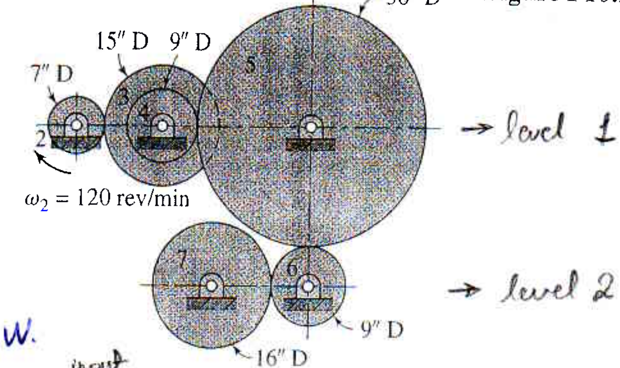
Convention : $\begin{cases} \omega \text{ is +ive for C.C.W} \\ \omega \text{ is -ive for C.W.} \end{cases}$

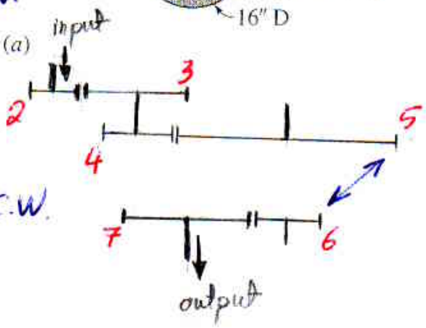
10.2 Part (*a*) of the figure gives the pitch diameters of a set of spur gears forming a train. Compute the kinematic coefficient of the train. Determine the speed and direction of rotation of gears 5 and 7.

$*$ Given: $\omega_2 = -120$ rpm $= 120$ rpm C.W.

Figure P10.2



(*a*)



Gears 5 & 6 are meshed externally

Kinematic coeff. of the train $e_{7/2}$:

- $e_{7/2} = \dfrac{\omega_7}{\omega_2} = \dfrac{\omega_7}{\omega_6} \cdot \dfrac{\omega_6}{\omega_5} \cdot \dfrac{\omega_5}{\omega_4} \cdot \cancelto{1}{\dfrac{\omega_4}{\omega_3}} \cdot \dfrac{\omega_3}{\omega_2}$

$$= \frac{-\cancel{D_6}}{D_7} \cdot \frac{-\cancel{D_5}}{\cancel{D_6}} \cdot \frac{-D_4}{\cancel{D_5}} \cdot 1 \cdot \frac{-D_2}{D_3}$$

$$= \frac{D_4 D_2}{D_7 D_3} = \frac{9 \cdot 7}{16 \cdot 15} = \frac{21}{80}$$

$\Rightarrow \omega_7 = e_{7/2}\, \omega_2 = \dfrac{21}{80} \cdot -120 = -31.5$ rpm $= 31.5$ rpm C.W.

- $e_{5/2} = \dfrac{\omega_5}{\omega_2} = \dfrac{\omega_5}{\omega_4} \cdot \cancelto{1}{\dfrac{\omega_4}{\omega_3}} \cdot \dfrac{\omega_3}{\omega_2} = \dfrac{D_4 D_2}{D_5 D_3} = \dfrac{9 \cdot 7}{30 \cdot 15} = \dfrac{7}{50}$

$\Rightarrow \omega_5 = e_{5/2}\, \omega_2 = \dfrac{7}{50} \cdot -120 = -16.8$ rpm $= 16.8$ rpm C.W.

10.8 In part (*a*) of the figure, shaft *C* is stationary. If gear 2 rotates at 800 rev/min ccw, what are the speed and direction of rotation of shaft *B*?

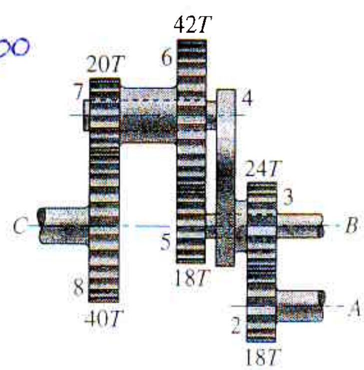
$\Rightarrow \omega_8 = 0$

no moving axes of rotation

- $e_{3/2} = \dfrac{\omega_3}{\omega_2} = -\dfrac{N_2}{N_3} \Rightarrow \omega_3 = \dfrac{-N_2}{N_3}\,\omega_2 = \dfrac{-18}{24} \cdot 800$

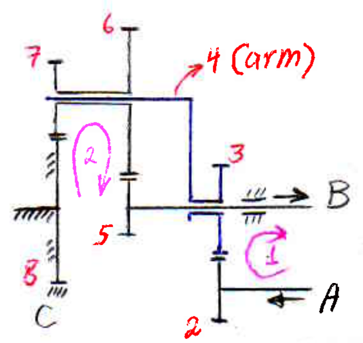
$$= -600 \text{ rpm}$$

$$\Rightarrow \omega_3 = 600 \text{ rpm C.W.}$$



(*a*)

Figure P10.8



$\omega_2 = 800$ rpm C.C.W

Arm 4 rotates with speed ω_3.

- $e_{8/5}\Big|_{\substack{\text{arm} \\ (\omega_3)}} = \dfrac{\omega_{8/3}}{\omega_{5/3}} = \dfrac{\omega_{8/3}}{\omega_{7/3}} \cdot \cancelto{1}{\dfrac{\omega_{7/3}}{\omega_{6/3}}} \cdot \dfrac{\omega_{6/3}}{\omega_{5/3}}$

$$= \frac{-N_7}{N_8} \cdot 1 \cdot \frac{-N_5}{N_6} = \frac{N_7 N_5}{N_8 N_6}$$

$$= \frac{20 \cdot 18}{40 \cdot 42} = \frac{3}{14} \quad \text{--- (1)}$$

But Also, $e_{8/5}\Big|_{\text{arm}} = \dfrac{\cancelto{0}{\omega_8} - \omega_3}{\omega_5 - \omega_3}$ --- (2)

$\Rightarrow \dfrac{-\omega_3}{\omega_5 - \omega_3} = \dfrac{3}{14} \Rightarrow \omega_5 = \dfrac{11}{3}\,\omega_3$

$\Rightarrow \omega_5 = 2200$ rpm (C.C.W)

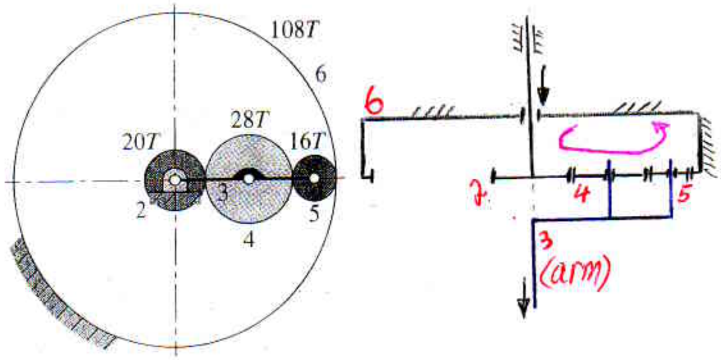
10.11 In part (*b*) of the figure, gear 2 is connected to the input shaft. If arm 3 is connected to the output shaft, what speed reduction can be obtained? What is the sense of rotation of the output shaft? What changes could be made in the train to produce the opposite sense of rotation for the output shaft?

* Gear 6 is fixed $\Rightarrow \omega_6 = 0$

Arm rotates with speed ω_3.

$$e_{6/2}\Big|_{arm} = \frac{\omega_{6/3}}{\omega_{2/3}} = \frac{\omega_{6/3}}{\omega_{5/3}} \cdot \frac{\omega_{5/3}}{\omega_{4/3}} \cdot \frac{\omega_{4/3}}{\omega_{2/3}}$$

$$= \frac{N_5}{N_6} \cdot \frac{-N_4}{N_5} \cdot \frac{-N_2}{N_4} = \frac{20}{108} = \frac{5}{27} \quad \text{--- (1)}$$



(*b*)

Also,

$$e_{6/2}\Big|_{arm} = \frac{\omega_6 - \omega_3}{\omega_2 - \omega_3} \quad \text{--- (2)}, \quad \omega_6 = 0$$

$$\Rightarrow \frac{-\omega_3}{\omega_2 - \omega_3} = \frac{5}{27} \Rightarrow \boxed{\omega_3 = \frac{-5}{22}\omega_2} \quad \text{--- (3)}$$

$\Rightarrow$ the output speed is reduced by

$$\frac{|\omega_2| - |\omega_3|}{\omega_2} \times 100 = \frac{\omega_2\left(1 - \frac{5}{22}\right)}{\omega_2} \times 100 = \frac{17}{22} \times 100$$

$$= 77.3\%$$

& the output shaft rotates in opposite sense to the input shaft (based on (3)).

* To have input & output shafts rotate in same sense, I need to have $e_{6/2}\big|_{arm}$ in negative. One way to do that is to replace gears 4 & 5 with one bigger gear, whose pitch diameter will be the sum of pitch diameters of gears 4 & 5. Since gears 2, 4 & 5 have same diametral pitch, then the replacement gear will have $28 + 16 = 44$ teeth.

[Diametral pitch P. Replacement gear has N teeth & $d_P = d_{P_4} + d_{P_5}$. Then, $N = P \cdot d_P = P(d_{P_4} + d_{P_5}) = N_4 + N_5$]

10.17 Shaft *A* in the figure is the output and is connected to the arm. If shaft *B* is the input and drives gear 2, what is the speed ratio?

ω_B: input speed. ω_A: output speed.

Arm rotates with speed $\omega_3 = \omega_A$.

$$e_{6/2}\Big|_{arm} = \frac{\omega_{6/A}}{\omega_{2/A}} = \frac{\omega_{6/A}}{\omega_{5/A}} \cdot \frac{\omega_{5/A}}{\omega_{4/A}} \cdot \frac{\omega_{4/A}}{\omega_{3/A}} \cdot \frac{\omega_{3/A}}{\omega_{2/A}}$$

$$= \frac{-N_5}{N_6} \cdot 1 \cdot \frac{-N_3}{N_4} \cdot \frac{-N_2}{N_3}$$

$$= \frac{-N_5 N_2}{N_6 N_4} = \frac{-18 \cdot 16}{50 \cdot 16} = \frac{-9}{25} \quad \text{--- (1)}$$

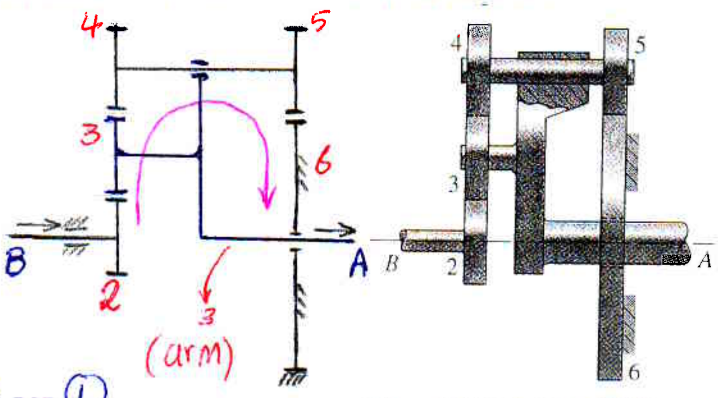


Figure P10.17 $N_2 = 16$ T, $N_3 = 18$ T, $N_4 = 16$ T, $N_5 = 18$ T, $N_6 = 50$ T.

$$\text{Also,} \quad e_{6/2}\Big|_{arm} = \frac{\omega_6 - \omega_A}{\omega_2 - \omega_A} \quad \text{--- (2)}, \quad \omega_6 = 0$$

$$(1) \ \& \ (2) \Rightarrow \frac{\omega_A}{\omega_2 - \omega_A} = \frac{9}{25} \Rightarrow \frac{25}{9}\omega_A = \omega_2 - \omega_A \Rightarrow \omega_A = \frac{9}{34}\omega_2$$

$$\therefore \text{speed reduction ratio} = \frac{9}{34}.$$