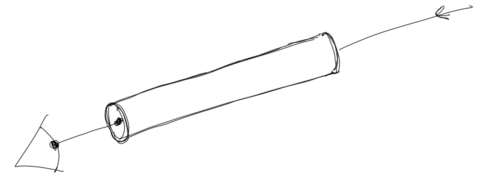
lecture 6

radiometry

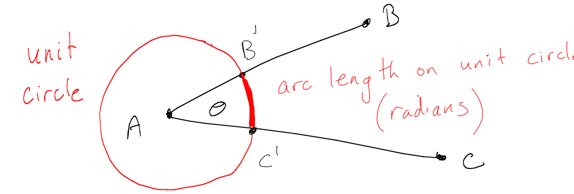
(illumination and reflectance)

How bright is a light ray?



How much power passes through a thin tube centered on the ray?

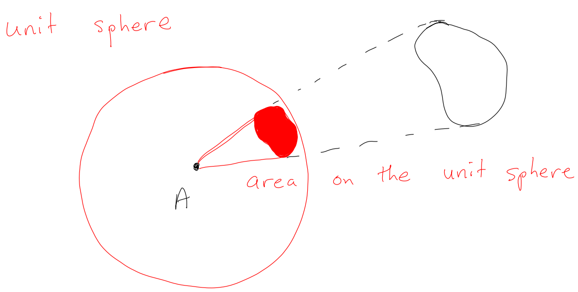
Angle (radians)



$$\theta \text{ radians} = \frac{\pi}{180} \times \theta \text{ degrees}$$

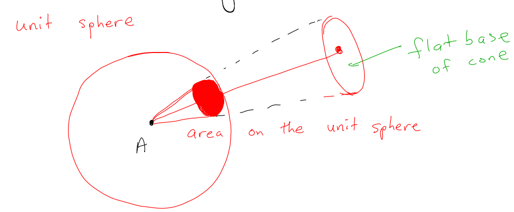
$$\approx 2 \arctan\left(\frac{B'C'}{2}\right)$$

Solid Angle (steradians)



Ω steradians (maximum 4π)

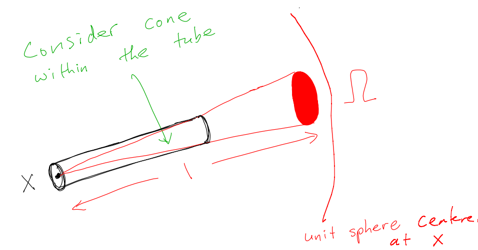
Solid Angle of a Cone



$$\Omega \approx \frac{\text{area of base of cone}}{(\text{height of cone})^2}$$

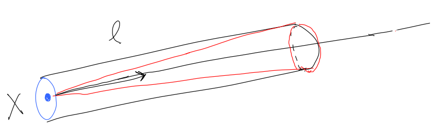
The approximation is due to the fact that a flat disk has different area than a slightly curved disk

Solid angle



$$\Omega \approx \frac{\text{cross sectional area of tube}}{(\text{length of tube})^2}$$

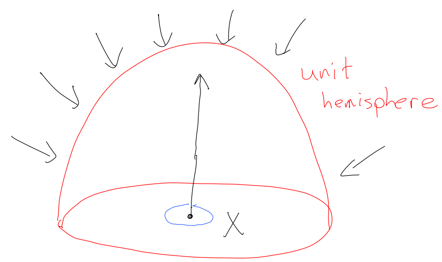
Radiance (of a ray)



$$L(X, \vec{l}) \equiv \frac{\text{light power through tube}}{(\text{area } A) * (\text{solid angle } \Omega)}$$

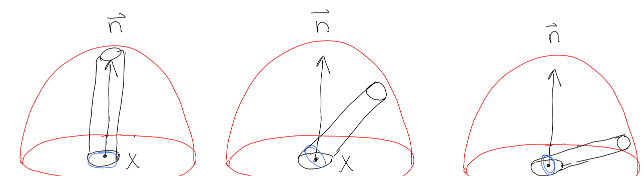
Assume A, Ω are both small.

Surface Irradiance E(X)

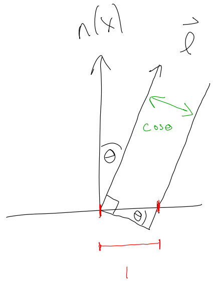


How much light power arrives at X per unit area?

Irradiance E(x)

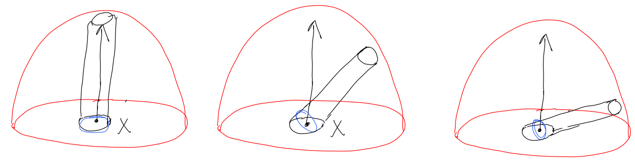


Contribution from direction $\vec{l}$ to irradiance at X must be weighted by $\vec{n} \cdot \vec{l}$ to account for foreshortening.



A unit area on the surface gives a cross sectional area of $\cos\theta = n \cdot \ell$ in the tube. (Only a slice through areas are shown.)

Irradiance $E(x)$



$$dE = L(x, \vec{\ell})\ \vec{n}(x) \cdot \vec{\ell}\ d\Omega$$

$$\therefore E(x) = \int_{\text{hemisphere}} L(x, \vec{\ell})\ \vec{n}(x) \cdot \vec{\ell}\ d\Omega$$

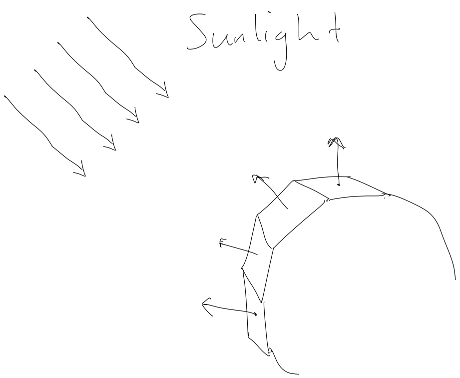
Example - Sunlight

$L(x, \ell)$ is very high in a small range of directions

$$E(X) = \int L(X, \vec{\ell})\ \vec{n}(X) \cdot \vec{\ell}\ d\Omega$$

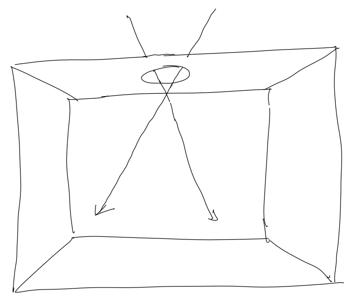
$$\approx L_{sun}\ \vec{n}(X) \cdot \vec{\ell}_{sun}\ \Omega_{sun}$$

Sunlight

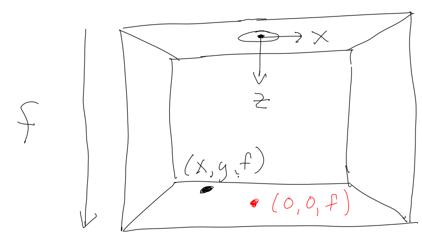


$$E(X) \approx L_{sun}\ \Omega_{sun}\ \vec{n}(X) \cdot \vec{\ell}_{sun}$$

Example: skylight (disk) window

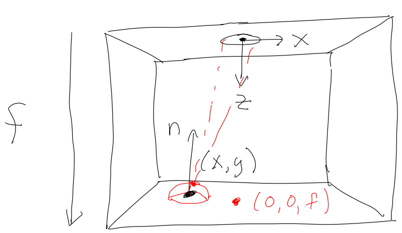


What is the irradiance $E(x)$ on the floor?



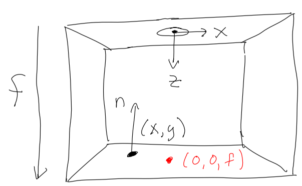
Assume
- radiance from sky is constant L_0 as on an overcast (cloudy) day
- center of disk skylight is (0,0,0)
- height of room is f

What is $E(x,y)$ on the floor?



$$\therefore E(x,y) = \int L(x,y,\vec{\ell})\ \vec{n}(x,y) \cdot \vec{\ell}(x,y)\ d\Omega$$

$$\approx L_0\ n(x,y) \cdot \vec{\ell}(x,y)\ \Omega$$



$$E(x,y) \approx L_0\, n(x,y) \cdot \vec{\ell}(x,y)\ \Omega$$

$$\Omega = \underbrace{\frac{A_{disk}}{x^2+y^2+f^2}}_{\text{recall def}^{n}\text{ of solid angle of a cone}}\ \underbrace{\vec{n} \cdot \vec{\ell}}_{\text{foreshorten the source disk}}$$

$$\ell(x,y) = \frac{(-x,-y,-f)}{\sqrt{x^2+y^2+f^2}}$$

$$n(x,y) = (0,0,-1)$$

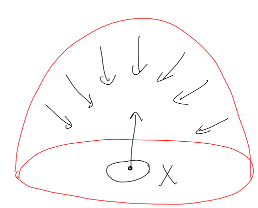
$$n \cdot \ell = \frac{f}{\sqrt{x^2+y^2+f^2}}$$

Thus,

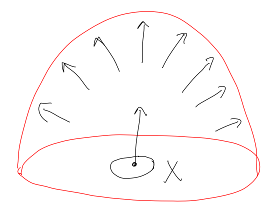
$$E(x,y) \approx L_0\, A_{disk}\, \frac{(n \cdot \vec{\ell})^4}{f^2}$$

Reflected Light

incoming (incident)



outgoing (exitant)



plastic

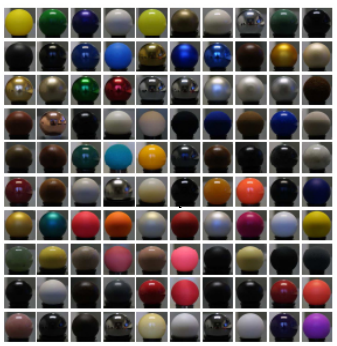
brushed metal

velvet

skin

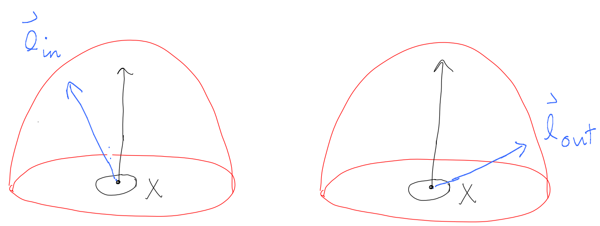
polish the apple
(change its reflectance)

leather

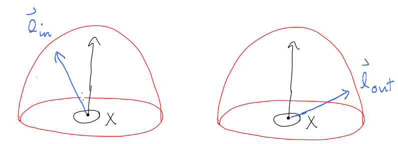


http://www.merl.com/brdf/

Modelling Surface Reflectance



How does light reflected in direction $\vec{l}_{out}$ depend on light arriving from direction $\vec{l}_{in}$?



$$E(X) = \int L(X, \vec{l}) \, \vec{n}(X) \cdot \vec{l} \; d\Omega$$

$$L(x, l_{out}) = \int \rho(x, \vec{l}_{in}, \vec{l}_{out}) \; L(x, \vec{l}_{in}) \, \vec{n}(x) \cdot \vec{l}_{in} \; d\vec{l}_{in}$$

called "bidirection reflectance distribution function" BRDF

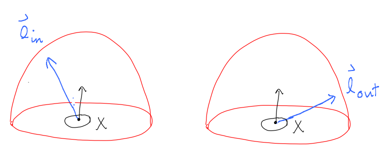
* property of surface material only !

BRDF's are heavily used in computer graphics

- Phong model (early 1970's)
- Ward model (early 1990's)
- BSSRDF (2001)

subsurface scattering in skin, marble

Special Case : Lambertian surface (matte)

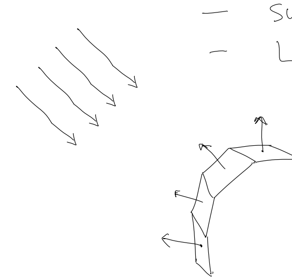


$$L(x, l_{out}) = \int \rho(x, \vec{l}_{in}, \vec{l}_{out}) \; L(x, \vec{l}_{in}) \, \vec{n}(x) \cdot \vec{l}_{in} \; d\vec{l}_{in}$$

$$= \int \rho(x) \; L(x, \vec{l}_{in}) \, \vec{n}(x) \cdot \vec{l}_{in} \, d\vec{l}_{in}$$

Special Case

- Sunlight
- Lambertian reflectance



$$L(x, l_{out}) \approx c \cdot \rho(x) \, n(x) \cdot \vec{l}_{sun}$$