Radiometry

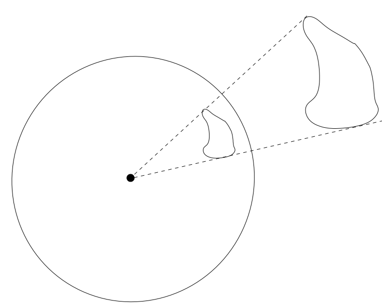
We have discussed how light travels in straight lines through space. We would like to be able to talk about how bright different light rays are. Imagine a thin cylindrical tube and consider the amount of light that passes through the tube without hitting the sides. We measure the power[1] of light through the tube.

The power of light that gets through the tube, depends on the length of the tube and on the cross-sectional area the tube (πr^2, where r is the tube radius). If you hold the area constant and increase the length of the tube, you decrease the amount of light that passes through the tube because you will have a tighter restriction on the direction of rays that can pass through the tube. Similarly, if you hold the length of the tube constant, and you decrease the cross-sectional area of the tube, you again decrease the number of rays that pass through the tube. Thus, the power of light that passes through the tube depends on the length and cross sectional area.

Angle vs. solid angle

For a thin tube, consider the angle subtended by the diameter at the far end of the tube, when seen from the axis at the opening of the near end of the tube. This angle is approximately the ratio $\frac{diameter}{tubelength}$. This angle is in units of *radians*, i.e. which is the arclength on the unit circle. Recall 1 radian is $\frac{1}{2\pi} \times 360 \approx 57$ degrees, $\frac{\pi}{2}$ radians is 90 degrees, etc.

To measure light, one needs to consider an angular measurement called *solid angle*. Solid angle is the area on a unit sphere that is subtended by an object, when that object is viewed from a point at the center of the sphere. The units of solid angle are *steradians*. The maximum value of solid angle is 4π, which is the entire area of the unit sphere. For example, if you are inside a room, then the solid angle subtended by the walls and ceiling and floor of the room is 4π steradians.



We said earlier that the power of light that passes through the tube depends on the length and width of the tube. Alternatively, we can express the power by the area of the near end of the tube and by the solid angle subtended by the far end of the tube when seen from near end. This leads us to the following definition.

Radiance

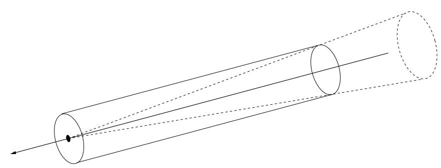
Place one end of a small hollow tube at some point $\mathbf{x}$ = (X,Y,Z) and point it in some direction $\mathbf{l}$. Measure the power of light passing through the tube. Define the *radiance* $L(\mathbf{x}, \mathbf{l})$ of the ray to be

[1]Namely, light energy per second, measured in Watts. I am ignoring color properties for now.

the light power[2] per unit cross section area at $\mathbf{x}$ per unit steradian of the cone of rays arriving at $\mathbf{x}$,

$$\text{radiance,} \quad L(\mathbf{x}, \mathbf{l}) \equiv \frac{\text{power}}{\text{cross section area } * \text{ solid angle}}$$

In trying to understand this definition, you should think about a thin tube, so that the cross sectional area and solid angle are both very small.

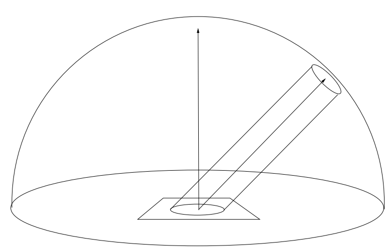


[ASIDE: If there is no scattering of light in space (e.g. fog), then *radiance is constant along a light ray.* Intuitively, if you place your thin tube at a point $\mathbf{X}$ in space and point it in some direction $\mathbf{l}$, and then move the tube in this direction, the power that you get through the tube will not change (i.e. the brightness of the light you will see through the tube will not change). Things that we look at do not get brighter when we walk toward them. They get bigger (in terms of their solid angle) but they don't get brighter. Note: small light sources *do* seem to get dimmer as you move away from them. But the reason is that the solid angle they subtend gets smaller – not that the radiance decreases.]

Irradiance

Another important concept is the total power of light that arrives *at a surface* per unit area. This is important for understanding how much illumination is received by points in the world, and also how much light arrives at a pixel on the sensor plane. Today we'll examine points in the world and next lecture we'll consider the sensor plane.

If you take a point $\mathbf{x}$ on a surface, then light can arrive at $\mathbf{x}$ from a hemisphere of directions. Usually one talks about the amount of light arriving in a small area containing that point, rather than about the set of rays arriving at precisely that point. The reason for talking about a small arrival area is that it allows one to capture the orientation (normal) of the surface at that point.



The *irradiance* $E(\mathbf{x})$ is defined as the light power arriving per unit area on the surface in a small neighborhood of $\mathbf{x}$. Light can arrive from a hemisphere of directions, so we divide the hemisphere

[2] energy per unit time

into solid angles $d\Omega_{in}$ and then add up the contribution of each one:

$$E(\mathbf{x}) = \int L(\mathbf{x}, \mathbf{l}_{in}) \; \mathbf{n}(\mathbf{x}) \cdot \mathbf{l}_{in} \; d\Omega_{in}$$

where $\mathbf{n}(\mathbf{x})$ is the unit normal vector of the surface at $\mathbf{x}$, and $d\Omega_{in}$ is a small solid angle centered in the direction $\mathbf{l}_{in}$.

The $\mathbf{n} \cdot \mathbf{l}_{in}$ component is needed because the radiance is defined per unit area *on the surface*. Consider a small tube centered on a ray in direction $\mathbf{l}_{in}$ which has one end at the surface point $\mathbf{x}$. If the incoming ray is oblique to the surface, then the surface will intersect the tube at an oblique angle. The light through the tube will be be spread out over an area proportional to $1/\mathbf{n} \cdot \mathbf{l}_{in} \geq 1$. Thus, to get the power per unit area *on the surface*, rather than the cross-sectional area of the tube, we need to multiply by $\mathbf{n} \cdot \mathbf{l}_{in} \leq 1$.

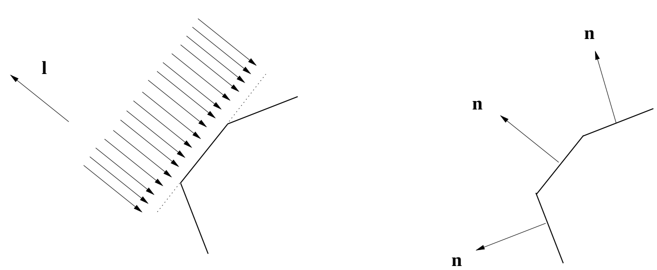
Special case 1: sunlight

Consider the special case of a "parallel source" such as sunlight. The sun spans a small solid angle Ω_{sun} and the radiance is approximately constant L_{sun} over that angle, so we can write

$$\begin{aligned} E(\mathbf{x}) &= \int_{sun} L(\mathbf{x}, \mathbf{l}_{in}) \mathbf{n}(\mathbf{x}) \cdot \mathbf{l}_{in} d\Omega_{in} \\ &\approx L_{sun} \mathbf{n}(\mathbf{x}) \int_{sun} \mathbf{l}_{in} d\Omega \\ &\approx L_{sun} \; \Omega_{sun} \; \mathbf{n}(\mathbf{x}) \cdot \mathbf{l}_{sun} \end{aligned}$$

where L_{sun} is the radiance of the source, Ω_{sun} is the solid angle subtended by the source, and $\mathbf{l}_{sun}$ is the direction to the center of the sun.

If you are struggling with the $\mathbf{n} \cdot \mathbf{l}$ effect, think of the (approximately) parallel rays from the sun and to notice that "number of rays" that reach a unit area patch of surface is proportional to $\mathbf{n}(\mathbf{x}) \cdot \mathbf{l}$, namely to the cosine of the angle between the surface normal and the light source. Consider the sketch below. The parallel vectors indicate the parallel rays arriving at the surface. The three boldface lines (bottom left) indicate three surfaces that having different surface normals. The two dotted lines illustrate that as the surface normal tilts away from the light source, the number of rays reaching a unit area of the surface decreases. (Only a slice is shown, but you should imagine this picture is the same in each depth slice.) The number of rays depends on the length of the dotted line, which by simple trigonometry is $\mathbf{n}(\mathbf{x}) \cdot \mathbf{l}$.



Special case 1: disk skylight in a room

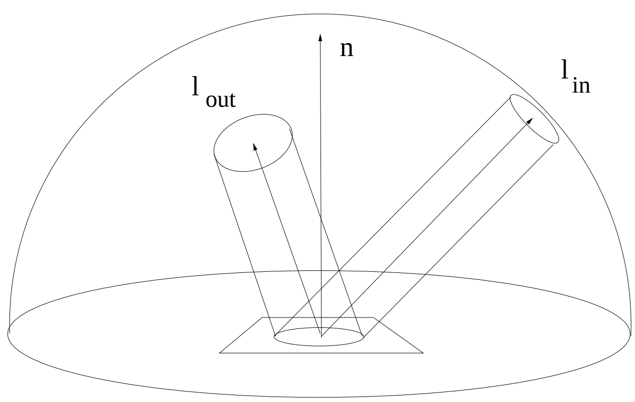
This case was discussed in class (see lecture slides). Details will be given in the lecture notes for lecture 7.

Surface reflectance

The above discussion was concerned with how much light *arrives* at a surface point $\mathbf{x}$ per unit surface area. To model the intensity of light *reflected* from a surface point $\mathbf{x}$, we must specify not just the irradiance (incoming light), but also say something about the surface material. There are many different kinds of surface materials and they differ in visual appearance in many ways. For example, some surfaces are shiny and others are not. Surfaces that are shiny can have very different appearances from each other: compare plastic, metal, velvet, leather, or skin, etc. Here I present only basic model for how people describe surface reflectance. This won't give you much understanding of the topic, but it at least exposes you to the existence of the topic!

Bidirectional reflectance distribution function (BRDF)

Suppose that we were to illuminate the surface near a point $\mathbf{x}$ from only a small set of directions centered at $\mathbf{l}_{in}$ which is a unit vector. This determines the component of the irradiance at $\mathbf{x}$ that is due to a small set of directions of incident light. We can then measure the radiance of the surface in some outgoing direction, say $\mathbf{l}_{out}$. The quantity of interest is how this outgoing radiance in direction $\mathbf{l}_{out}$ depends on the incoming radiance $\mathbf{l}_{in}$. If we know this dependence for all pairs $\mathbf{l}_{in}$ and $\mathbf{l}_{out}$, then we have characterized the reflectance properties of the surface.



One captures this dependence by defining a *bidirectional reflectance distribution function* (BRDF):

$$\rho(\mathbf{x}, \mathbf{l}_{in}, \mathbf{l}_{out})$$

which is defined such that:

$$L(x, \mathbf{l}_{out}) = \int \rho(\mathbf{x}, \mathbf{l}_{in}, \mathbf{l}_{out}) \ L(\mathbf{x}, \mathbf{l}_{in}) \mathbf{n}(\mathbf{x}) \cdot \mathbf{l}_{in} \ d\Omega_{in}$$

A BRDF can be measured in a laboratory by sequentially illuminating a surface at $\mathbf{x}$ with lights from each direction $\mathbf{l}_{in}$, and then measuring the resulting outgoing radiance in each direction $\mathbf{l}_{out}$. (Details omitted.)

Measuring and modelling BRDF's is a very important problem in computer vision. The applications are mainly in computer graphics, where one would like to be able to *render* surfaces using realistic BRDFs models that are obtained from real surfaces, instead of the oversimplified models (e.g. Phong) that were used in classical computer graphics.

Diffuse (matte, Lambertian) reflectance

For many surfaces, the BRDF $\rho(\mathbf{x}, \mathbf{l}_{in}, \mathbf{l}_{out})$ is independent of $\mathbf{l}_{out}$ and $\mathbf{l}_{in}$. In particular, for such a surfaces, the radiance leaving a surface point $\mathbf{x}$ is modelled as

$$L(x, \mathbf{l}_{out}) = \rho(\mathbf{x}) \int L(\mathbf{x}, \mathbf{l}_{in}) \mathbf{n}(\mathbf{x}) \cdot \mathbf{l}_{in} \, d\Omega_{in}$$

The constant $\rho(\mathbf{x})$ specifies whether the surface is light or dark colored. For a black surface, $\rho(\mathbf{x})$ would be near zero.

Notice that the outgoing radiance for Lambertian surfaces does not depend on $\mathbf{l}_{out}$. Such surfaces, for which the reflected radiance is the same in all directions, are called *Lambertian*[3], or *matte*.

Many computer vision methods assume the surfaces are approximately Lambertian. The main reason is that this condition helps one to find matching points in images taken from different viewpoints. Why? If a pixel has a particular intensity value from camera position and we wish to find a corresponding pixel when the same camera is moved to another position, then we only need to consider points that have the same image intensity.

[3] after an 18th century scientist Johann Lambert who wrote a treatise on light