 - R_{BC}^2}{2R_{CA}R_{BC}}\right) = \begin{cases} \underline{143.1^\circ = \theta_3} \\ \underline{-143.1^\circ = \theta_3'} \end{cases}$$

$$R_{CA} - R_{BA}\cos\theta_2 = -R_{BC}\cos\theta_3$$

$$R_{BA}\sin\theta_2 = R_{BC}\sin\theta_3$$

$$R_{CA}^2 + R_{BA}^2 - 2R_{CA}R_{BA}\cos\theta_2 = R_{BC}^2 \longrightarrow \theta_2 = \cos^{-1}\left(\frac{R_{CA}^2 + R_{BA}^2 - R_{BC}^2}{2R_{CA}R_{BA}}\right) = \begin{cases} \underline{36.9^\circ = \theta_2} \\ \underline{-36.9^\circ = \theta_2'} \end{cases}$$

$$\vec{R}_{PA} = \vec{R}_{BA} + \vec{R}_{PB} \longrightarrow R_{PA}e^{j\theta_P} = R_{BA}e^{j\theta_2} + R_{PB}e^{j\theta_3} \longrightarrow \left.\begin{array}{l} R_{PA}\cos\theta_P = R_{BA}\cos\theta_2 + R_{PB}\cos\theta_3 \\ R_{PA}\sin\theta_P = R_{BA}\sin\theta_2 + R_{PB}\sin\theta_3 \end{array}\right\}$$

$$R_{PA}^2 = R_{BA}^2 + R_{PB}^2 + 2R_{BA}R_{PB}\cos(\theta_2 - \theta_3) \longrightarrow \underline{R_{PA} = 30\text{ mm}}$$

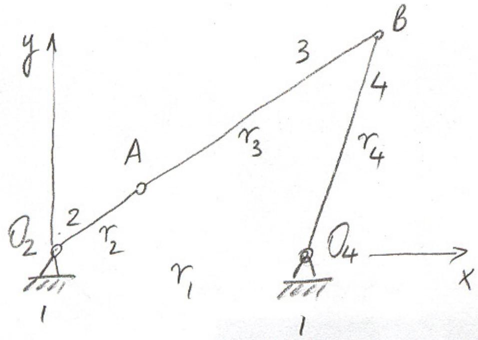
$$\theta_P = \tan^{-1}\left(\frac{R_{BA}\sin\theta_2 + R_{PB}\sin\theta_3}{R_{BA}\cos\theta_2 + R_{PB}\cos\theta_3}\right) \longrightarrow \theta_P = \begin{cases} \underline{90^\circ = \theta_P} \\ \underline{-90^\circ = \theta_P'} \end{cases} \longrightarrow$$

$$\longrightarrow \underline{X_P = 0 \quad Y_P = 30\text{mm}}$$

$$\underline{X_P' = 0 \quad Y_P' = -30\text{mm}}$$

Note: graphical solution is accurate

Problem 2



$r_1 = 400mm$

$r_2 = 200\ mm$

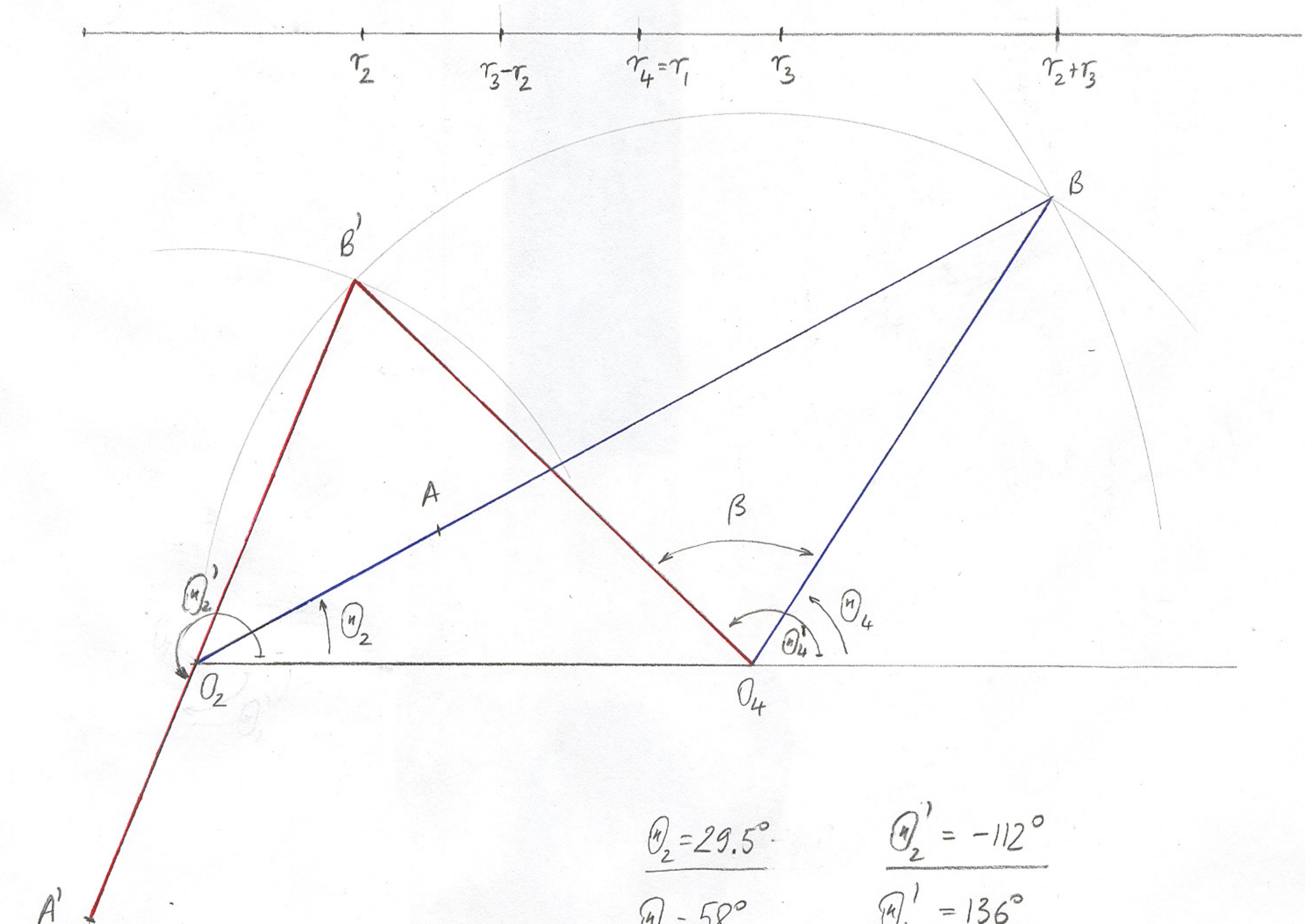
$r_3 = 500\ mm$

$r_4 = 400\ mm$

input information: links 2 and 3 form a straight line
→ angle between them: 180° or 0°

Loop closure equation: $\overset{\checkmark o}{\vec{R}_{BO_2}} = \overset{\checkmark\checkmark}{\vec{R}_{O_4O_2}} + \overset{\checkmark o}{\vec{R}_{BO_4}}$

(a) Graphical solution: scale: 1cm/5cm



$\underline{\Theta_2 = 29.5°}$ $\underline{\Theta_2' = -112°}$

$\underline{\Theta_4 = 58°}$ $\underline{\Theta_4' = 136°}$

Total rocking angle of link 4, $\underline{\beta = 78°}$

(b) Analytical solution using complex numbers

$$\vec{R}_B = \vec{R}_{O_4} + \vec{R}_{BO_4}$$

$$R_B e^{i\theta_B} = R_{O_4} e^{i\theta_1} + R_{BO_4} e^{i\theta_4}$$

two possibilities: 1. $R_B = r_2 + r_3$, $\theta_B = \theta_2$
2. $R_B = r_3 - r_2$ $\theta_B = \theta_2 + 180°$

$$\downarrow \; / e^{i\theta_1} \qquad \theta_1 = 0$$

$$\left.\begin{aligned} R_B \cos\theta_B &= R_{O_4} + R_{BO_4} \cos\theta_4 \\ R_B \sin\theta_B &= R_{BO_4} \sin\theta_4 \end{aligned}\right\}$$

$$\downarrow$$

$$R_B^2 = R_{O_4}^2 + R_{BO_4}^2 + 2 R_{O_4} R_{BO_4} \cos\theta_4 \longrightarrow \underline{\theta_4} = \cos^{-1}\left(\frac{R_B^2 - R_{O_4}^2 - R_{BO_4}^2}{2 R_{O_4} R_{BO_4}}\right)$$

rearranging the two equations:

$$R_{BO_4}^2 = R_{O_4}^2 + R_B^2 - 2 R_{O_4} R_B \cos\theta_B \longrightarrow \underline{\theta_B} = \cos^{-1}\left(\frac{R_{O_4}^2 + R_B^2 - R_{BO_4}^2}{2 R_{O_4} R_B}\right)$$

1. $R_B = r_2 + r_3 \longrightarrow \underline{\theta_2 = 28.95°}$, $\underline{\theta_4 = 57.9°}$

2. $R_B = r_3 - r_2 \longrightarrow \theta_B' = 67.97°$ $\underline{\theta_4' = 135.95°}$

$$\hookrightarrow \underline{\theta_2'} = 67.97° - 180° \cong \underline{-112°}$$

The total rocking angle of link 4: $\underline{\beta = \theta_4' - \theta_4 = 78.04°}$

Note: the graphical solution also gave relatively accurate resuts.

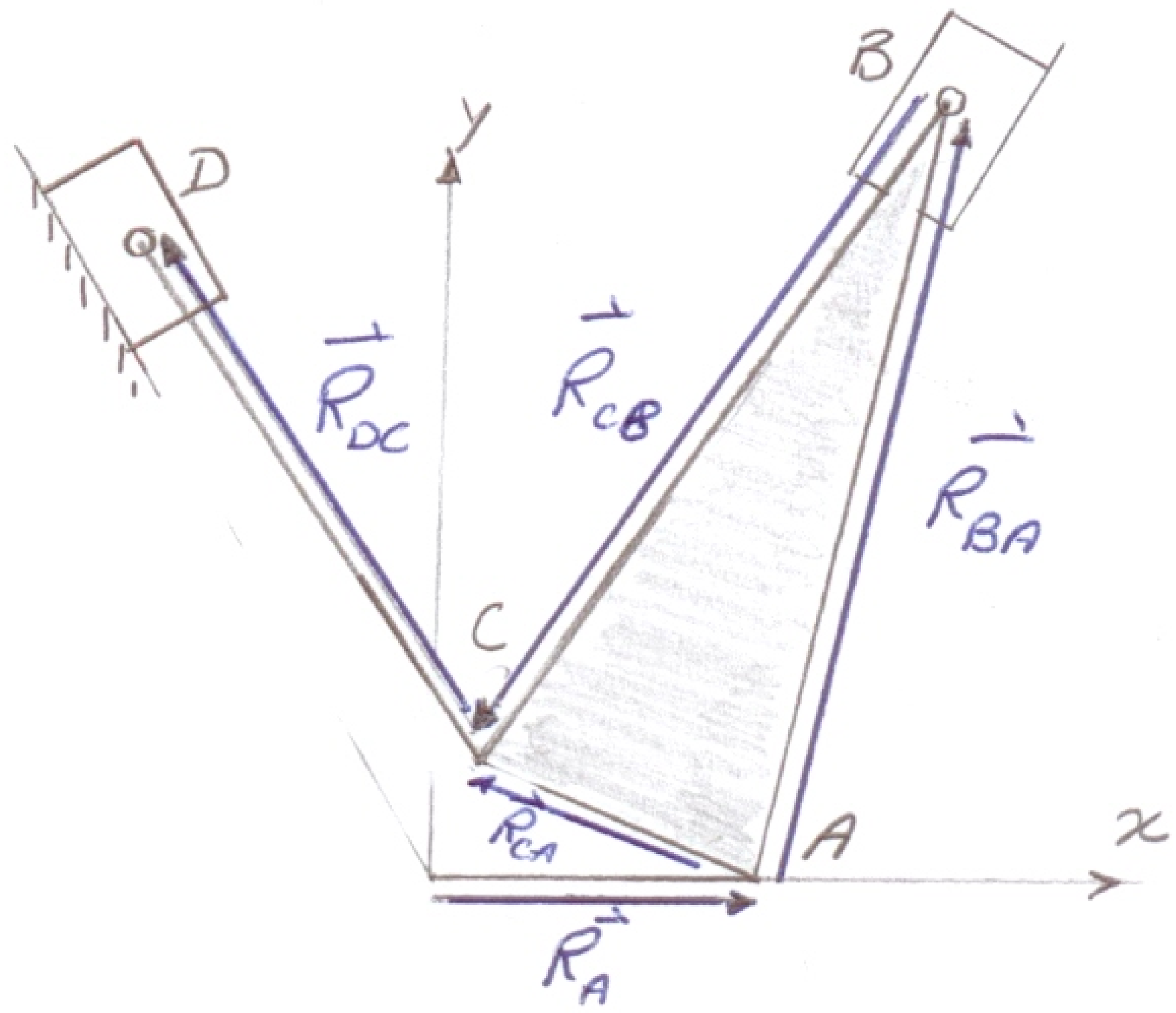
Problem 3 (Figure 3.23 from the book J.J. Wicker , et. al.)

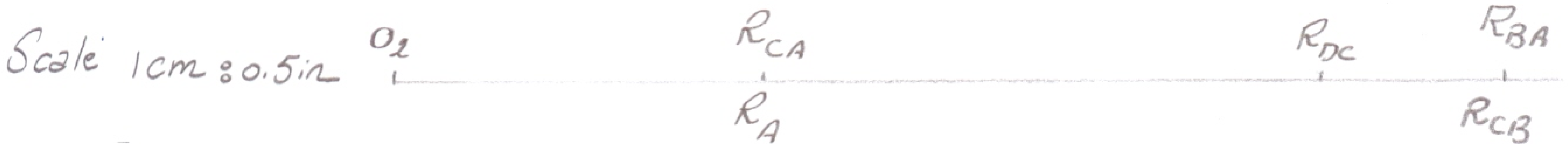
Given: $R_{AO_2} = 2\,in$

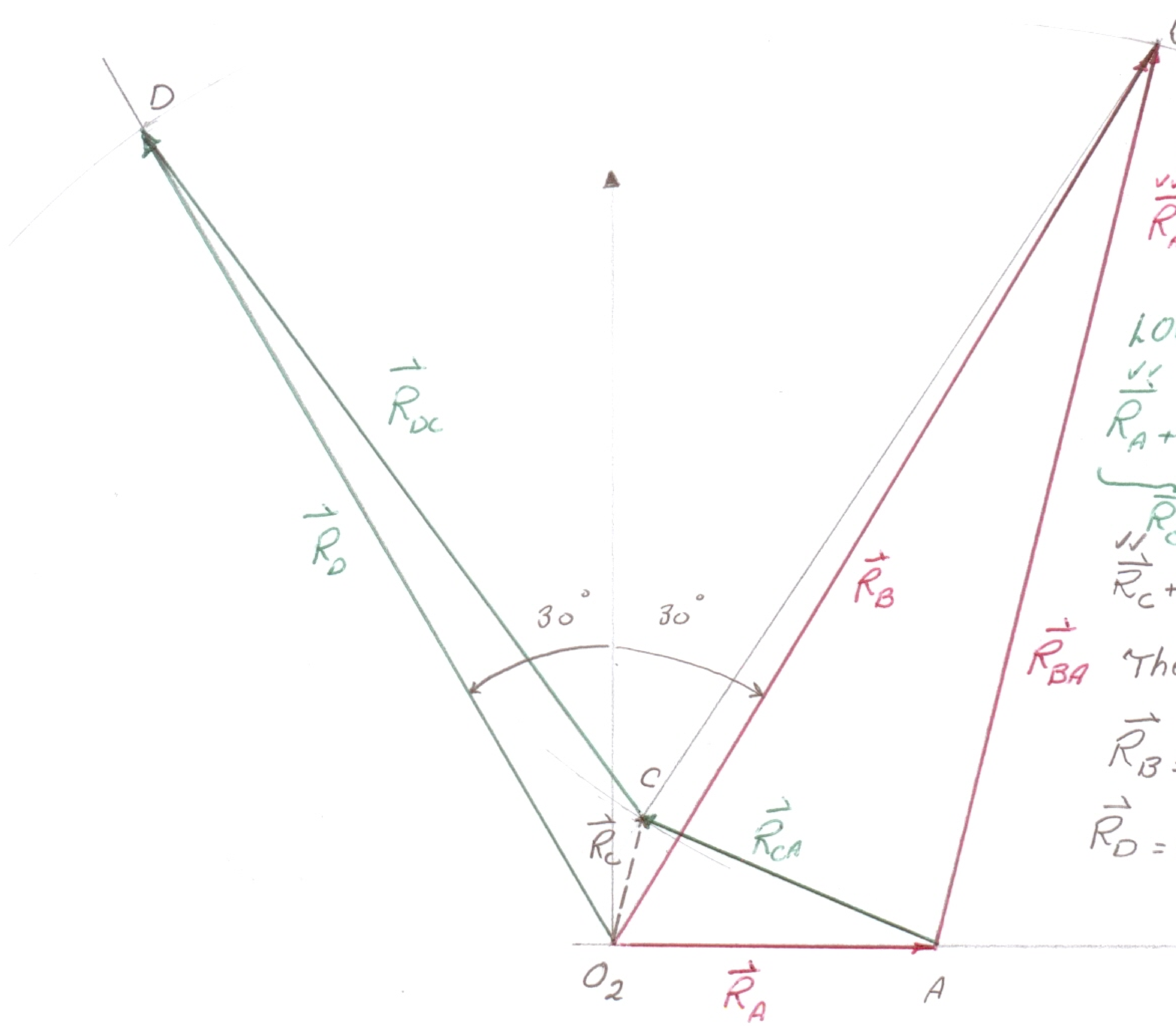
$R_{BA} = R_{CB} = 6\,in$

$R_{CA} = 2\,in$

$R_{DC} = 5\,in$







LOOP1:

$$\overset{\checkmark\checkmark}{\vec{R}_A} + \overset{\checkmark o}{\vec{R}_{BA}} = \overset{o\checkmark}{\vec{R}_B}$$

LOOP2:

$$\underbrace{\overset{\checkmark\checkmark}{\vec{R}_A} + \overset{\checkmark\checkmark}{\vec{R}_{CA}}}_{\vec{R}_C} + \overset{\checkmark o}{\vec{R}_{DC}} = \overset{o\checkmark}{\vec{R}_D}$$

$$\overset{\checkmark\checkmark}{\vec{R}_C} + \overset{\checkmark o}{\vec{R}_{DC}} = \overset{o\checkmark}{\vec{R}_D}$$

Therefore:

$\vec{R}_B = 6.75\,in \angle 60°$

$\vec{R}_D = 5.55\,in \angle 120°$