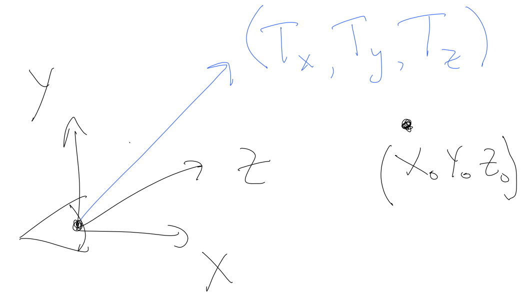
lecture 2

camera translation

Moving Observer



DEMO

CAMERA TRANSLATION

$$\begin{pmatrix} X(t) \\ Y(t) \\ Z(t) \end{pmatrix} = \begin{pmatrix} X_0 - T_x t \\ Y_0 - T_y t \\ Z_0 - T_z t \end{pmatrix}$$

Recall $(x, y) = \left(\frac{X_0}{Z_0}, \frac{Y_0}{Z_0} \right) f$

$$\Rightarrow \quad (x(t), y(t)) = \left(\frac{X_0 - T_x t}{Z_0 - T_z t}, \frac{Y_0 - T_y t}{Z_0 - T_z t} \right) f$$

Image Velocity at $t=0$

$$(x(t), y(t)) = \left(\frac{X_0 - T_x t}{Z_0 - T_z t}, \frac{Y_0 - T_y t}{Z_0 - T_z t} \right) f$$

$$(v_x, v_y) = \frac{d}{dt} \left(x(t), y(t) \right) \Big|_{t=0}$$

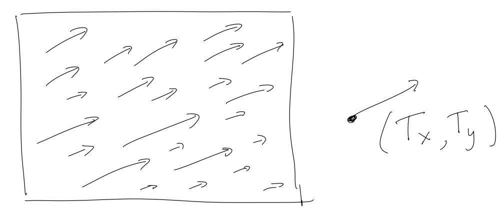
$$v_x = \frac{dx}{dt}\Big|_{t=0} = \frac{-T_x Z_0 + T_z X_0}{Z_0^2} \cdot f$$

$$v_y = \frac{dy}{dt}\Big|_{t=0} = \frac{-T_y Z_0 + T_z Y_0}{Z_0^2} \cdot f$$

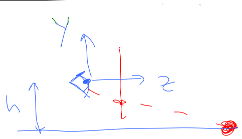
Special Case: $T_z = 0$ (lateral motion)

$$v_x = \frac{dx}{dt}\Big|_{t=0} = \frac{-T_x Z_0 + \cancel{T_z X_0}}{Z_0^2} \cdot f = -\frac{f T_x}{Z_0}$$

$$v_y = \frac{dy}{dt}\Big|_{t=0} = \frac{-T_y Z_0 + \cancel{T_z Y_0}}{Z_0^2} \cdot f = -\frac{f T_y}{Z_0}$$



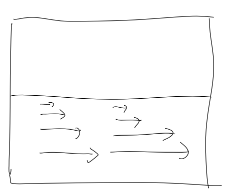
Lateral motion + Ground Plane



Recall $y = \frac{-fh}{Z}$

$T_y = T_z = 0$

$$\Rightarrow \quad v_x = \frac{-T_x f}{Z_0} = \frac{T_x}{h} y$$

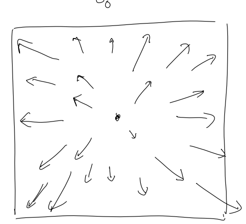


Special Case: forward translation

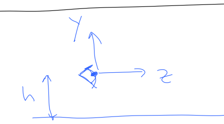
$T_x = T_y = 0, \quad T_z \neq 0$

$$v_x = \frac{dx}{dt}\Big|_{t=0} = \frac{-T_x Z_0 + T_z X_0}{Z_0^2} \cdot f = \frac{T_z X_0 f}{Z_0^2} = \frac{T_z}{Z_0} x$$

$$v_y = \frac{dy}{dt}\Big|_{t=0} = \frac{-T_y Z_0 + T_z Y_0}{Z_0^2} \cdot f = \frac{T_z Y_0 f}{Z_0^2} = \frac{T_z}{Z_0} y$$



Forward Translation + Ground Plane

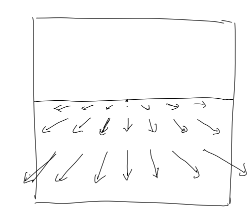


Recall $y = \frac{-fh}{Z}$

$T_y = T_x = 0$

$$v_x = -\frac{T_z}{Z_0} x = \frac{T_z}{fh} xy$$

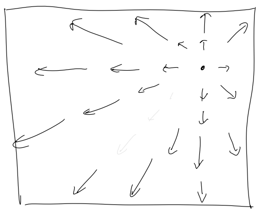
$$v_y = -\frac{T_z}{Z_0} y = \frac{T_z}{fh} y^2$$



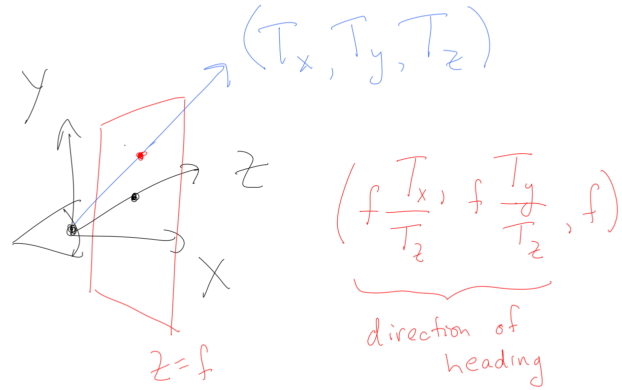
General Translation ($T_z \neq 0$)

$$v_x = \frac{-T_x Z_0 + T_z X_0}{Z_0^2}\cdot f = \frac{T_z}{Z_0}x - \frac{T_x f}{Z_0} = \frac{T_z}{Z_0}\left(x - \frac{T_x}{T_z}f\right)$$

$$v_y = \frac{-T_y Z_0 + T_z Y_0}{Z_0^2}\cdot f = \frac{T_z}{Z_0}y - \frac{T_y f}{Z_0} = \frac{T_z}{Z_0}\left(y - \frac{T_y}{T_z}f\right)$$

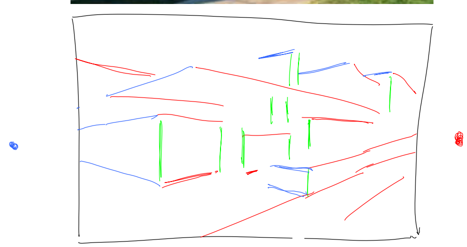


$f\left(\frac{T_x}{T_z}, \frac{T_y}{T_z}\right)$ is called the "direction of heading"

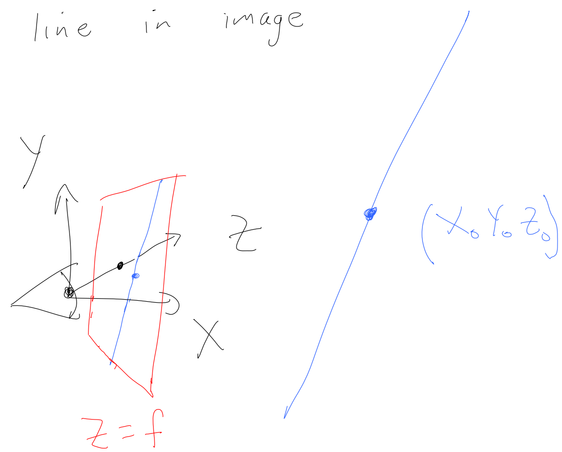


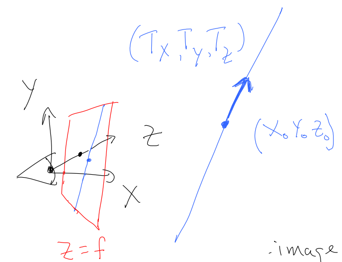
Vanishing Points

Parallel lines in 3D project (typically) to non-parallel lines in the image.



Line in world projects to line in image

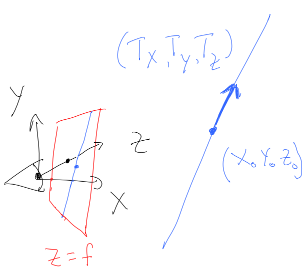




$$\begin{pmatrix} X(t) \\ Y(t) \\ Z(t) \end{pmatrix} = \begin{pmatrix} X_0 + T_x t \\ Y_0 + T_y t \\ Z_0 + T_z t \end{pmatrix}$$

image line is:

$$(x(t), y(t)) = \left(\frac{X_0 - T_x t}{Z_0 - T_z t}, \frac{Y_0 - T_y t}{Z_0 - T_z t}\right) f$$



$$\begin{pmatrix} X(t) \\ Y(t) \\ Z(t) \end{pmatrix} = \begin{pmatrix} X_0 + T_x t \\ Y_0 + T_y t \\ Z_0 + T_z t \end{pmatrix}$$

Corresponding image line is:

$$(x(t), y(t)) = \left(\frac{X_0 - T_x t}{Z_0 - T_z t}, \frac{Y_0 - T_y t}{Z_0 - T_z t}\right) f$$

Let $t \to \infty$ gives

$$\underbrace{(x(\infty), y(\infty)) = \left(\frac{T_x}{T_z}, \frac{T_y}{T_z}\right)\cdot f}_{\text{vanishing point } (T_z \neq 0)}$$

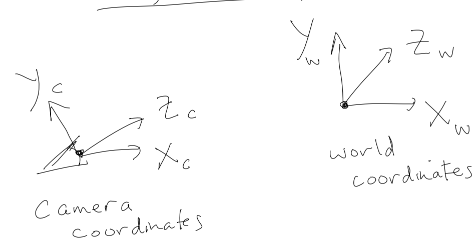
$$\begin{pmatrix} X(t) \\ Y(t) \\ Z(t) \end{pmatrix} = \begin{pmatrix} X_0 + T_x t \\ Y_0 + T_y t \\ Z_0 + T_z t \end{pmatrix}$$

$$(x(t), y(t)) = \left(\frac{X_0 - T_x t}{Z_0 - T_z t}, \frac{Y_0 - T_y t}{Z_0 - T_z t}\right) f$$

Vanishing point: let $t \to \infty$.

- Vanishing point is independent of (X_0, Y_0, Z_0).
- If $T_z = 0$, then letting $t \to \infty$ gives a point at infinity and in direction $(-T_x, -T_y)$

1-, 2-, 3-point perspective



- X, Y, Z axes of world coordinates each define a vanishing point.
- n point perspective means there are n finite vanishing points.

1-point perspective

2-point perspective

3-point perspective