

# lecture 12

## Scale space

### Scale

Canny edge detection

$$I(x) * f(x) \text{ vs } I(x) * f_s(x)$$

Edge/Corner detection & Registration

$$\sum_{(x,y) \in N_{gd}(x_0,y_0)} (I(x+h_x, y+h_y) - J(x,y))^2$$

Blurred Images  $I(x) * G(x, \sigma)$



368 x 550 pixels



$\sigma = 2$



$\sigma = 4$



$\sigma = 8$

### ASIDE



There are formal relationships between blurring and subsampling. (This topic is omitted because of time constraints.)

- KEYWORDS:
- image pyramids
  - multiresolution
  - wavelets

### Notation

$$f_s(x) \equiv f\left(\frac{x}{s}\right)$$

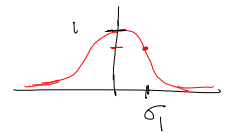
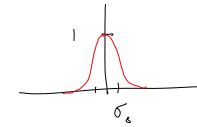
I will change the notes from lecture 9 (Canny)

where we used  $f_s(x) = f(sx)$

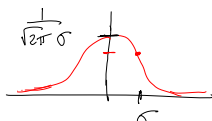
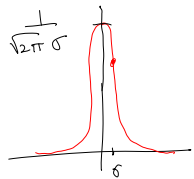
$$g(x) = e^{-\frac{x^2}{2}}$$

$$g_\sigma(x) = g\left(\frac{x}{\sigma}\right) = e^{-\frac{x^2}{2\sigma^2}}$$

$$g_\sigma(\sigma) = \frac{1}{\sqrt{e}}$$



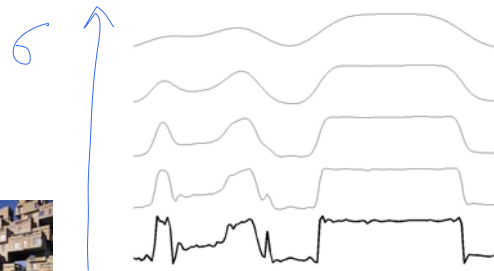
$$G(x, \sigma) = \frac{1}{\sqrt{2\pi} \sigma} \cdot e^{-\frac{x^2}{2\sigma^2}} = \frac{1}{\sqrt{2\pi} \sigma} \cdot g_\sigma(x)$$



### Gaussian Scale Space

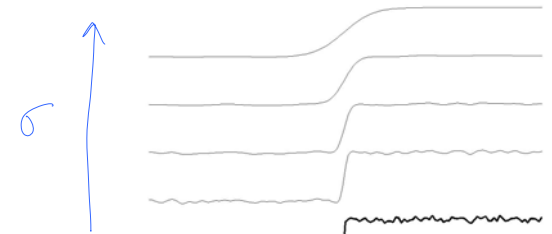
$$I(x, \sigma) = I(x) * G(x, \sigma)$$

blurred scan line (sigma of Gaussian = 0, 1, 2, 4, 8)



$$I(x, \sigma) = I(x) * G(x, \sigma)$$

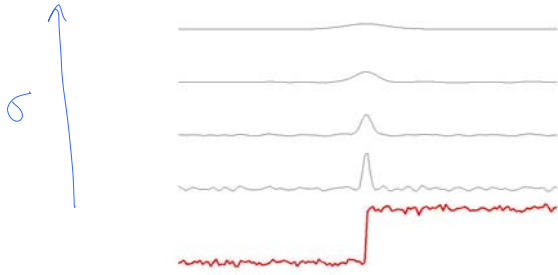
blurring a noisy edge (sigma of Gaussian = 0, 1, 2, 4, 8)



$$I(x) = u(x-x_0) + n(x)$$

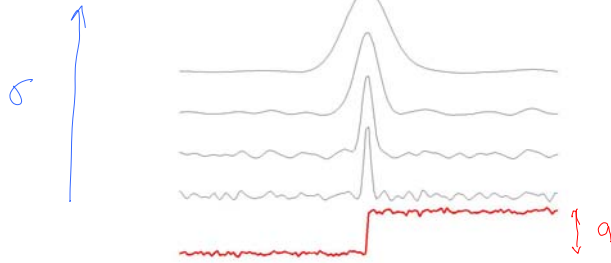
$$I(x) * \frac{d}{dx} G(x, \sigma)$$

blurring a noisy edge with first derivative of  $G_{\text{Gauss}}$



$$I(x) * \frac{d}{dx} g_{\sigma}(x)$$

blurring a noisy edge with first derivative of  $G_{\text{Gauss}}$



$$I(x) * \frac{d}{dx} g_{\sigma}(x)$$

$$= a u(x-x_0) * \frac{d}{dx} g_{\sigma}(x)$$

$$= a \delta(x-x_0) * g_{\sigma}(x)$$

$$= a g_{\sigma}(x-x_0)$$

Response at  $x=x_0$  is  $a$   
(independent of  $\sigma$ ).

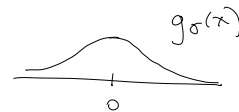
Edge detection

Look for peaks of  
 $I(x) * \frac{d}{dx} g_{\sigma}(x)$

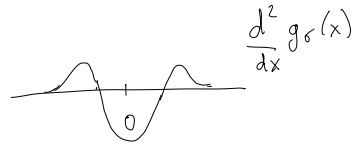
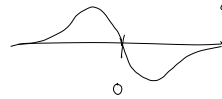
$\therefore$  Look for zero crossings of

$$\frac{d}{dx} I(x) * \frac{d}{dx} g_{\sigma}(x)$$

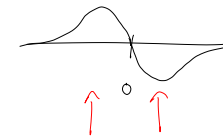
$$= I(x) * \frac{d^2}{dx^2} g_{\sigma}(x)$$



$$\frac{dg_{\sigma}(x)}{dx} = u(x) * \frac{d^2 g_{\sigma}(x)}{dx^2}$$



$$\frac{dg_{\sigma}(x)}{dx} = u(x) * \frac{d^2 g_{\sigma}(x)}{dx^2}$$



Where do the peaks occur?  
What are the heights of the peaks?

Where do peaks of  $u(x-x_0) * \frac{d^2}{dx^2} g_{\sigma}(x)$  occur?

$$u(x-x_0) * \frac{d^2}{dx^2} g_{\sigma}(x)$$

$$= \delta(x-x_0) * \frac{dg_{\sigma}(x)}{dx}$$

$$= \frac{d}{dx} g_{\sigma}(x-x_0)$$

$$= \frac{d}{dx} e^{-\frac{(x-x_0)^2}{2\sigma^2}}$$

$$= -\frac{(x-x_0)}{\sigma^2} e^{-\frac{(x-x_0)^2}{2\sigma^2}}$$

take derivative  
and set to 0.

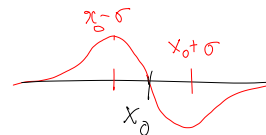
Where do the peaks occur?

$$\frac{d}{dx} \left( \frac{d}{dx} g_{\sigma}(x-x_0) \right)$$

$$= -\frac{d}{dx} \left( \frac{(x-x_0)}{\sigma^2} e^{-\frac{(x-x_0)^2}{2\sigma^2}} \right)$$

$$= \frac{1}{\sigma^2} \left( 1 - \frac{(x-x_0)^2}{\sigma^2} \right) e^{-\frac{(x-x_0)^2}{2\sigma^2}}$$

$$= 0 \quad \text{when} \quad x = x_0 \pm \sigma$$



Height of peak of  $I(x) * \frac{d^2 g_{\sigma}(x)}{dx^2}$ ?

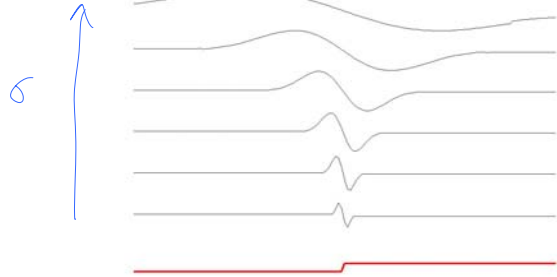
$$-\frac{(x-x_0)}{\sigma^2} e^{-\frac{(x-x_0)^2}{2\sigma^2}} \Big|_{x=x_0 \pm \sigma} = \pm \frac{1}{\sigma} e^{-\frac{1}{2}}$$

If we use  $\sigma \frac{dg_{\sigma}(x)}{dx}$  then

height of peak of  $u(x-x_0) * \sigma \frac{dg_{\sigma}(x)}{dx}$   
will be  $e^{-\frac{1}{2}}$  (independent of  $\sigma$ ).

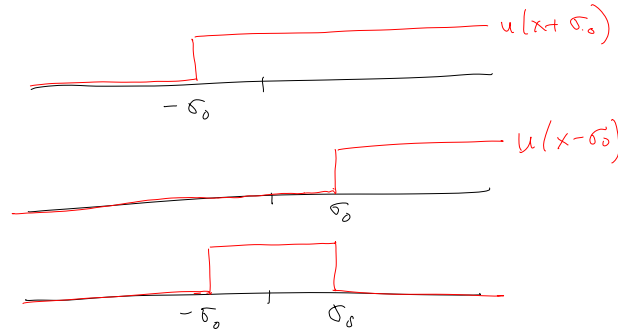
$$u(x-x_0) * \sigma \frac{d^2}{dx^2} g_\sigma(x)$$

blurring a step edge with sigma \* d^2 g\_sigma / dx^2

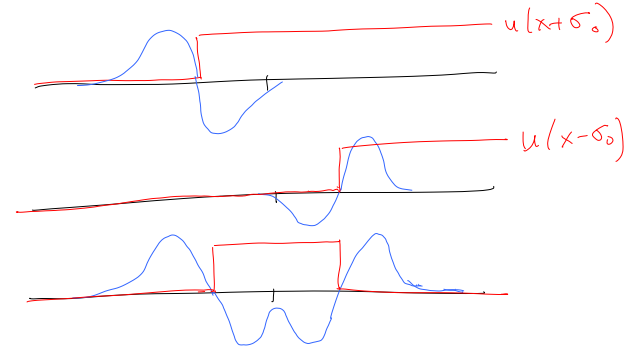


Box function

$$I(x) = u(x+\sigma_0) - u(x-\sigma_0)$$

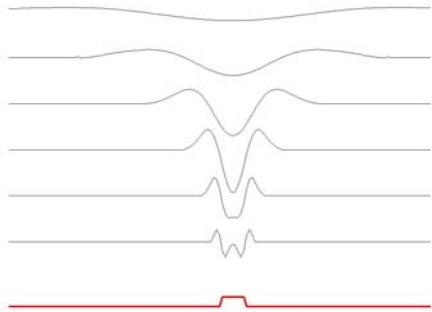


$$(u(x+\sigma_0) - u(x-\sigma_0)) * \sigma \frac{d^2 g_\sigma(x)}{dx^2}$$



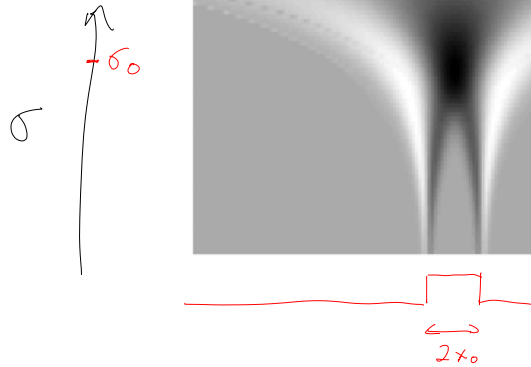
$$(u(x+\sigma_0) - u(x-\sigma_0)) * \sigma \frac{d^2 g_\sigma(x)}{dx^2}$$

blurring a (optional noisy) box function with sigma \* second derivative of g\_sigma



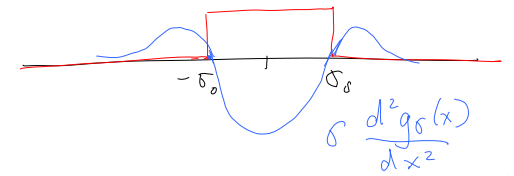
$$(u(x+\sigma_0) - u(x-\sigma_0)) * \sigma \frac{d^2 g_\sigma(x)}{dx^2}$$

normalized 2nd deriv Gaussian



Box function

$$I(x) = u(x+\sigma_0) - u(x-\sigma_0)$$



Minimum occurs when zeros of shifted  $\sigma \frac{d^2 g_\sigma(x)}{dx^2}$  align with edges.

## Summary

$I(x) * G(x, \sigma) \sim$  blur an image

$I(x) * \frac{d}{dx} g_\sigma(x) \sim$  edge detection

$I(x) * \sigma \frac{d^2}{dx^2} g_\sigma(x) \sim$  box detection