Interreflections

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Synonyms

- mutual illumination

Related Concepts

- radiance
- shape from shading
- diffuse reflection
- global illumimination
- volumetric scattering
- subsurface scattering
- bas relief ambiguity

Definition

Interreflections are reflections of light from one surface to another surface.

Background

Surfaces are illuminated not just by light sources, but also by each other. These interflections can provide a significant component of surface illumination, especially in concavities or enclosures. Numerical methods for computing interreflections were developed in the early 20th century to solve problems in heat transfer such as in furnace design. The methods were developed further by the computer graphics community in the 1980's to render global illumination. for Lambertian surfaces and then scenes with specular components [7,8].

Theory

Interreflections can be described mathematically in several equivalent ways. One way is to write the reflected light as a sum of the light that is due to the illumination that arrives at a surface directly from the light source, plus the light that arrives from other surfaces in the scene via interreflections. Suppose the scene is composed of Lambertian surfaces with albedo $\rho(\mathbf{x})$ varying across surfaces. Let $L_s(\mathbf{x})$ be the component of $\mathbf{x}$'s outgoing radiance that is due to direct illumination from the source. Then the total radiance $L(\mathbf{x})$ leaving $\mathbf{x}$ is the sum of $L_s(\mathbf{x})$ and the radiance that is due to interreflections:

$$L(\mathbf{x}) \;=\; L_s(\mathbf{x}) \;+\; \rho(\mathbf{x}) \int_{\mathcal{S}} L(\mathbf{y})\; K(\mathbf{x},\mathbf{y})\; d\mathbf{y}\ . \tag{1}$$

Here the integral is taken over all surface points $\mathbf{y} \in \mathcal{S}$ in the scene, and the function $K(\mathbf{x},\mathbf{y})$ is a symmetric weighting function that depends on the surface normals at $\mathbf{x}$ and $\mathbf{y}$ and on the distances between $\mathbf{x}$ and $\mathbf{y}$. $K(\mathbf{x}, \mathbf{y})$ is zero if the $\mathbf{x}$ and $\mathbf{y}$ are not visible to each other.

It is common to approximate Eq. (1) by using a polygonal mesh surface whose facets have constant radiance,

$$\mathbf{r} \;=\; \mathbf{r}_s \;+\; \mathbf{PKr} \tag{2}$$

where $\mathbf{r}$ and $\mathbf{r}_s$ are the vectors of direct and total radiance, respectively, $\mathbf{P}$ is a diagonal matrix of albedos, and $\mathbf{K}$ is called the *form factor* matrix. The above equations can be generalized to non-Lambertian surfaces as well[7,8].

A second approach is to consider the eigenfunctions of $\mathbf{K}$ which are radiance functions that are invariant to interreflections [11,9]. These eigenfunctions are concentrated in surface concavities and at points of contact between surfaces [10].

A third approach is to use ray tracing to follow the light emitted from the source through successive reflections or bounces in the scene. The n^{th} reflection serves as the source for the $n + 1^{st}$ reflection, and the sum of all reflections gives an infinite series. For any scene geometry and reflectance, it is possible to construct a linear operator that can be applied iteratively to decompose the interreflections into their n bounce components[15]. Understanding the various bounces is especially important for making finite approximations. For example, a two-bounce model has been used to model how surface microfacets can account for non-Lambertian reflection [14] and how color bleeding occurs between surfaces in a concavity [5].

Applications

Standard shape from shading and photometric stereo methods consider only the direct illumination component [6]. When interreflections are present these methods produce erroneous results [4]. It is possible to extend these methods to account for interreflections by first ignoring interreflections to obtain an approximate solution and then iteratively updating the solution to account for interreflections. This idea has been applied to photometric stereo [12], and to shape from shading for the special case that the surface is an unfolded book in a photocopier [16].

The above applications assume the light sources are known. But what if they are unknown? An important fact that applies in this situation is the *bas relief ambiguity* [1]. When a Lambertian surface is illuminated by direct illumination only, there exists a family of scenes (shape, albedo, lighting) that all produce the same image. With interreflections present, the bas relief ambiguity no longer exists [3]. One can estimate the surface shape and reflectance similarly to above, namely by applying a photometric stereo method that is designed for unknown lighting [17] and then iteratively updating the shape and reflectance to account for interreflections [3].

Interreflections also arise in projector-camera systems. An image that is projected on a concave screen will suffer from interreflections that will lower the image contrast. This contrast reduction can be compensated for, to some extent, by modifying the originally projected pattern [2]. A related example which involves active illumination of a 3D scene is to obtain a small number of images of the scene by illuminating it with a set of high frequency projection patterns such as checkerboards [13]. The interreflection components of the scene will have relatively low spatial frequency and will be similar in the images. This property allows one to decompose the image of a fully illuminated sceone into its direct component and its interreflection component. Unlike most methods for interreflections which assume Lambertian scenes, this method allows for non-Lambertian surfaces and other forms of reflections such as volume scattering.

Recommended Readings

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