

TRIPLE 3D PENDULUM

Model Features

An inverted Triple 3D Pendulum is represented in an inertial frame (x, y, z) as a three-body system :

- The lower body is an axially symmetric rigid link whose end-points are referred to as the "foot" and the "hip", respectively. There is an additional lumped mass placed at the "hip" and the "foot" is rigidly fixed to the ground understood to be the (x, y) plane.
- The middle body is an axially symmetric rigid link whose end-points are referred to as the "hip" and the "shoulder" with an additional lumped mass placed at the "shoulder". The upper and middle bodies are attached to one another at the "hip".
- The upper body is also an axially symmetric rigid link with an additional lumped mass placed at its tip that represents the "head". The upper and middle bodies are attached to one another at the "shoulder".
- The joint at the "foot" is assumed to be spherical, but is realized here as three consecutive revolute joints

permitting three rotations about the x, y, z axes. The revolute joints at the "foot" are NOT actuated.

- The joints at the "hip" and at the "shoulder" are assumed to be universal and each is realized as two consecutive revolute joints permitting rotations about the x and y axes. The universal joints at the "hip" and at the "shoulder" are actuated fully and freely, i.e. there is no constraint on the actuation power on the four rotational force moments.
- All joints are frictionless and the triple pendulum is placed in a gravity potential field.

In this simulation, but without the loss of generality, the geometry and inertia matrices for the upper and middle bodies are assumed the same while the upper link has shorter length. The spatial triple pendulum system is modeled as a seven link kinematic chain in the framework of the Spatial Operator Algebra (SOA),

<http://dshell.jpl.nasa.gov/SOA/index.php>.