lecture 19 (part 1)

Camera Calibration

Projection Matrix

$$\underset{3\times4}{P} = K R [I \mid -c]$$

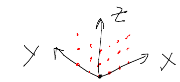
$$\begin{bmatrix} wx \\ wy \\ w \end{bmatrix} = \begin{bmatrix} P_{11} & P_{12} & P_{13} & P_{14} \\ P_{21} & P_{22} & P_{23} & P_{24} \\ P_{31} & P_{32} & P_{33} & P_{34} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

What are the rows and columns ?
See Exercises 1 Question 10.

Suppose I have an object with known positions $(X_i\, Y_i\, Z_i)$ in scene coordinates and labelled pixels $(x_i\, y_i)$ for $i = 1, \ldots N$.

Compute a projection matrix P that best fits the $\{(X_i\, Y_i\, Z_i\, x_i\, y_i)\}$

$X_i\, Y_i\, Z_i\, x_i\, y_i$



$$\begin{pmatrix} w\, x_i \\ w\, y_i \\ w \end{pmatrix} = \begin{pmatrix} P_{11} & P_{12} & P_{13} & P_{14} \\ P_{21} & P_{22} & P_{23} & P_{24} \\ P_{31} & P_{32} & P_{33} & P_{34} \end{pmatrix} \begin{pmatrix} X_i \\ Y_i \\ Z_i \\ 1 \end{pmatrix}$$

$$\therefore \; (P_{31} X_i + P_{32} Y_i + P_{33} Z_i + P_{34}) \cdot x_i = P_{11} X_i + P_{12} Y_i + P_{13} Z_i + P_{14}$$

$$(P_{31} X_i + P_{32} Y_i + P_{33} Z_i + P_{34}) \cdot y_i = P_{21} X_i + P_{22} Y_i + P_{23} Z_i + P_{24}$$

$$\begin{bmatrix} X_i & Y_i & Z_i & 1 & 0 & 0 & 0 & 0 & -X_i x_i & -Y_i x_i & -Z_i x_i & -x_i \\ 0 & 0 & 0 & 0 & X_i & Y_i & Z_i & 1 & -X_i y_i & -Y_i y_i & -Z_i y_i & -y_i \end{bmatrix} \begin{bmatrix} P_{11} \\ P_{12} \\ P_{13} \\ P_{14} \\ P_{21} \\ P_{22} \\ P_{23} \\ P_{24} \\ P_{31} \\ P_{32} \\ P_{33} \\ P_{34} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

2 equations for each i

$\Rightarrow \; 2N \times 12$

How many N ?

We have defined a least squares problem. But is it the problem we want to solve ?

Analogy: When fitting a line to a set of points in a plane, which do we minimize ?

- $y = mx + b$
- $x \cos\theta + y \sin\theta = r$

Geometric interpretations are different.

Let

$$w \begin{bmatrix} \hat{x}_i \\ \hat{y}_i \\ 1 \end{bmatrix} = P \begin{bmatrix} X_i \\ Y_i \\ Z_i \\ 1 \end{bmatrix}$$

Suppose we want to minimize

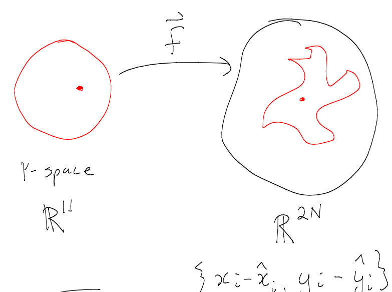
$$\sum_i \| (x_i - \hat{x}_i \,,\, y_i - \hat{y}_i) \|^2$$

that is, pixel errors.

$$\sum_i \| (x_i - \hat{x}_i \,,\, y_i - \hat{y}_i) \|^2$$

$$= \sum_i \left[\left(x_i - \frac{P_{11} X_i + P_{12} Y_i + P_{13} Z_i + P_{14}}{P_{31} X_i + P_{32} Y_i + P_{33} Z_i + P_{34}} \right)^2 + \left(y_i - \frac{P_{21} X_i + P_{22} Y_i + P_{23} Z_i + P_{24}}{P_{31} X_i + P_{32} Y_i + P_{33} Z_i + P_{34}} \right)^2 \right]$$

This is now a non-linear problem. Use the previous solution (linear) to get an initial estimate of P.



eg.

$$P_{11} = \sqrt{1 - \sum_{\substack{i \neq 1 \\ j \neq 1}} P_{ij}^2}$$

How to decompose P ?

$$P = K R [I \mid -c]$$

$$= \begin{bmatrix} \alpha_x & s & p_{ox} \\ 0 & \alpha_y & p_{oy} \\ 0 & 0 & 1 \end{bmatrix} R \, [I \mid -c]$$

$$11 = 5 + 3 + 3$$

$$\underset{3\times4}{P} = [\, \underset{3\times3}{\tilde{P}} \mid \underset{3\times1}{*} \,]$$

$$= [\, K R \mid K R (-C) \,]$$

How to decompose $\tilde{P}$ into K R ?

$$\tilde{P} = \begin{bmatrix} * & * & * \\ 0 & * & * \\ 0 & 0 & * \end{bmatrix} \begin{bmatrix} \text{rotation} \\ \text{\& reflection} \end{bmatrix}$$

unconstrained

positive diagonals

Decompose $\underset{3\times3}{\tilde{P}}$ into $\underset{3\times3}{K}\, \underset{3\times3}{R}$.

This is similar to the "RQ" decomposition in linear algebra. Commented code for doing it will be given to you in A4 (and details found in Appendix to today's notes).

Camera Calibration - Summary

1) Linear least squares solution for P.

2) (Optional) Use (1) as initial estimate for non-linear least squares solution.

3) Decompose P in camera internals & externals.

$$P = K R [I \mid -c]$$

$$= \begin{bmatrix} \alpha_x & s & p_x \\ 0 & \alpha_y & p_y \\ 0 & 0 & 1 \end{bmatrix} R \, [I \mid -c]$$

What if we wish to constrain the solution eg. $s = 0$, $\alpha_x = \alpha_y$?

Take previous solution, set $s = 0$, $\alpha = \frac{\alpha_x + \alpha_y}{2}$.

Given $\underset{3}{c}, \underset{3}{R}, \underbrace{\alpha, p_{ox}, p_{oy}}_{3}$, compute $P = KR[I \mid -c]$

$\Rightarrow$ Compute $\hat{x}_i\; \hat{y}_i$ and minimize sum of squares