MECH 576
Geometry in Mechanics

November 10, 2009

Two Degree of Freedom Planar Parallel Manipulators and the Burmester Problem

Abstract

Three 2dof parallel manipulator designs are introduced. The intent is to reveal possible compromise solutions to the classical planar five-pose Burmester problem if the conventional four-bar solution engenders branching, *i.e.*, the two coupled serial dyads are configured such that not all of the poses specified can be attained without disassembling the closed chain. Notions discussed herein may be useful as well in achieving greater precision in fully mobile parallel manipulators, *i.e.*, the more usual 3dof planar variety, by detent-locking one or other actuator for certain specific tasks.

keywords design compromise, accuracy, planar parallel robot

1 Introduction

For the ASME-DETC2002 conference in Montréal Professor M. McCarthy issued a challenge, *i.e.*, to design a planar mechanism to guide -as closely as possible- a rigid body through a sequence of eleven specified poses; solutions to be presented in a conference workshop. This had two effects.

- Renewed interest in the Burmester problem [1] resulted in a flurry of publications aimed at improving the methodology to design planar four-bars to precisely move the coupler to five poses [2, 3], to identify the need for special architecture [4], *e.g.*, a slider-crank or trammel configuration and to determine whether all five are in the same assembly mode or *branch* [5].
- Taken together with these contributions Professor P. Larochelle set a "time-bomb" [6] by inferring two questions that germinated the ideas expressed in this article. His solution was a doubly actuated five-link closed $\underline{R}RR\underline{P}R$ chain working model.
- The questions are
  1. Are there any general advantages to be gained with two degree of freedom (2dof) parallel manipulators?
  2. How may these be implemented?
- Partial answers are
  1. They invoke the usual advantages, far too often stated in comparing parallel *-vs-* serial mechanisms, while reducing architectural complexity of a complete 3dof implementation.

2. When used to augment a 2dof serial dyad, specifically chosen to implement a particular Burmester problem, virtually all conceivable two- and three-legged designs will overcome failure due to branching if it is encountered in a simple four-bar chain solution.

2 A Branching Four-Bar

We begin by introducing the rocker-crank four-bar mechanism, shown in Fig. 1, wherein one of the revolute joints on the coupler may move on two finite circular arcs designated by limits MM and $M'M'$, respectively. Thus it is easy to pick five poses, of the rigid body represented by the coupler link, such that three occur with a coupler R-joint centre in the arc MM while the remaining two happen with this coupler joint between the limits labeled $M'M'$. The 16 poses, from which five are to be chosen, shown in Fig. 1 are generated by placing the crank-pin so as to assume the four corresponding rocker arc limits and the four with the pin conveniently at 3, 6, 9 and 12 o'clock. Finding the limits MM and $M'M'$, after configuring a tentative four-bar candidate, so as to ensure that a fairly nice "straw-man" problem example will be formulated, is a simple exercise that reveals some interesting twists. It is outlined in **VI. Appendix**.

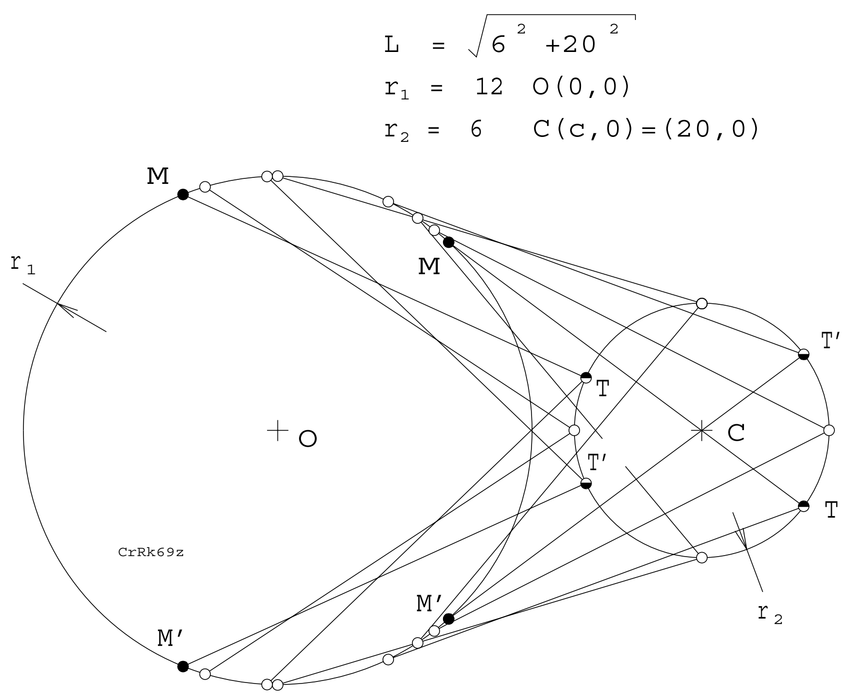

Figure 1: Rocker-Crank Mechanism

The "interesting twist" comes in answer to the question, "What is the underlying structure of a problem described by the intersection of these four (special) quadrics?" To see this we invoke a simple nonlinear mapping, called *Zyklographie* [7], of circles in the plane to points in a three dimensional space where two Cartesian coordinates define circle centres and the third specifies radii. This mapping, applied to the example problem, reveals the intersection of vertical axis cylinder and cone of revolution. The cylinder represents all circles centred on the circumference of the given circle (O, r_1) while the cone of apex angle $\pi/2$ contains all the point-mapped circles tangent to the given circle (C, r_2). Finally, all circles of radius L become points in the horizontal planes $r = L \pm r_2$. The apparently higher order problem degenerates to finding the intersections of two circular sections on the cylinder and cone, respectively, cut by two horizontal planes. Zyklographic mapping of the example problem is illustrated in Fig. 8.

3 Two-Legged Varieties

The "straw man" demonstration example involves selecting, say, three solution poses with the coupler bearing $M \rightarrow M$ = south-east and two bearing north-east, in the other branch. Note the insertion of a prismatic (P-) actuator in addition to the revolute (R-) actuator on the basal joint of the link r_2, resulting in a five-bar closed chain to create a two-legged platform. Poses as shown in Fig. 2 are numbered in the desired sequence they are to be visited. This chain has the architecture of the device exhibited by Larochelle. A second model, with the prismatic actuator replaced by a rotary one, is immediately suggested but is obviously equivalent to the first.

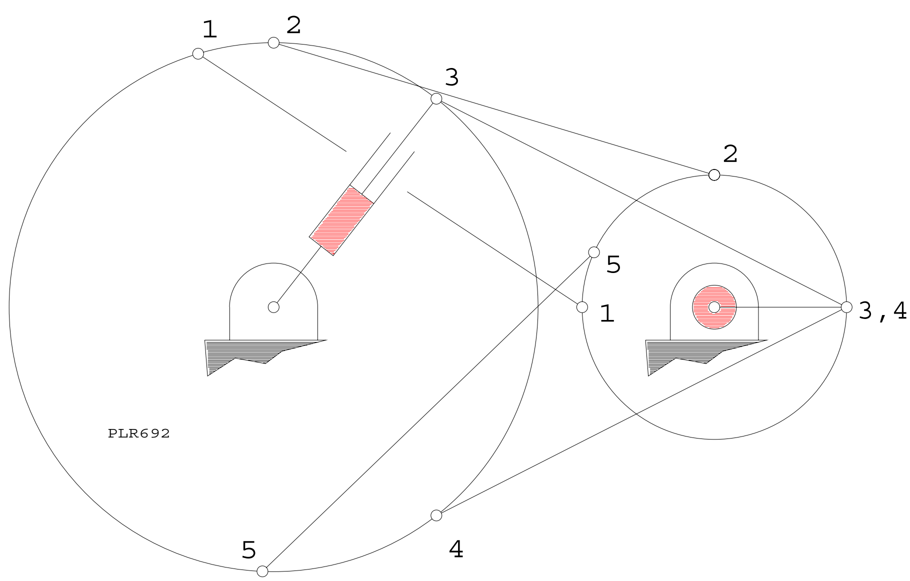

Figure 2: Branching

3.1 The <u>R</u>RR<u>P</u>R Chain

The branching problem that occurs between poses 3 and 4 may be overcome by shrinking the actuated P-leg $r_1 \rightarrow r_2$ while backing the R-actuated leg counterclockwise. This crank-crank configuration can then be rotated clockwise again with the expectation that the coupler will be inertially carried, through the dead-centre singularity, into the lower branch where the P-actuator will restore its leg to length r_1; not an entirely satisfactory state of affairs from a strictly kinematic point of view. The image space map of this design involves a single hyperboloid of one sheet, representing a surface region accessible to the dyad containing the R-actuator, and a solid region between two such hyperboloids representing the surfaces that the other RR dyad generates at the actuation limits of the P-joint.

3.2 The RRRPR Chain

A more satisfactory transition is shown in Fig. 3. This architecture is preferable because both actuated R-joints are basal. Furthermore the transition from pose 3 to pose 4 can be accomplished without moving the R-actuator on the right, attached to the leg of length r_2. This suggests a principle, albeit heuristic, concerning how to change branches efficiently. "Keep the coupler, *i.e.*, platform moving, as closely as possible, with respect to a *fixed* pole." A planar rigid body displacement due to rotation about a fixed pole describes a geodesic path in the image space. What constitutes a good criterion of proximity in this regard will remain an open question to be debated forever, for the entertainment of kinematicians! No confusion arises with respect to the chosen example. It engenders a fixed pole. With this architecture the image space map contains a ruling circle on the single hyperboloid that intersects the solid bounded, as determined by P-joint limits, by two hyperbolic paraboloids.

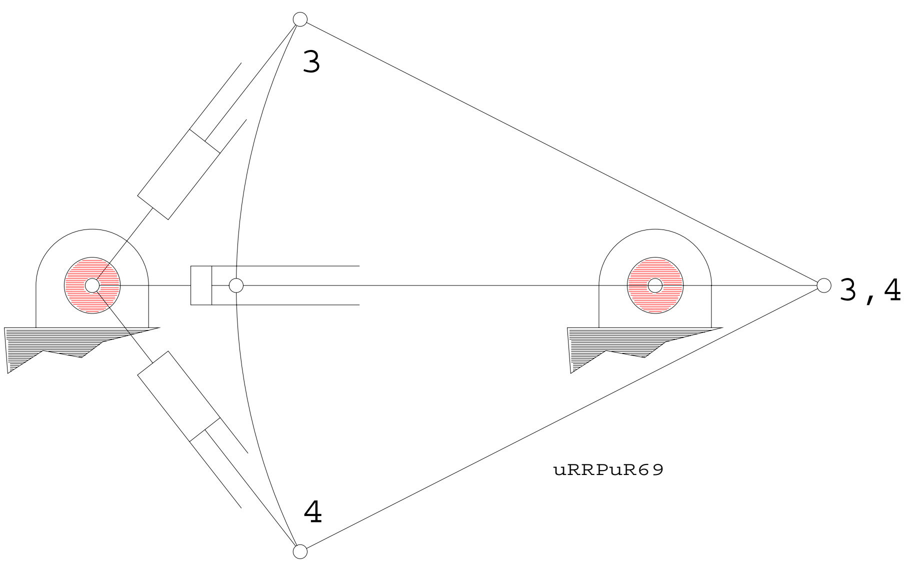

Figure 3: A Better Way to Handle Branching

3.3 Concerning Two-Legged 2dof Manipulators

These mechanisms are marginally more complicated than a simple four-bar configured to form two dyads that satisfy a given Burmester problem. Indeed, one of the dyads is selected to remain intact while the other dyad, still containing the common coupler link, is augmented by furnishing its basal link with an intermediate joint that may or may not be chosen to be the additional actuated one. Complexity, compared to the simple four-bar, involves additional link, joint and actuator in order to resolve a branching problem while retaining the stiffness of two legs as opposed to merely one, with an open chain, serial dyad. What can be gained with a three-legged implementation?

4 Three-Legged Varieties

Starting by introducing three-legged planar parallel manipulator specimens that have less than 3dof, a 2dof three-legged design is then proposed as a reliable and not-too-expensive solution to Burmester problems with branching defects.

4.1 Existing 1dof Designs

First, an existing, though decrepit, example of a *one*-dof manipulator of this sort will be illustrated. It is quite different from the time-honoured steam locomotive multiple drive wheel connecting side-rod type. It used to crush tea leaves in a “grow-op” near Ouro Preto, Brazil, in preparation for packing and shipping to Germany during and between two World Wars. Thanks to the Royal Navy, Indian, even Chinese, teas were not readily procurable. Fig. 4 shows this museum piece while Fig. 5 shows the operating principle whereby points on the rim of the inner cylinder translate in circular paths, crushing the tea against the inner surface of the outer, fixed cylinder.

4.2 A Branching Defect Solution Proposal

Consider the six-link $\underline{\text{P}}$PRRP$\underline{\text{P}}$ closed chain shown in Fig 6.

The dyad CTM has been retained, with the original four-bar coupler illustrated in its second chosen position, (2). The two actuated P-joints are represented by the pair of rack-and-pinion combinations shown in Fig. 6. However neither the R-joint at C nor that at T is actuated.

Nevertheless the point T is supported by a 2dof leg with a free P-joint attached. The other, actuated, basal P-joint on the leg provides translation in a direction perpendicular to the free P-joint. A similar PP leg is attached at M but its actuated basal P-joint moves in a direction perpendicular to the other leg’s actuated joint. In principle, this arrangement allows excursion of the coupler rigid body on MT to reach *any* point on the hyperboloid of one sheet that represents motion of MT, constrained by the dyad CTM, in the planar kinematic image space.

5 Conclusion

Various issues concerning planar parallel manipulators with less than 3dof have been raised. The primary intent was to show how such manipulators, containing an embedded dyad, or two-link kinematic chain, designed to solve a particular five pose Burmester problem specification, may overcome a branching defect inherent in the classical closed four-bar linkage solution. Essentially, the placement of additional link and joint hardware, required in any of the three proposed parallel augmentations of the appropriate Burmester dyad, constitutes a custom design exercise in itself, especially as regards P-joints. Moreover the two-legged designs incur an additional actuated joint and link -five joints as opposed to four- compared to a simple four-bar implementation while the

Figure 4: Tea Leaf Crusher

three-legged design requires a total of six joints. As regards extensions to this work, it is perceived that the nature, *i.e.*, common characteristics, as determined in the kinematic image space, of fixed polar displacements, occurring between adjacent poses, may be investigated to some advantage. Note that transition between poses (3) and (4) is rotation about T which is stationary during this displacement.

6 Appendix

The task is to find the two Cartesian coordinates $x_M = x_{M'}$ of the points M and M' in Fig. 1. Because the four quadratic constraint equations in variables $M(x_M, y_M)$, $T(x_T, y_T)$ combine to yield a sixteenth order univariate polynomial in x_M that decomposes into a product of four quadratic factors of which two are trivial and two are conjugate there are but two solutions so the problem can be solved by construction due of its underlying quadratic nature. The constructive solution is illustrated in Fig. 7. Analytically, formulation and solution, Eq. 5, proceed as follows. Three of

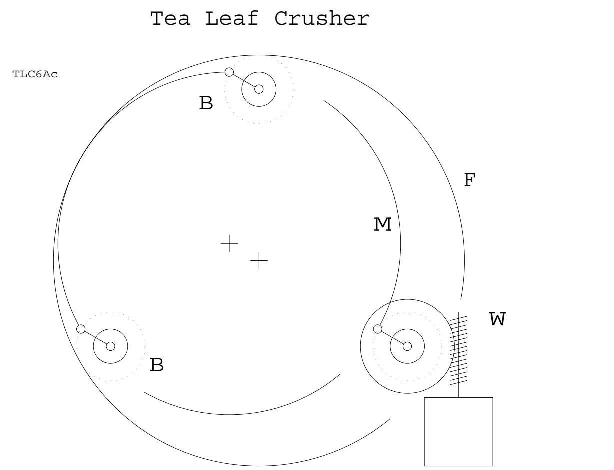

Figure 5: Principle of Operation

the constraints consist of the crank and rocker circles and a circle centred on M and of radius L, the length of the coupler. The fourth states that the circle of radius L be tangent to the circle of radius r_2.

$$x_M^2 + y_M^2 - r_1^2 = 0 \tag{1}$$
$$(x_T - c)^2 + y_T^2 - r_2^2 = 0 \tag{2}$$
$$(x_T - x_M)^2 + (y_T - y_M)^2 - L^2 = 0 \tag{3}$$

The fourth equation, Eq. 4, is the vanishing descriminant produced by the tangency condition of the intersection of Eq. 2 and Eq. 3.

$$16y_M^2[x_M^2 + y_M^2 - 2cx_M + c^2 - (L - r_2)^2] = 0 \tag{4}$$

Solving for x_M

$$256(x_M - r_1)^2(x_M + r_1)^2 \left[2cx_M - c^2 - r_1^2 + (L - r_2)^2\right]^2 \left[2cx_M - c^2 - r_1^2 + (L + r_2)^2\right]^2 = 0 \tag{5}$$

The top and front view drawing in Fig. 8 shows the intersection of three surfaces.

- The horizontal plane $r = L$ that represents *all* circles of radius L,
- The vertical axis cylinder of revolution that represents *all* circles centred on the circumference of the rocker arm circle of radius r_1, centred on O and

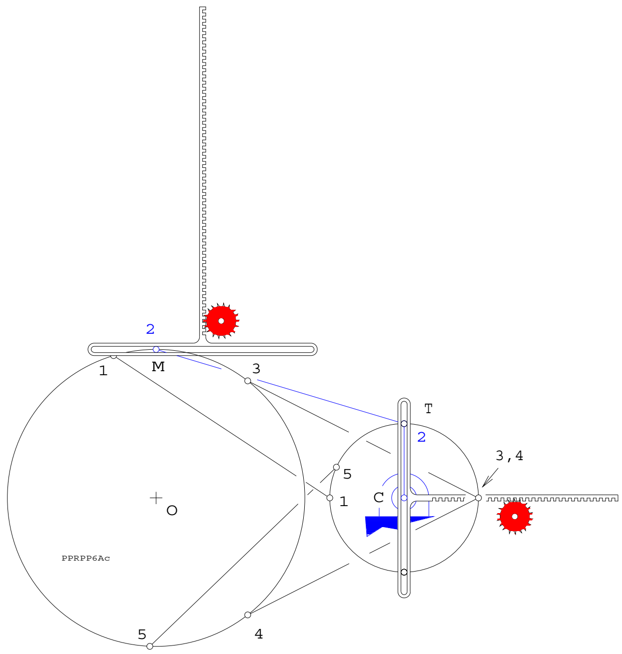

Figure 6: A Three-Legged 2dof Design

- *Two* concentric, vertical axis, $\pi/2$ apex angle cones of revolution whose sections on plane $r = 0$ are on the crank circle of radius r_2 centred on C.

  1. The cone whose apex $-X$ is a distance r_2 *beneath* plane $r = 0$ represents all tangent circles osculating with the crank circle while
  2. The cone whose apex $+X$ is a distance r_2 *above* represents all tangent circles that embrace or are embraced by the crank circle.

The entire problem to find MM and $M'M'$ is solved in the plane $r = L$ as the intersection of two pairs of circles.

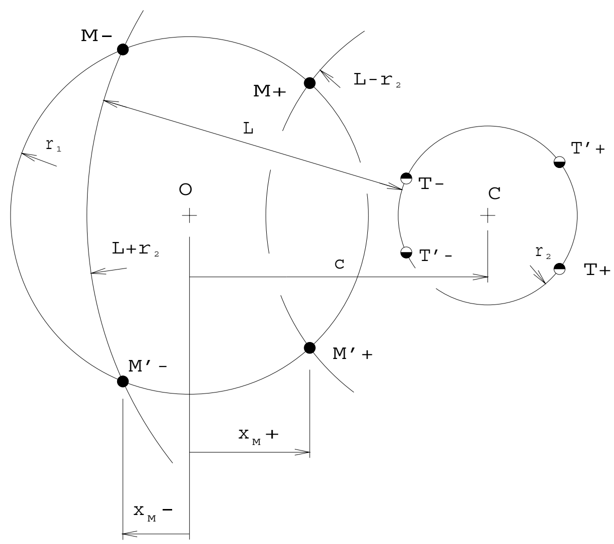

Figure 7: Finding Tangent Circle Centre Coordinates x_M

References

[1] Burmester, L. *Lehrbuch der Kinematik*. A. Felix, Leipzig, 1888.

[2] Hayes, M.J.D. and Zsombor-Murray, P. Solving the Burmester Problem Using Kinematic Mapping. *Proc. ASME-DETC2002*, MECH-34378, Montréal, September, 2002.

[3] Hayes, M.J.D. and Husty, M. On the kinematic Constraint Surfaces of General Three-Legged Planar Robot Platforms. *Mechanism and Machine Theory*, 38(5):379–394, 2003.

[4] Brunnthaler, K., Pfurner, M. and Husty, M. Synthesis of Planar Four-Bar Mechanisms *Transactions of The Canadian Scociety for Mechanical Engineering*, 30(2):297–313, June,

[5] Schröcker, H.-P., Husty, M. and McCarthy, M. Kinematic Mapping Based Evaluation of Assembly Modes for Planar Four-Bar Synthesis. *Proc. ASME-DETC2005*, MECH-85037, Long Beach, September, 2005.

[6] Larochelle, P. Two actuator solution to "The McCarthy Challenge" using a five-link closed planar chain. *ASME-DETC2002 Workshop on Parallel Mechanisms*, Montréal, September, 2002.

[7] Müller, E. (Edited by Krames, J.L.) *Vorlesungen über darstellende Geometrie -II Band, Die Zyklographie-*. Franz Deuticke, Leipzig u. Wien, September, 1928.

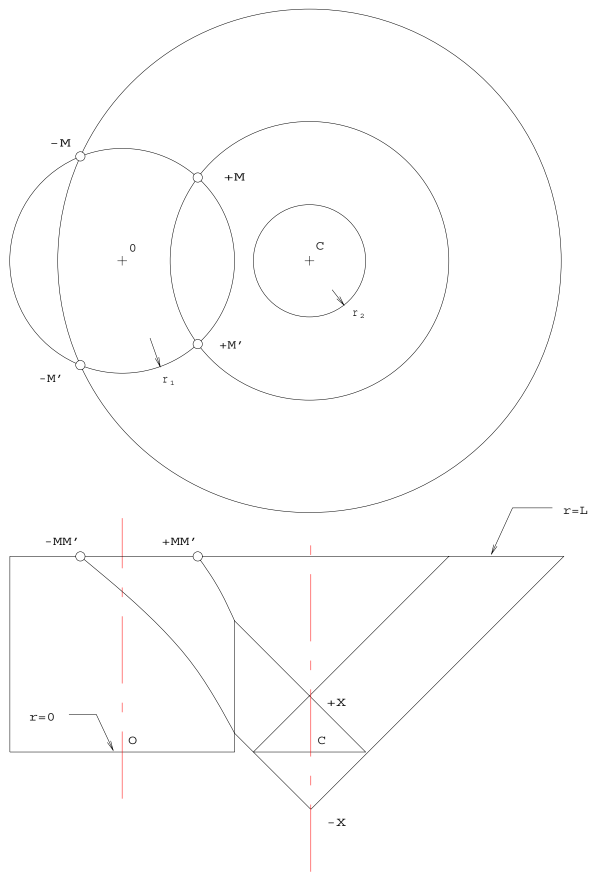

Figure 8: Finding Rocker-Arc Limits with *Zyklographie*