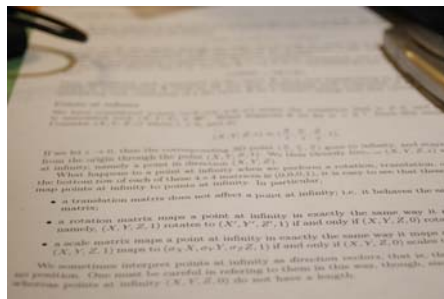
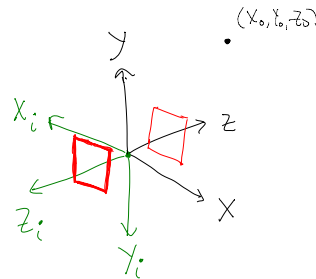


lecture 5

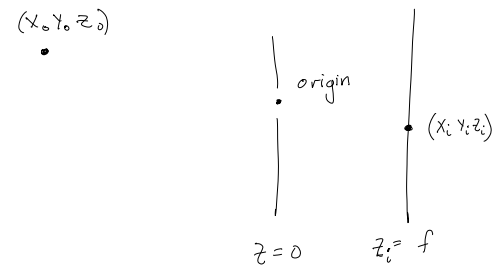
thin lens model, focus



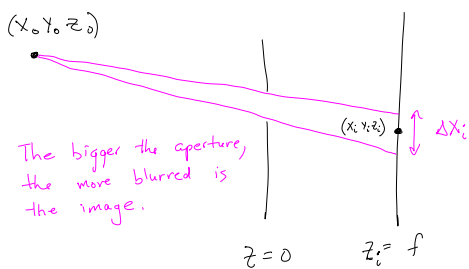
Real projection planes are behind the center of projection



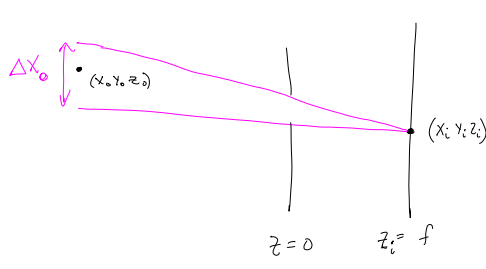
Non-pinhole camera (finite aperture)



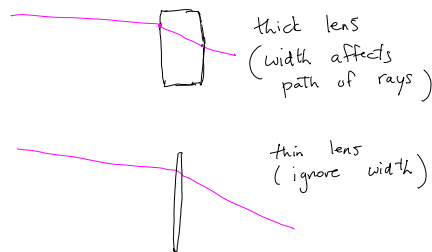
Forward projection



Reverse Projection

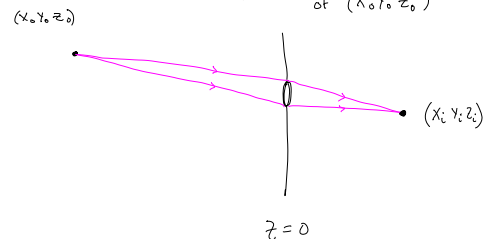


Thin lens



Conjugate Points

For all rays that leave (X_0, Y_0, Z_0) and pass through the lens, these rays all meet at some (X_i, Y_i, Z_i) , called the **conjugate point** of (X_0, Y_0, Z_0) .



The path of a ray through a lens does not depend on the direction \leftarrow vs. \rightarrow , so I will typically not specify the direction explicitly.

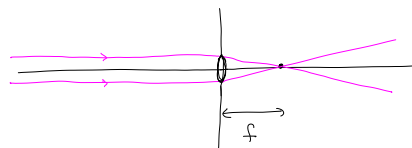
Our goal is to find a relationship between X_0, Y_0, Z_0 and X_i, Y_i, Z_i

(thin lens equation)

$$\frac{1}{f} = \frac{1}{z_0} + \frac{1}{z_i}$$

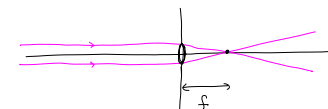
Focal length, f , of a lens

$$(X_0, Y_0, z_0) = (0, 0, \infty) \quad (X_i, Y_i, z_i) = (0, 0, f)$$

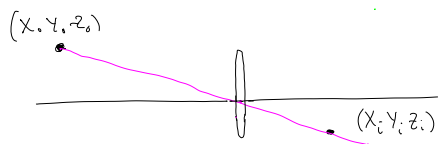
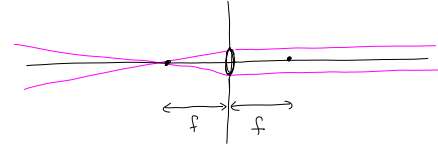


f is called the focal length.
Note: if sensor plane is a distance f from the lens, then objects at ∞ will be "in focus" in the image.

$$(X_0, Y_0, z_0) = (0, 0, \infty) \quad (X_i, Y_i, z_i) = (0, 0, f)$$

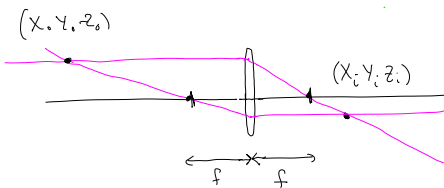


$$(X_0, Y_0, z_0) = (0, 0, f) \quad (X_i, Y_i, z_i) = (0, 0, \infty)$$

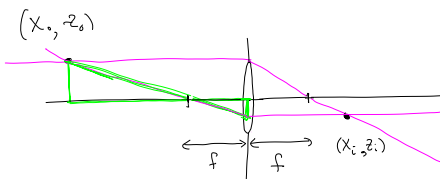


By similar triangles

$$\frac{X_i}{z_i} = \frac{X_0}{z_0}$$

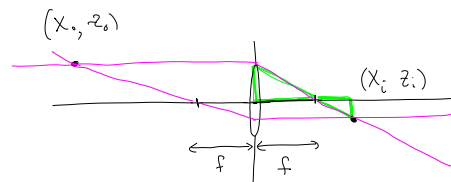


Note: this geometric argument only works for points very close to optical axis. A more complicated analysis is required for general points (omitted).



By similar triangles:

$$\frac{X_0}{z_0 - f} = \frac{X_i}{f}$$



By similar triangles:

$$\frac{X_i}{z_i - f} = \frac{X_0}{f}$$

$$\left. \begin{aligned} \frac{X_o}{z_o - f} &= \frac{X_i}{f} \\ \frac{Y_o}{z_o - f} &= \frac{Y_i}{f} \end{aligned} \right\} \begin{aligned} \frac{X_i}{X_o} &= \frac{f}{z_o - f} = \frac{z_i - f}{f} \\ \frac{Y_i}{Y_o} &= \frac{f}{z_o - f} \\ \frac{1}{f} &= \frac{1}{z_o} + \frac{1}{z_i} \end{aligned}$$

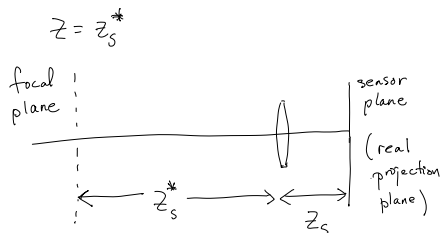
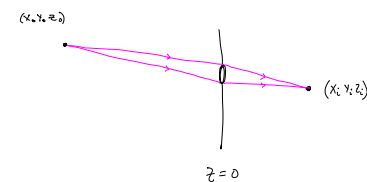
THIN LENS EQUATION

$$\left. \begin{aligned} \frac{X_o}{z_o - f} &= \frac{X_i}{f} \\ \frac{Y_o}{z_o - f} &= \frac{Y_i}{f} \\ \frac{1}{f} &= \frac{1}{z_o} + \frac{1}{z_i} \end{aligned} \right\} \begin{aligned} X_i &= f \cdot \frac{X_o}{z_o - f} \\ Y_i &= f \cdot \frac{Y_o}{z_o - f} \\ z_i &= f \cdot \frac{z_o}{z_o - f} \end{aligned}$$

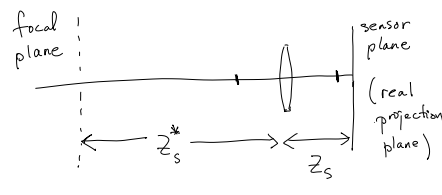
$$\begin{aligned} X_i &= f \cdot \frac{X_o}{z_o - f} \\ Y_i &= f \cdot \frac{Y_o}{z_o - f} \\ z_i &= f \cdot \frac{z_o}{z_o - f} \end{aligned}$$

$$\begin{bmatrix} X_i \\ Y_i \\ z_i \\ 1 \end{bmatrix} = \begin{bmatrix} f X_o \\ f Y_o \\ f z_o \\ z_o - f \end{bmatrix} = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & f & 0 \\ 0 & 0 & 1 & -f \end{bmatrix} \begin{bmatrix} X_o \\ Y_o \\ z_o \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & f & 0 \\ 0 & 0 & 1 & -f \end{bmatrix} \begin{bmatrix} X_o \\ Y_o \\ z_o \\ 1 \end{bmatrix} = \begin{bmatrix} f^2 & 0 & 0 & 0 \\ 0 & f^2 & 0 & 0 \\ 0 & 0 & f^2 & 0 \\ 0 & 0 & 0 & f^2 \end{bmatrix}$$

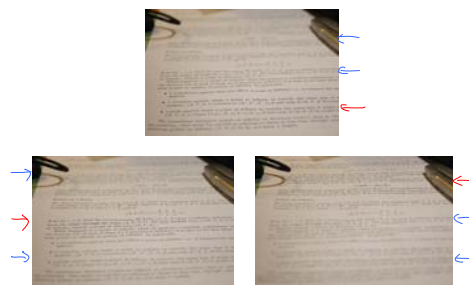


$$\frac{1}{f} = \frac{1}{z_s} + \frac{1}{z_s^*}$$

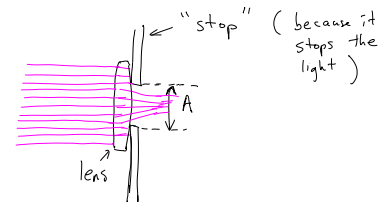


What if $z_s = f$?

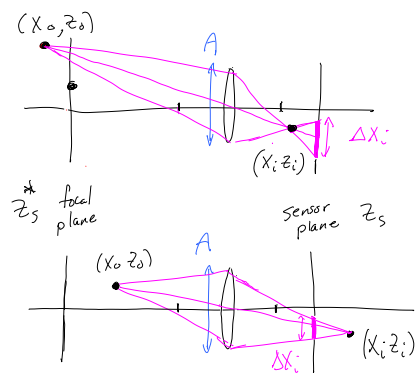
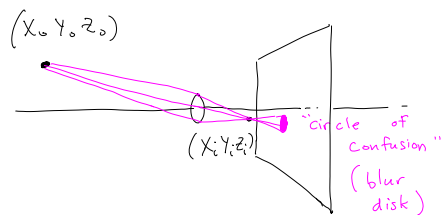
Moving the lens with respect to the sensor plane causes different parts of the scene to be in or out of focus.



Aperture



In future diagrams, I will not draw the stop.



$$\frac{\Delta X_i}{z_s - z_i} = \frac{A}{z_i}$$

$$\Delta X_i = A \left(\frac{z_s}{z_i} - 1 \right) = A \left(z_s \left(\frac{1}{f} - \frac{1}{z_o} \right) - 1 \right)$$

z_s^* focal plane

sensor plane z_s

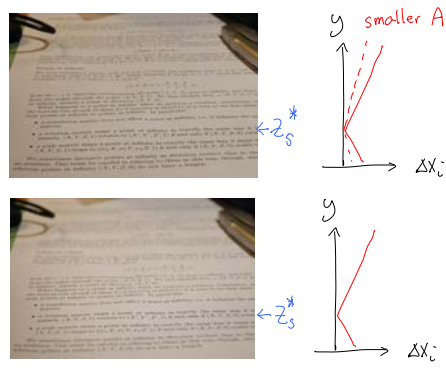
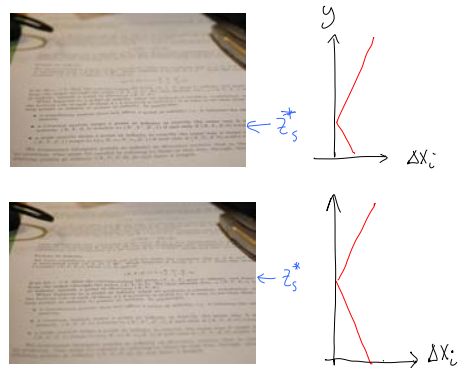
$$\frac{\Delta X_i}{z_i - z_s} = \frac{A}{z_i}$$

$$\Delta X_i = A \left(\frac{z_i - z_s}{z_i} \right) = A \left(1 - z_s \left(\frac{1}{f} - \frac{1}{z_o} \right) \right)$$

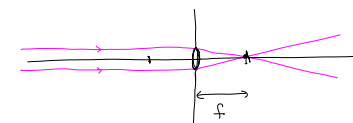
$$\Delta X_i = A \left| z_s \left(\frac{1}{f} - \frac{1}{z_o} \right) - 1 \right|$$

Recall that if you have an a plane $aX + bY + cZ = 1$ in the scene, then $ax + by + cz = \frac{f}{z}$.

So the blur width ΔX_i varies linearly with x, y .



"f-number" $N \equiv \frac{f}{A}$



- Aperture A often written "f/#"
- eg. $f = 80 \text{ mm}$, $N = 5.6$, $f/\# = \frac{80}{5.6} \text{ mm}$
- f-number is (angle subtended by lens)⁻¹