Scale space

Canny edge detection
\[ I(x) \star f(x) \text{ vs } I(x) \star f_5(x) \]

Edge/Corner detection & Registration
\[ \sum_{(x,y) \in N(y)} (I(x+h_x, y+h_y) - I(x,y))^2 \]

ASIDE

There are formal relationships between blurring and subsampling. (This topic is omitted because of time constraints.)

KEYWORDS: image pyramid, multiresolution

Notation

\[ f_5(x) = f \left( \frac{x}{5} \right) \]

I will change the notes from lecture 9 (Canny) where we used \[ f_5(x) = f(5x) \]

Gaussian Scale Space

\[ I(x, \sigma) = I(x) \star G(x, \sigma) \]

\[ G(x, \sigma) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{x^2}{2\sigma^2}} = \frac{1}{\sqrt{2\pi\sigma^2}} \cdot g_0(x) / \sigma \]

Blurred images

\[ I(x) \star G(x, \sigma) \]

\[ I(x) = u(x-x_0) + n(x) \]
Edge detection

Look for peaks of:

\[ I(x) \ast \frac{d}{dx} g_\sigma(x) \]

- Look for zero crossings of:

\[ \frac{d}{dx} I(x) = \frac{d}{dx} g_\sigma(x) \]

where do the peaks occur?

\[ u(x-x_0) \ast \frac{d^2}{dx^2} g_\sigma(x) \]

\[ d g_\sigma(x) \]

\[ \frac{d g_\sigma(x)}{dx} = u(x) \ast \frac{d^2 g_\sigma(x)}{dx^2} \]

\[ \frac{d^2 g_\sigma(x)}{dx^2} = u(x) \ast \frac{d^2 g_\sigma(x)}{dx^2} \]

What are the heights of the peaks?

Height of peak of \( I(x) \ast \frac{d^2 g_\sigma(x)}{dx^2} \):

\[ \frac{d}{dx} \left( \frac{d}{dx} g_\sigma(x-x_0) \right) \]

\[ \frac{d}{dx} \left( \frac{d}{dx} g_\sigma(x-x_0) \right) = \frac{1}{\sigma^2} \left[ \frac{(x-x_0)^2}{\sigma^2} \right] - \frac{2}{\sigma^3} \left( \frac{(x-x_0)^3}{\sigma^4} \right) \]

\[ = 0 \text{ when } x = x_0 \pm \sigma \]

Response at \( x = x_0 \) is \( \delta \) (independent of \( \sigma \)).
Summary

\[ I(x) * G(x, \sigma) \sim \text{blur an image} \]

\[ I(x) * \frac{d}{dx} g_{\sigma}(x) \sim \text{edge detection} \]

\[ I(x) * \sigma \frac{d^2}{dx^2} g_{\sigma}(x) \sim \text{box detection} \]