TRIPLE 3D PENDULUM

Control Problem

Goal in this experiment:

Partial, semi-global stabilization to the unstable upright, inverted equilibrium.

- Partial stabilization is defined as stabilization to a subspace in the phase space of the system that excludes the rotational motion of the lower body around the vertical z-axis. Spinning of the pendulum around the vertical axis is permitted, albeit not observed in this control design experiment.

- Semi-global stabilization is defined as stabilization from a maximally large set of initial positions of the pendulum.

Assumptions:

- There are no constraints on the available actuation power;

- The ground, or floor, is not a motion obstacle, i.e. the pendulum is free to move about its "foot".

Characteristics & Difficulty of the Problem:

- The system has several relative equilibria with different stability characteristic;
- Due to assumed loss of friction in the joints, the uncontrolled system exhibits sustained chaotic behavior;
- The system is under-actuated and is represented by 14 states of which 12 need to be stabilized by the action of only 4 control variables.
- Due to the presence of gravity, the system can be considered as a non-holonomic system of order two with acceleration constraints that are fully non-integrable – the constraints cannot be integrated to equivalent velocity constraints.
- The system cannot be globally partially stabilized due to its non-holomomy and absence of a global atlas on the configuration space $SO(3) \times SO(3) \times SO(3)$.
- From a structural point of view, the 3D inverted pendulum stabilization problem is very different from the corresponding planar 2D inverted pendulum stabilization problem as the 3D system does not have kinetic symmetries. The kinetic energy of the system is not cyclic with respect to the un-actuated variables.

For this reason planar approaches cannot be applied directly.

- Symmetry reduction techniques, energy shaping and passivity based control have never been attempted.

A Few Hints Towards a Computationally Feasible Stabilizing Control Design Approach

- The $x, y-$projections l_x, l_y of the angular momentum of the pendulum w.r.t. its "foot", when adopted as the system output, has vector relative degree $(3, 3)$ with respect to the $x, y-$components of the reaction force, f_x, f_y at the foot. A similar fact is true for the middle and upper body treated as a single "free body" while controlled by the reaction forces at the "hip" and actuation torques at the shoulder with the output defined as the $x, y-$projections of the angular momentum of the free body w.r.t. the "hip".

- The pendulum system may then be viewed from the perspective of a two-body system consisting of mutually dependent subsystems: (i) the pendulum as a whole and (ii) the free-body subsystem consisting of the middle and upper links only. The partially linearized subsystems can then be controlled similarly as the Double 3D Pendulum while their relative motion must be coordinated and the resulting zero dynamics must be asymptotically stabilized by way of the action of the vertical components of the reaction forces at the foot and at the hip. Such coordination and zero dynamics stabilization can again be achieved by an overall energy-work balance effectuated by suitable nonlinear correction terms in the controls.